

# Efficient Exploration Algorithms for Rough Terrain Modeling Using Triangular Mesh Maps

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**Abstract**— The objective of this paper is to exploit the potential of considering the information gain in greedy mapping strategies based on a triangular mesh map for automatic modeling of a large rough outdoor environment. An energy cost function is used to represent the travel cost and two methods to estimate possible new terrain in one spot using a 3D image sensor are described. For the first method, assuming a partly known environment, the information gain is estimated by applying the ray tracing algorithm to the known part of the environment. For the second method, the new information gain is calculated using polygon clipping in an unknown environment. Simulation results in a typical rough agricultural field showed that the exploration strategy, which was incorporated with energy consumption and the information gain estimation with a ray tracing algorithm using a coarse map, had an advantage over other policies in terms of energy consumption and the path length.

**Keywords**—coverage path planning, rough environment, exploration, Dijkstra shortest path, energy cost

## I. INTRODUCTION

The main objective of exploration is to create an accurate map of an unknown area. To create an accurate map, a robot needs to know where it is, plan where to go next, collect the terrain information with sensor readings, and build the map to represent the environment. Therefore, the exploration task requires mobile robots to meet the objectives of both localization accuracy and exploring efficiency [1]. This work mainly focused on the exploring efficiency problem.

Different exploration strategies have been employed for environmental modeling tasks. One group of exploration strategies is to choose the closest point as the next-best viewpoint among frontiers extracted from the boundary between the known and unknown areas [2]-[4]. The robot moved to the nearest frontier by the shortest path, took a scan and updated the environmental map. The mapping procedure repeated this iterative step until the entire area had been explored. Another family of methods [5]-[6] chooses the next-best viewpoint considering safety factors.

A large body of studies [1], [7]-[12] has centered on information-theoretic methods based on the use of information as a measure of utility for making exploration control actions. Bourgault et al. [1] attempted to maximize both the expected Shannon information gain and localization accuracy. Simmons et al. [9] investigated an explore algorithm based on next-best viewpoints which would provide the maximum new

information gain and minimum driving cost using multiple robots. Feder et al. [11] proposed an information metric, named Fisher information, which was used to plan next sensing positions to maximum the terrain information gain and minimize expected dead-reckoning errors. Tovar et al. [12] developed optimal exploration strategies using a utility function which integrated the travel distance, size of the unexplored space, robot configuration uncertainty, landmark identification probability, and ability to see features like corners.

Some researchers have started to investigate outdoor exploration tasks. Moorehead et al. [8] proposed a multiple-information-metrics exploration planner to integrate multiple sources of information in order to solve complex planetary exploration tasks. An information map was used to store multiple information sources for a 3D environment. This method enabled the robot explorer to maximize the total information gained while minimizing costs such as driving, sensing, and planning. The algorithm was demonstrated by creating traversability maps and exploring cliffs. Sujan and Dubowsky [10] developed an information-based visual robotic mapping approach based on a 3D occupancy grid map in an unstructured environment. The robot was controlled to maximize geometric knowledge gained about its environment using an evaluation function based on Shannon's information theory. They firstly used the field of view of the camera to measure the new information gained in the exploration task.

The aim of this project was to develop a next-best viewpoint algorithm for the construction of topographic maps for partially-known rough agricultural fields using a 3D image sensor. Agricultural environments have enough peculiarities to make the proposed development project challenging. In agricultural environments, there may be a commercial low-resolution topographic map available. Agricultural fields are usually rough and large. The approach in this paper to agricultural environmental modeling has two important contributions. First, energy consumption is used to represent the travel cost rather than the path distance, which has been used in previous work. Second, a visibility analysis method based on frustum culling and ray casting is applied to estimate the new terrain information gain using a coarse triangular mesh map.

## II. TRIANGULAR MESH MAP

Several environmental models have been implemented, including certainty grids [8], polygonal layouts [6], topological

maps [13], and triangular mesh maps [14]. The triangular mesh map was used in this research to model the agricultural field surface because it allows a smoother path, compared with the square grid map [15].

The triangular mesh map is incrementally built using laser sensor readings based on Delaunay triangulation [16]. The Visualization Toolkit [17] has been used to implement the triangulation in this work. Figure 1 shows an example of the triangular mesh map of an agricultural field. To find the boundary and holes in a triangular mesh map, a connectivity graph was built by adding all the boundary edges according to their connectivity relationships. By visiting all the edges of the connectivity graph, all the loops, including the outer boundary and holes, will be found. The loop of edges with the largest area is the outer boundary, while holes inside the outer boundary have smaller areas.

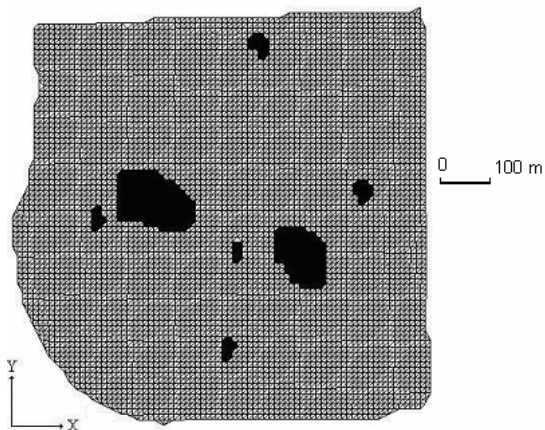


Figure 1. Triangular mesh map of an agricultural field environment (the black blobs represent obstacles).

### III. IMAGE SENSOR MODEL

A 3D camera model was used in this task to deal with the rough terrain visibility problem, in which one part of the terrain may occlude other parts. The image sensor's capacity is defined by a viewing frustum. The viewing frustum shown in Fig. 2 is described by six planes, which are named the near, far, left, right, top, and bottom planes. The viewing frustum defines the visibility of every triangle in the terrain for each viewpoint, and triangles inside the viewing frustum are visible to the viewer. Frustum culling was used to process the triangular level before the individual pixel was handled in the visibility analysis. Hence, the object level, the triangle, can be rejected quickly in the simulation. The procedure is much faster than a ray casting method [18]. To cull the models, the six planes of the viewing frustum were dynamically generated in accordance with the sensor's posture. These planes were calculated from the view and perspective projection matrices in the camera system. In order to determine whether a triangle within the mesh is inside the frustum, it was necessary to check that all the vertices of the triangle were located inside the volume of the frustum.

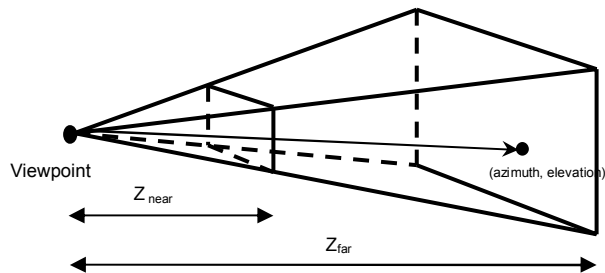


Figure 2. Image sensor viewing frustum..

The next step in the vision sensor simulation was to check the visibility of every triangle contained in the frustum using a ray casting algorithm. A ray, as shown in Fig. 2, is a straight line extending from the viewpoint to a pixel in the far plane of the viewing frustum. The algorithm begins by shooting a ray from the viewpoint to the screen (the far plane of the viewing frustum), then every triangle inside the viewing frustum is tested to see if the given ray intersects any of them. One ray may intersect more than one triangle when a triangle is behind another. From the point of intersection, the triangle nearest to the viewpoint is visible through this ray, while other triangles which intersect with the ray are shadowed. In this way, the visible triangles can be identified as those which intersect contiguous rays with the shortest distance to the viewpoint.

## IV. EXPLORATION ALGORITHM

### A. Overview

To create a topological map using a 3D image sensor, the robot used the iterative greedy method to plan the next-best viewpoints. The objective of the greedy approach is to find an optimal path to minimize the travel cost and maximize new terrain information gain. To begin each step, the robot extracts the frontiers [4] from the triangles close to the boundary between the known and unknown area. It constructs a voting scheme using a utility function that includes the estimated energy cost for it to travel to the frontiers and the estimated new terrain information gain for the frontiers. The robot visits the frontier at the point of the maximum utility and takes a scan using its image sensor. The map is updated by combining the new data. The robot plans the next best viewpoint with the new map until it reaches the goal of exploration or it is blocked.

The use of a triangular mesh to represent terrain allows the use of a graph search to easily find the next best viewpoint. The triangular mesh map is stored in the computer as a directed weighted graph. Once the graph is constructed, an optimal path between the current rover location and a destination can be planned by Dijkstra's shortest path algorithm [19].

Previous work focused on path distance as the only source of the travel cost for the exploration. However, travel distance alone is unsuitable to represent travel cost in the rough terrain of an outdoor unstructured environment. Energy consumption is very important for exploration tasks such as planetary exploration or agricultural applications. In this work, the energy cost function not only considered the travel distance, but also included the energy required to change elevation and the rolling resistance of the terrain during exploration.

### B. Frontiers Extraction

The outer boundary and the hole boundary edges are identified after finding all the loops in the connectivity graph extracted from the triangular mesh map. The candidate frontiers in the map are defined as those triangles that have a specific distance from the boundary and have never acted as a viewpoint previously. To reduce the number of candidate locations on the frontier, the distance between two candidate frontiers must satisfy the minimum-distance requirement.

### C. Utility Function

The goal of this work was to maximize terrain information gain while minimizing energy consumption; therefore, the utility was constructed simply using a linear combination of new terrain information and the energy cost. The utility function can be described by the formula

$$Utility = \alpha_i \times IG - (1 - \alpha_i) \times k \times Cost + \omega_n, \quad (1)$$

where,

$\alpha_i$  = information gain weight (dimensionless decimal fraction),

IG = estimated new terrain information gain (m<sup>2</sup>),

Cost = estimated energy consumption (N·m),

K = information gain coefficient (m<sup>2</sup>/(N·m)), and

$\omega_n$  = utility offset (m<sup>2</sup>).

### D. Energy Cost

The total energy requirement for the vehicle to reach a goal location from a starting location would be predicted by the integration of energy (E) in the piecewise path.

Assuming that soil hardness of the field is known and uniform, the energy requirement can be calculated by the following formula [20]:

$$E_{total} = W * \sum_{i=1}^n [\Delta z_i + d_{hi} * \mu_i], \quad (2)$$

where,

$E_{total}$  = the energy requirement (N·m),

$W$  = the weight of the robot (N),

$\Delta z_i$  = slope height of the  $i_{th}$  segment of a piecewise path (m),

$d_{hi}$  = horizontal distance of the  $i_{th}$  segment of a piecewise path (m),

$\mu_i = \frac{d_{hi}}{l_i} * B + 0.04$ , rolling resistance coefficient

(dimensionless),

$n$  = the number of the segments of the path,

$B$  = constant (dimensionless), related with the robot weight, soil hardness, and the tire size and

$l_i$  = the Euclidean distance of the  $i_{th}$  segment of a piecewise path (m).

The energy consumption is proportional to the elevation change,  $\Delta z_i$ . When driving uphill,  $\Delta z_i$  is positive and the slope resistance is in the opposite direction of the vehicle's tractive force. When driving downhill,  $\Delta z_i$  is negative, and the slope resistance is in the same direction as the vehicle's tractive force. This manuscript assumed the vehicle travels at a constant speed; therefore, brake energy will be required when the negative elevation change,  $\Delta z_i$ , is excessive.

The energy consumption is also proportional to the horizontal trip distance of the vehicle. The contribution of travel distance to energy consumption will vary considerably in relation to the rolling resistance coefficient,  $\mu_i$ . A tire's rolling resistance coefficient depends on the soil hardness, terrain slope, and wheel parameters.

The vehicle dynamics and the safety factor are also of concern in this work. In this work, the maximum climbing slope for the robot was set as 30° and the maximum downhill slope was 35°. When the uphill slope is greater than 30° or downhill slope is greater than 35°, the slope factor will be set to infinity because of the vehicle limitation.

The soil hardness might be highly variable in one field considering soil type and moisture content can cause significant changes in CI values. While it wasn't considered in this work, a soil strength map with site-specific CI value could be used to calculate the variable rolling resistance at any point in the field.

### E. Information Gain

Two different methods to estimate the new terrain information gain have been developed. To address the case of exploration without a coarse map *a priori*, the overlap of the sensor's 2D footprint with the map was used to find how much new terrain area might be found in the next step. A 3D visibility analysis method based on frustum culling and ray casting was employed to estimate the new terrain information gain when a low-resolution map is available in advance.

#### 1) Information gain estimation method 1

The Sutherland-Hodgman algorithm [18] was used to calculate the overlap between the terrain boundary and the sensor foot print. The algorithm uses a divide-and-conquer strategy to attack the problem. First, it identifies the intersection between the triangles within the viewing frustum and triangles within the outer boundary. Any area within the viewing frustum that does not coincide with triangles within the boundary is neglected. The resulting polygon will include triangles about which information is known and those which are to be explored as shown in Fig. 3. The terrain information gained can be estimated by calculating unknown area of unexplored terrain within the polygon.

#### 2) Information gain estimation method 2

With a partially known map and 3D camera model, visibility analysis can be used to estimate which triangle is visible from every candidate viewpoint of the image sensor. The information gain can be calculated by applying frustum culling and a ray tracing algorithm to the known part of the environment.

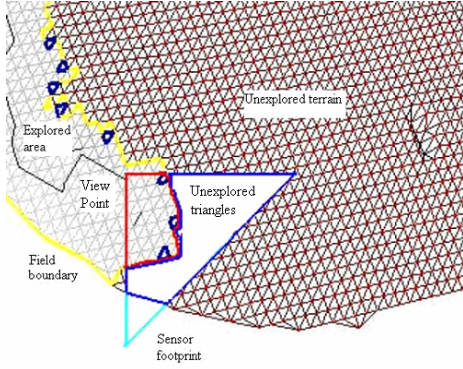


Figure 3. The sensor's footprint used to estimate the information gain.

## V. SIMULATION SETUP

A four-wheel drive robot (mass: 16 kg; length: 50 cm; width: 49 cm; height: 26 cm) with four identical wheels (wheel diameter: 25.2 cm; wheel width: 7.5 cm) was used in the simulation. A 3D laser sensor model with a  $90^\circ$  field of view, 50 meter depth of field, and 1:1 aspect ratio was used as the vision system. It was assumed that the robot traveled at a steady speed of 3 m/s. Two different methods of estimating information gain were tested on the field. The starting location was varied to investigate its affect on algorithm performanc. By combining three different information gain weights with two utility functions, five exploration strategies ( $\alpha_i=0$  results in identical functions) were tested:

- (1) minimum energy consumption ( $\alpha_i=0$ ),
- (2) considered both energy requirement and information gain 1 (method 1,  $\alpha_i=0.5$ ),
- (3) maximum information gain 1 (method 1,  $\alpha_i=1$ ),
- (4) considered both energy requirement and information gain 2 (method 2,  $\alpha_i=0.5$ ), and
- (5) maximum information gain 2 (method 2,  $\alpha_i=1$ ).

## VI. RESULTS AND DISCUSSIONS

The trajectory paths generated using the five methods are given in Fig. 4(a) through Fig. 4(e), respectively. The black lines represent the path and the arrows show the vehicle's travel direction, while the cross marks represent viewpoints where the robot stopped to take a scan. Figure 4 shows that strategies considering only information gain have more overlap in their trajectory plots. However, the other three strategies, which considered the minimum energy cost or integrated both the energy cost and information gain, have less overlap in the path generated in the simulation.

Figures 5(a) – 5(e) show the relationship of the fraction of the environment mapped and the energy requirement, distance traveled, time requirement (including planning time and navigation time), and scan number by the robot exploration of agricultural field 1 starting from location A for the five exploration strategies. As shown in Figs. 5(a) and 5(b), it is apparent that the energy requirement and path length of the

methods that considered energy consumption were substantially smaller than those that considered only

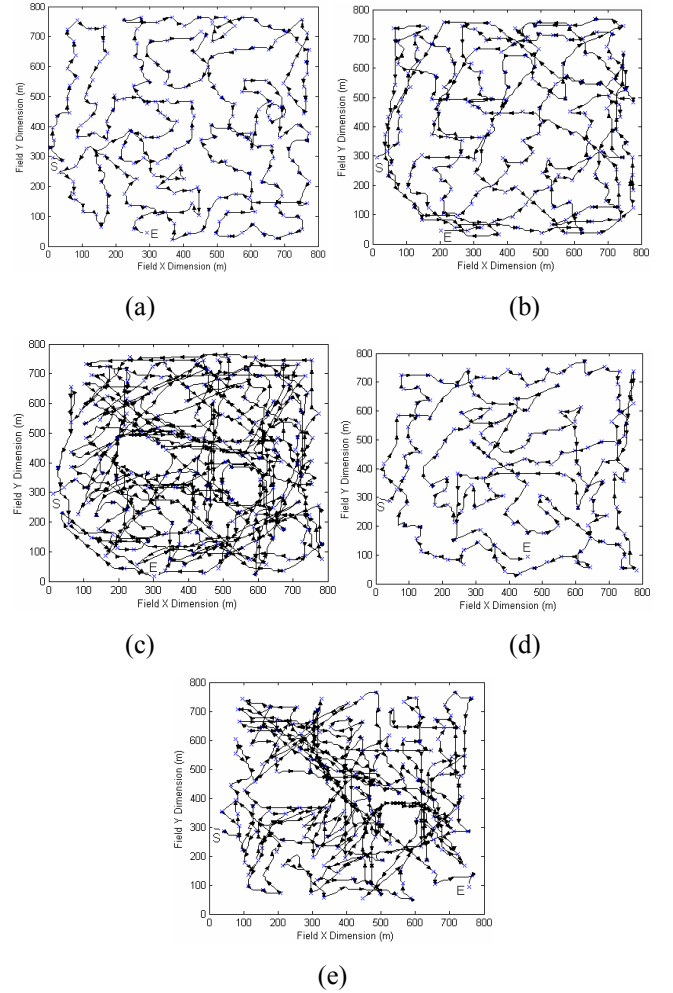


Figure 4. Trajectories generated in the simulation (arrows represent travel direction; cross marks represent viewpoints; started at S (starting point A) and ended at E): (a) minimum energy consumption; (b) considered both energy requirement and information gain 1; (c) maximum information gain 1; (d) consider both energy requirement and information gain 2, (e) maximum information gain 2.

information gain, while the fraction of the terrain explored was greater in the former. The strategies using both information gain and energy consumption resulted in more efficient energy usage and shorter path length during the beginning of the exploration tasks, while the minimum energy requirement method required slightly more energy and traveled further in the earlier stages of the exploration task. For the three methods that considered the energy consumption or both energy consumption and information gain, they resulted in almost the same path length and energy consumption for 90 percent of the exploration task. After that, the efficiency of the exploration policy using information gain method 1 decreased greatly in terms of energy usage and path length.

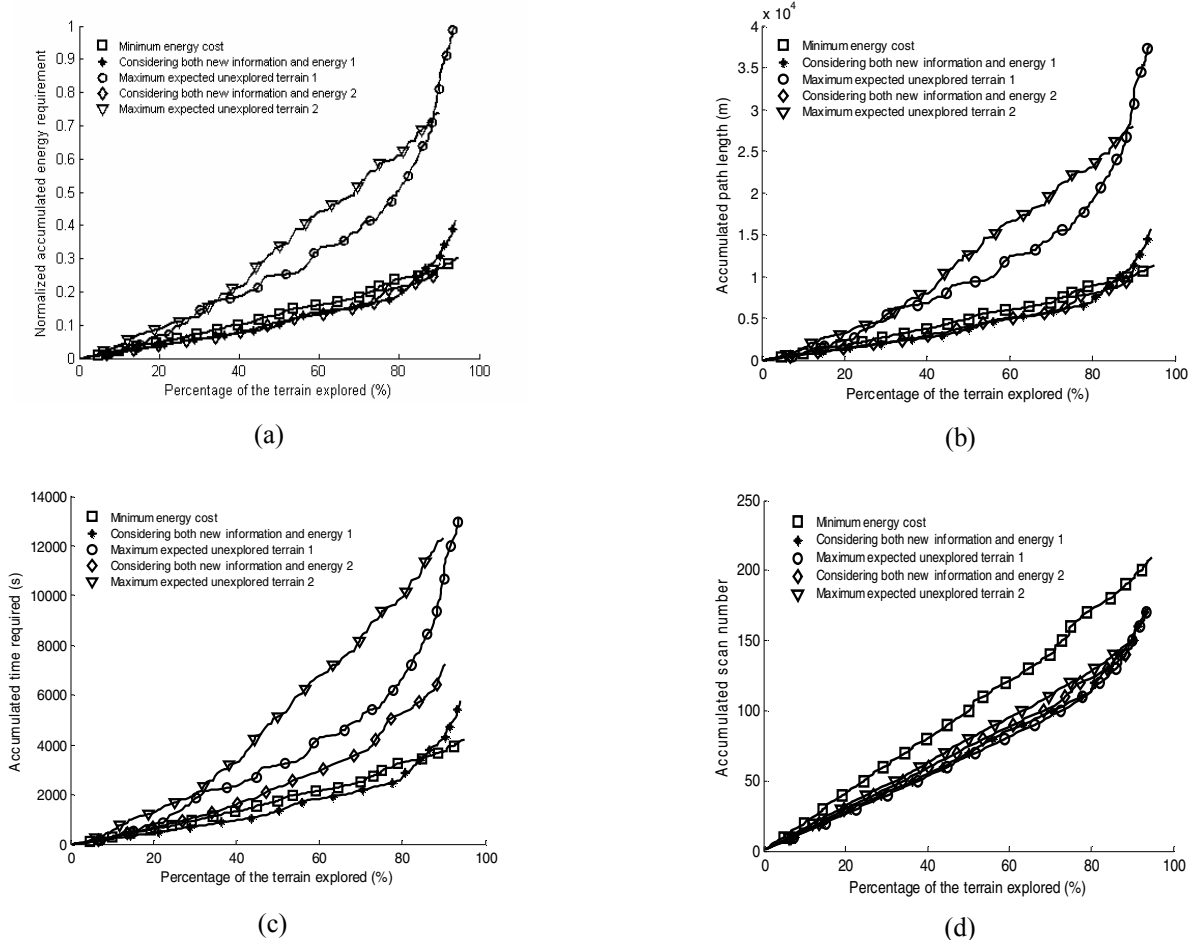


Figure 5. Result of the autonomous mapping of the agricultural field: (a) normalized accumulated energy requirement (the nominal energy is divided by the maximum energy) of the exploration as a function of fraction of explored terrain; (b) accumulated path length traveled by the robot as a function of fraction of explored terrain. (c) accumulated time required as a function of fraction of explored terrain; (d) accumulated number of scans as a function of fraction of explored terrain.

Figure 5(c) shows that the minimum energy requirement method greatly outperformed other methods in terms of time requirement, although it required more energy and traveled further to complete the same tasks than the method that considered both energy requirement and information gain 2. This is because the information gain methods required much more time for planning the path. It also shows that the method using frustum culling and a ray casting algorithm required much more time than the method using the sensor footprint to estimate new information gain, because the ray casting algorithm needs a relatively large amount of time to do calculations.

The method, considered both energy requirement and information gain 2 (method 2,  $\alpha_i = 0.5$ ), performed the best among the 5 methods in terms of total energy requirement and path length for 90% of the exploration task. Because there is a coarse map available for this method to estimate the information gain method, the robot could plan a better next-best viewpoint by maximizing both the information gain and minimizing the travel cost in each step; therefore, the information gain method 2 performed best in terms of total

energy consumption and path length. These results demonstrate a great advantage of exploration with the ray tracing using a coarse map over other situations, and the advantages of the ray casting algorithm over the simple sensor footprint method to estimate the new terrain information are apparent.

Figure 5(e) shows that the number of scans for the minimum energy cost method, which did not consider the information gain, was drastically larger than those methods that considered the information gain.

## VII. SUMMARY AND CONCLUSIONS

Information-based exploration algorithms were presented to address the problem of the next-best viewpoint in modeling large rough unstructured environments. A triangular mesh map was used to represent a 3D rough environment. An energy cost function was proposed to represent the travel cost. Two methods of estimating new terrain information gain were developed. The first method of estimating information gain involved polygon clipping. A terrain visibility analysis based on a viewing frustum model and ray casting algorithm was proposed in the second method to address the information gain

estimation for exploration with an *a priori* coarse map. Simulation results using a typical western Canadian agricultural field were presented.

The exploration strategy, which incorporated the energy consumption and the information gain with a ray tracing algorithm using a coarse map, had an advantage over other policies in terms of the total energy consumption and the path length. However, the frustum culling and the ray casting algorithms required more planning time than the method that used the sensor footprint to estimate new information gain.

Path length and energy requirement of the methods that considered energy consumption were substantially less than those for the methods that consider only information gain. The maximum-information-gain methods required more energy than the minimum energy and other two methods, considering both energy requirement and the information gain.

The number of scans for the greedy method that did not consider information gain was larger than for those methods that considered information gain. These results show the effectiveness of the algorithm considering both the energy consumption and travel cost.

Future work will aim at implementing the algorithm on an experimental robot. Some practical problems need to be addressed in the future. First, the energy cost function developed in this paper assumed that the soil hardness is uniform in the whole field. The soil hardness might be highly variable in one field. The soil hardness map should be integrated in the energy cost function if a soil strength map is available in future work. Second, it will require significant research effort to complete the whole robotic navigation system. A robotic platform equipped with a 3D image sensor and a position sensor should be developed. A map stitching algorithm which combines a variety of sensor readings into a triangular mesh map should also be developed.

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