

Optimality Criteria for the Design of Manipulators

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Abstract—In this paper design criteria for manipulators are discussed and formulated for a multi-objective optimization problem. Optimality aspects are identified as suitable for numerical computations in a kinematic design procedure by looking at main aspects for analysis and design purposes.

Keywords—Robot Design, Manipulators, Analysis, Evaluation

I. INTRODUCTION

Robotic manipulations are widely used in industrial applications and even in non-industrial environments, since manipulators are used to help human beings and/or to execute manipulative tasks. Several analysis results and design procedures have been proposed in a very rich literature in the last two decades.

Robot performance is studied in terms of analysis that is also aimed to identify criteria as indices of merit. Evaluation can be computed by using synthetic numerical values of those criteria, even for comparative and catalogue purposes. Thus, there has been and still there is great attention for formulating performance criteria with numerically efficient algorithms. In addition, performance analysis is often formulated with numerical procedures that have been deduced for specific robot architectures. But then they have been extended to serial and parallel manipulators as general indices of merit. Those performance criteria are often used with suitable adaptations in design algorithms. Recently, optimal design procedures have been proposed for manipulators by using performance criteria as objective functions with the availability of commercial packages of numerical techniques for solving optimization problems. Illustrative examples of those approaches are [1-7] in which design procedures are proposed as single-objective or multi-objective optimization problems by using optimality criteria from performance analysis.

Since the beginning of 1990's at LARM: Laboratory of Robotics and Mechatronics in Cassino, a research line has been dedicated to the development of analysis formulation of manipulator performance that could be used in design algorithms and even in proper optimization problems by taking advantage of the peculiarity of solving techniques in commercial software packages. Recent results are reported in [8-14] as regarding serial and parallel manipulators, just to cite illustrative experiences.

A well-trained person is usually characterized for manipulation purpose mainly in terms of positioning skill, arm mobility, arm power, movement velocity, and fatigue limits. Similarly, robotic manipulators are designed and selected for manipulative tasks by looking mainly at workspace volume, payload capacity, velocity performance, and stiffness. Therefore, it is quite reasonable to consider those aspects as fundamental criteria for manipulator design and operation characterization.

In this paper, we have addressed our attention to performance analysis of manipulators with the aim to identify and formulate optimality criteria that can be useful both for evaluation and design purposes. Main aspects have been surveyed to propose efficient numerical computations for the basic manipulator characteristics regarding workspace, singularity, path planning, lightweight design, power consumption, and stiffness.

II. THE DESIGN PROBLEM

The design problem for manipulators consists in several phases whose the first one concerns with the kinematic design of the structure in terms of architecture, mobility, and size. The architecture can be chosen as open chain or parallel structure. In addition, different solutions can be selected within each structure as depending on manipulative tasks. In this paper, we have addressed attention to the problem of dimensional design that include mobility and size of a manipulator, [15]. In general, kinematic dimensional design is aimed to compute values of design parameters that characterize and size the kinematic structure of a manipulator. Several aspects can be considered in a design procedure and since they may give contradictory results, a formulation of multi-objective optimization problem can be useful to consider them simultaneously toward a suitable optimal solution. This can be formulated in a very general form as

$$\min \mathbf{F}(\mathbf{X}) \quad (1)$$

subject to

$$\mathbf{G}(\mathbf{X}) \leq 0 \quad (2)$$

$$\mathbf{H}(\mathbf{X}) = 0$$

where \mathbf{X} is the vector whose components are the design parameters; \mathbf{F} is the objective function vector whose

components are the expressions of mobility criteria; $\mathbf{G}(\mathbf{X})$ is the vector of constraint functions that describes limiting conditions, and $\mathbf{H}(\mathbf{X})$ is the vector of constraint functions that describes design prescriptions.

In general, the design parameters in Eq.(1) are the sizes and mobility angles of manipulators architectures. The formulation of the design problem as an optimization problem gives the possibility to consider contemporaneously several design aspects that can be contradictory for an optimal solution. Thus, optimality criteria are of fundamental interest even for efficient computations in solving optimization problems for manipulator design. In this case, the analysis of manipulator performance must be aimed to computational algorithms that can be efficiently linked to the solving technique of the highly non-linear optimal design problem of manipulators.

III. OPTIMALITY CRITERIA

A choice of a design criterion can be made according to two aspects, namely computational efficiency and possibility to be used both for serial and parallel manipulators. Flexibility for computational issues can be reached by using a general formulation that can have even computational complexity or can require certain computational efforts.

Alternatives in formulating and choosing optimality criteria are always possible depending of the designer experience, design goals, and manipulator applications. Indeed, any choice of optimality criteria can be questionable when considering the above-mentioned aspects. In fact, many different indices and/or their computations have been proposed in a rich literature on manipulators both for analysis and design purposes. Those indices can be used and they have been used with proper formulation as optimality criteria in specific algorithms for optimal design of specific manipulators. Of course, any optimality criterion as well as its formulation can suffer drawbacks in terms of conceptual aim and numerical efficiency. In this paper we have focused attention on performance versatility. Within such aspects, computation of the proposed optimality criteria can require computational efforts, like for example those for time-consuming iterative or scanning processes, and they can need additional numerical constraints, like for example those for avoiding singular numerical situations and bounding the feasible ranges for design parameters. The generality of a formulation, which can ensure a successful application to any manipulator topology, has been treated as much as possible by using standard expressions with adequate precisions with the aim to give a fairly simple numerical evaluation of the proposed optimality criteria and to obtain numerical versatility of the algorithms for efficiency within the overall design procedure.

Therefore, considering the above-mentioned aspects we have proposed significant optimality criteria in term of workspace analysis, singularity avoidance, path planning, lightweight design, power consumption, and stiffness features.

A. Workspace criterion

The workspace is one of the most important kinematic properties of manipulators, because of its impact on

manipulator design and its location in a work cell.

A manipulator workspace can be identified as a set of reachable positions by a reference point at the manipulator's extremity. This is referred as position workspace. Similarly, orientation workspace can be identified as a set of reachable orientations by a reference point at the manipulator's extremity. Interpreting the orientation angles as workspace coordinates permits to treat the determination of the orientation workspace likewise the determination of the position workspace when a Cartesian space is considered in the computations.

The analytic mapping for the forward Kinematics of a n-DOF manipulator with r-dimensional task-space can be expressed in the form

$$\mathbf{k} : \mathfrak{R}^n \rightarrow \mathfrak{R}^r, \mathbf{p} = \mathbf{k}(\mathbf{q}) \quad (3)$$

A general numerical evaluation of the workspace can be deduced by formulating a suitable binary representation of a cross-section in the task-space, as described in [16]. A cross-section can be obtained with a suitable scan of the computed reachable positions and orientations \mathbf{p} , once the forward kinematic problem has been solved to give \mathbf{p} as function of the kinematic input joint variables \mathbf{q} . A binary matrix \mathbf{P} can be defined in a cross-section plane for a cross-section of the workspace as follows: if the (i, j) grid pixel includes a reachable point, then $P_{ij} = 1$; otherwise $P_{ij} = 0$, as shown in Fig. 1. Therefore, a binary mapping for a workspace cross-section can be given as

$$P_{ij} = \begin{cases} 0 & \text{if } P_{ij} \notin W(H) \\ 1 & \text{if } P_{ij} \in W(H) \end{cases} \quad (4)$$

where $W(H)$ indicates workspace region; \in stands for 'belonging to' and \notin "not belonging to".

The workspace volume V can be computed considering the cross-sections areas A_z and the number of slices n_z that have been considered for the workspace volume evaluation, according to scheme of Fig. 1, as

$$V = \sum_{z=1}^{n_z} \left(\sum_{i=1}^{i_{\max}} \sum_{j=1}^{j_{\max}} (P_{ij} \Delta x \Delta y) \right) \Delta z \quad (5)$$

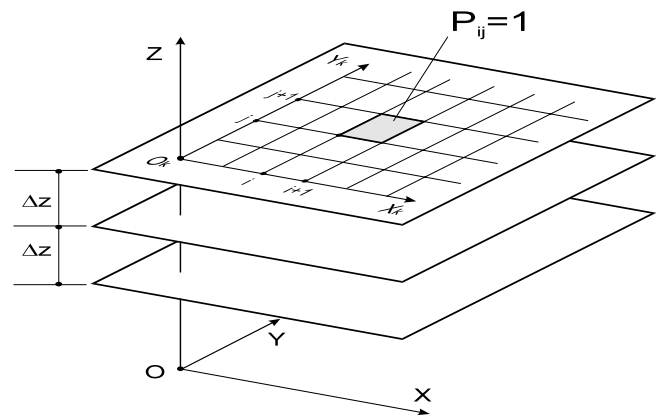


Figure 1. A general scheme for a binary representation and evaluation of manipulator workspace.

Similarly, the orientation workspace can be analyzed by using a suitable binary representation with another binary matrix for a workspace region that can be described in term of orientation angles, whose values can be considered in the reference axes of a grid mesh. Consequently, a numerical evaluation of orientation workspace can be carried out by using the formulation of Eqs. (3) to (4) in order to identify a corresponding binary matrix and to compute the corresponding orientation performance measures cross-sections areas A_{ϕ} , and orientation workspace volume V_{or} , when a 3D representation of the orientation capability is obtained by using three angular coordinates as Cartesian coordinates.

One can use Eqs (3) to (4) in order to evaluate any cross-section by properly adapting the formulation to the cross-section plane and intervals of a scanning process. Therefore, an optimum design problem with objective functions regarding workspace characteristics can be formulated as finding the optimal design parameters values to obtain the position and orientation workspace volumes that are as close as possible to prescribed ones in the form

$$f_{PW}(\mathbf{X}) = \left| 1 - \frac{V_{pos}}{V_{pos}'} \right| \quad (6)$$

$$f_{OW}(\mathbf{X}) = \left| 1 - \frac{V_{or}}{V_{or}'} \right| \quad (7)$$

where $|\cdot|$ is the absolute value; the subscripts pos and or indicate position and orientation, respectively; and prime refers to prescribed values.

B. Singularity avoidance criterion

Design requirements can also be focused conveniently on a free singularity condition. In fact, it is desirable to ensure a given workspace volume within which the manipulator extremity can be movable, controllable, and far enough from singularities. The singularity analysis both for serial and parallel manipulators can be performed by means of Jacobian matrices. The instantaneous relationship between the velocity in the task-space and active joint velocity can be expressed as

$$\mathbf{J}(\mathbf{q}) : \mathfrak{R}^n \rightarrow \mathfrak{R}^r, \quad \mathbf{B}\mathbf{t} = \mathbf{A}\dot{\mathbf{q}} \quad (8)$$

For serial manipulators B is the identity matrix, while for many parallel manipulators including the modified Gough-Stewart platform design, A is the identity matrix. For manipulator designs for which neither A nor B become the identity matrix, a single Jacobian can be defined as either $\mathbf{A}^{-1}\mathbf{B}$ or $\mathbf{B}^{-1}\mathbf{A}$ provided A or B is invertible, the difference being the direction from task-space to joint space and vice-versa, in which the Jacobian is defined. Indeed, Eq. (8) can be conveniently expressed, as

$$\mathbf{t} = \mathbf{J} \dot{\mathbf{q}} \quad (9)$$

Matrix J represents the analytic Jacobian of a manipulator at the configuration $\mathbf{q} \in \mathfrak{R}^n$ and it indicates how infinitesimal

changes of the configuration \mathbf{q} translate into infinitesimal end-effector motions in the vicinity of $\mathbf{k}(\mathbf{q})$. Vector $\dot{\mathbf{q}}$ represents the joint rates, and \mathbf{t} is the twist array containing the linear velocity vector \mathbf{v} and the angular velocity vector $\boldsymbol{\omega}$ of the moving platform.

In general, the conditions for identifying singular configurations can be represented by surfaces in the n-dimensional Joint Space and they can be obtained by vanishing the determinant of the Jacobian matrices (for square matrices).

In particular, if Eq.(8) is considered, matrix A gives the Inverse Kinematics singularities; and B gives the Direct Kinematics singularities. Direct Kinematics singularities can be defined for parallel manipulators only, and they occur inside the workspace. In such configurations a parallel manipulator loses its rigidity, becoming locally movable, even if the actuated joints are locked.

The concept of singularity has been extensively studied and several classification methods have been defined. Manipulator singularities can be classified into three main groups. Parallel manipulator singularities arise whenever A, B, or both, become singular; serial manipulator singularities arise if A becomes singular. Thus, a distinction can be made among three types of singularities, by considering Eq. (8), namely:

- the first type of singularity occurs when A becomes singular but B is invertible, being

$$\mathbf{A} \Rightarrow \text{not full rank and } \mathbf{B} \Rightarrow \text{full rank} \quad (10)$$

- the second type of singularity occurs only in closed kinematic chains and arises when B becomes singular but A is invertible, i.e.

$$\mathbf{A} \Rightarrow \text{full rank and } \mathbf{B} \Rightarrow \text{not full rank} \quad (11)$$

- the third type of singularity occurs when A and B are simultaneously singular, while none of the rows of B vanish.

Under this type of singularity, the movable platform can undergo finite motions even if the actuators are locked or, equivalently, it cannot resist forces or moments into one or more directions over a finite portion of the workspace, even if all actuators are locked. The Jacobian matrix is pose dependent and non-isotropic. Consequently, it is important to consider the Jacobian in a rational design procedure, also because of those influences. Indeed, one can propose an objective function f_J that can be deduced by analyzing the analytical expression of the determinant of matrix J in the form

$$f_J = \frac{\min |\det(\mathbf{J})|}{|\det(\mathbf{J})|_0} \quad (12)$$

with the condition

$$\det(\mathbf{J}) \neq 0 \quad (13)$$

that can take into account somehow all the situations in a singularity analysis, when the initial guess value J_0 is considered to let f_J be adimensional. Furthermore, since Jacobian matrices are pose dependent, the minimum value has

been considered convenient to obtain a single value function f_j .

C. Optimal path planning

Path planning can be regarded as the way to obtain a constrained trajectory when initial and final points are given. In order to determine a path for the end-effector from a start point to a goal point, different trajectories can be performed by the actuator actions. Polynomial splines are used to represent the joint position as function of travelling time. Cubic Splines are widely used for interpolation since they assure speed and acceleration continuity.

A path planning optimality criterion can be formulated by considering the optimal traveling time, which takes the robot to perform a trajectory between two points in Cartesian coordinates. Thus, one can write

$$f_{pp}(X) = 1 - \left(\frac{t_{\text{path}}}{t_{\text{straight}}} \right) \quad (14)$$

where t_{path} is the time that takes the robot to perform the trajectory and t_{straight} is the time that the robot will take to perform a straight line trajectory. The position of the motors can be considered that evolve in time as a polynomial given by

$$q_i(t) = a_{0j} + a_{1j}t + a_{2j}t^2 + a_{3j}t^3 \quad (15)$$

where a_{ij} are constants, which can be calculated as based on boundary conditions on position, velocity and acceleration in the joint space at initial and goal positions that are obtained by means of the inverse kinematics.

A reasonable choice is to assume as start point a robot configuration in which all actuators are in zero position ($q(i)=0$). Then, goal point can be chosen as a point, which is located from starting point at a fixed distance d given by

$$d = \sqrt{(x_g - x_s)^2 + (y_g - y_s)^2 + (z_g - z_s)^2} \quad (16)$$

One should also ensure that the whole trajectory from the start point to the goal point should belong to robot the workspace. Therefore, additional constraint equations can be added to Eq.(14) accordingly. On the other hand, restriction in the actuators movement must be fulfilled as related to maximum speed, acceleration and jerk that cannot overcome the prescribed maximum values of joint velocities, accelerations, and jerks, respectively.

Summarizing an optimality criterion for path planning has been proposed in term of the path time that can be computed both in simulation procedures and experimental tests as one of the most used merit index for productivity of robot manipulation.

D. Lightweight design

Lightweight design is desirable in order to have a light mechanical structure for safety reasons and at the most for a general suitable maneuverability, installation, and location of

the robot.

A reasonable and computationally efficient expression of the lightweight design criterion can be given by

$$f_L(X) = \left| 1 - \frac{M_T}{M_d} \right| \quad (17)$$

as referred to M_T which is the overall mass of a robot and to M_d which is the desired overall mass of the same robot. The robot mass, M_T can be computed as the sum of the mass of links M_i , the mass of actuators M_j , and the mass of cables and sensors M_k , in the form

$$M_T = \sum_{i=1}^{n_{\text{link}}} M_i + \sum_{j=1}^{m_{\text{actuator}}} M_j + \sum_{k=1}^{l_{\text{component}}} M_k \quad (18)$$

It is worth noting that the most critical aspect for obtaining a lightweight mechanical design is to reduce the weight of links. In fact, cables and sensors are usually market components with given size and mass. Although actuators are usually market components their size and mass mainly depend on the desired performance in terms of output power and they can be properly selected. Thus, the objective function in Eq.(17) will minimize the masses of links and actuators.

The mass of the links can be easily related with their volumes and density. Thus, a minimization process will attempt to reduce link lengths and cross section sizes. Nevertheless, a constraint should be added in the form

$$A_i - A_{i_{\min}} < 0 \quad (19)$$

where A_i is the cross section area of i -th link and $A_{i_{\min}}$ is the minimum acceptable cross section area for i -th link. The constraints given by Eq.(19) are needed for obtaining cross sections of links greater than a minimum area whose value depends on manufacturing constraints and strength conditions.

E. Power consumption

Power consumption is a critical issue at the most, but not only in the design of battery powered manipulators for service tasks. Power consumption is a very important design issue for other robots due to the fast growing of energy costs. Power consumption is strongly related also to lightweight design and path planning aspects. In fact, a lightweight design can significantly reduce power consumption and can improve path planning performance by reducing the overall mass and inertia.

A design criterion for minimizing power consumption can be written in the form

$$f_{pc}(X) = \left| 1 - \frac{\Delta E_D}{\Delta E_A} \right| \quad (20)$$

where ΔE_D is the total dissipated mechanical energy and ΔE_A is a given available mechanical energy for the robot system.

In a manipulator, positive work of motors ΔL_{mr} is

necessary for increasing the kinetic energy during the motion with increasing angular speeds. But, this kinetic energy that is stored in the system will be dissipated by the system in order to stop the input shafts at the end of the motion. If one neglects the effects of potential energy, one can consider the value of total dissipated mechanical energy ΔE_D close to the value of the positive work ΔL_{mr} during the motion with increasing angular speeds. This positive work ΔL_{mr} can be computed as

$$\Delta L_{mr} = \sum_{k=1}^N \left[\sum_{c=1}^m \tau_c(t) \Delta \dot{q}_c(t_c) \Delta t_c \right]_k \quad (21)$$

where m is the total number of control points of the trajectory with increasing angular speeds; τ_c is the actuator torque on the k -th input shaft of the N actuators at the c -th control point; $\Delta \alpha_c$ is the k -th joint variable at the c -th control point; Δt_c is the time step between the c -th control point and the $(c-1)$ -th control point.

The kinetic energy of a robot strongly depends on inertial characteristics of the moved links in terms of mass and inertia moment. Thus, inertia characteristics pay an important role for power consumption and they should be carefully sized and included as design parameters or even as explicit optimality criteria for manipulator design within the performance evaluations in Eqs.(20) and (21).

F. Stiffness criterion

Stiffness and accuracy of a robotic manipulator are strongly related to each other since positioning and orientating errors are due to compliant displacements and clearances as well as to control, construction and assembling errors. The last errors can be evaluated by a kinematic analysis (calibration) by considering uncertainties in the kinematic parameters due to tolerances of construction and assembling of a robotic manipulator mechanism.

The stiffness properties of a manipulator can be defined through a matrix that is called ‘Cartesian stiffness matrix K ’. This matrix gives the relation between the vector of the compliant displacements $\Delta \mathbf{S} = (S_x, S_y, S_z, S_\phi, S_\psi, S_\theta)$ occurring at the movable platform when a static wrench $\mathbf{W} = (F_x, F_y, F_z, T_x, T_y, T_z)$ acts upon it, and \mathbf{W} itself in the form

$$\mathbf{K}(\mathbf{q}) : \mathfrak{R}^6 \rightarrow \mathfrak{R}^6, \quad \mathbf{W} = \mathbf{K} \Delta \mathbf{S} \quad (22)$$

The stiffness matrix can be numerically computed by defining a suitable model of the manipulator, which takes into account lumped stiffness parameters of links and motors.

The proposed stiffness models with lumped parameters can take into account both the compliance of actuators and links along and about X, Y, and Z directions. They are based on the assumption of small compliant displacements. Under this assumption the superposition principle holds. Thus, the compliance of each link and actuator can be considered as an additive term to the overall compliance. Moreover, also the effects of tension/compression, bending and torsion stiffness of

a link can be considered as an additive term to the stiffness of the link itself. These additive terms can be defined as lumped parameters and they can be represented as linear or torsion springs. For example, a planar beam under an axial load along X axis can be represented with lumped stiffness parameter as shown in Fig.2a). In the model of Fig.2a) the symbol k_C is the lumped stiffness parameter of the compression/tension stiffness of the beam that is represented by a linear spring. Similarly, a scheme of a planar beam with a bending force F_B can be represented as in Fig.2b). In the model of Fig.2b) the symbol k_B is the lumped stiffness parameter of the bending stiffness of the beam. Thus, a torsional spring will represent effects of torsion and bending.

The stiffness matrix can take into account the effects of tension/compression, bending and torsion. Simplified stiffness models with lumped parameters can be defined by considering only the non negligible terms and therefore, corresponding linear and torsional springs can be identified as related to the non zero entries. only. In addition, the stiffness of the actuators can be taken into account in a similar manner and they can be represented as linear or torsion springs too. Indeed the superposition principle can be applied by combining contributions into single springs. Thus, each spring can model compliance both of links and actuators.

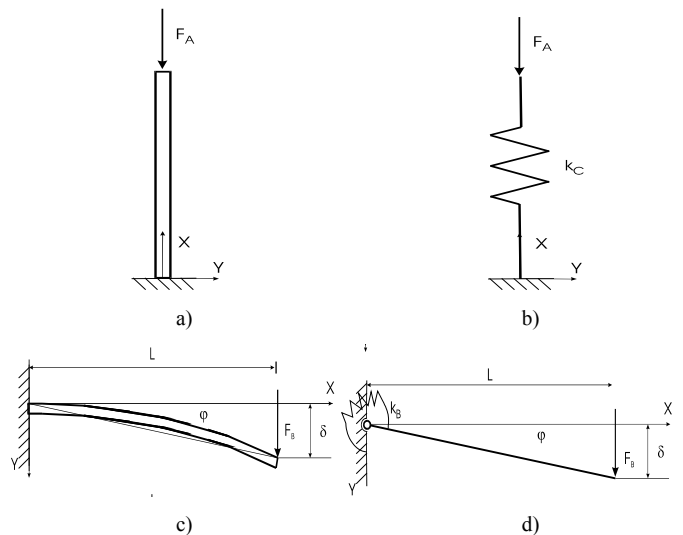


Figure 2. Models for stiffness evaluation of a planar beam: a) the case with axial load F_A acting along X-axis and its model with lumped stiffness parameter k_C ; b) the case with a bending force F_B and its model with lumped stiffness parameter k_B .

The stiffness matrix K can also be used to compute accuracy performance. In fact, the vector of compliant displacements $\Delta \mathbf{S} = [\Delta U, \Delta Y]^T$ can be computed with its translating component ΔU and rotational component ΔY by using Eq. (22) once the matrix K is determined when a static wrench acting on the movable platform is given.

From the above-mentioned considerations two objective functions that take into account stiffness performance can be defined as

$$f_{PS}(\mathbf{X}) = \left| 1 - \frac{|\Delta \mathbf{U}_d|}{|\Delta \mathbf{U}_g|} \right| \quad (23)$$

$$f_{OS}(\mathbf{X}) = \left| 1 - \frac{|\Delta \mathbf{Y}_d|}{|\Delta \mathbf{Y}_g|} \right| \quad (24)$$

where $|\cdot|$ is the operator for obtaining positive absolute values; $\Delta \mathbf{U}_d$ and $\Delta \mathbf{U}_g$ are maximum compliant displacements along X, Y, and Z-axes; $\Delta \mathbf{Y}_d$ and $\Delta \mathbf{Y}_g$ are vectors whose components are the maximum compliant rotations φ , θ and ψ about X, Y, and Z axes, respectively; d and g subscripts stand for design and given values, respectively.

Criteria f_{PS} and f_{OS} of Eqs. (23) and (24) can be considered separately or in a single objective function component, according to specific requirements. But, because of the definition in Eq. (20) this formulation needs the condition

$$\det \mathbf{K} \neq 0 \quad (25)$$

that can be used as additional constraint.

IV. CONCLUSIONS

Optimality criteria for manipulators design have been proposed as suitable for numerical computation. Performance aspects are outlined and formulations are derived for design purposes in a frame for a multi-objective optimization problem.

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