New Method for Approximating Vague Sets to Fuzzy Sets based on Voting Model

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Abstract*— By analyzing Vague Sets voting model, we bring forth a new method for approximating Vague Sets to Fuzzy Sets, and its general process is presented in the article. In a voting model, firstly, we suppose that the abstainers must vote for once more, and the results are close studied. Then the randomicity, uncertainty, and conformity of voting are found. As we know, an abstainer may favor somebody, oppose somebody, or just abstain. In this article, we suppose the distribution of results is consistent with a normal distribution. So, we advance the new approximation method based on Gauss Distribution. *(Abstract)

Keywords*—Vague Sets, uncertain information, Fuzzy Sets, Gauss distribution, Fuzzy approximation. *(key words)

I. INTRODUCTION

In 1993 Gau and others proposed Vague Sets [1], which is a promotion of Fuzzy Sets [2], and it is also known as an intuitive Fuzzy Sets[3-6]. In a Vague Sets, there is a real membership function $t_V(x)$ and a false membership function $f_V(x)$ to describe the border of the membership. The two borders, within [0,1], constitute a sub-interval [$t_V(x)$),1- $f_V(x)$]. Concretely, $t_V(x)$ stands for the degree of support, $f_V(x)$ represents the degree of opposition, and $1-t_V(x)-f_V(x)$ represents the degree of uncertainty respectively. In a traditional Fuzzy Sets, one element must belong to it or not. But sometime the reality is that: besides affirming and denying the element, there is a range of between positive and negative uncertainty. This makes Vague Sets work better to deal with uncertainty information than the traditional Fuzzy Sets. Currently, Vague Sets have infiltrated the fuzzy control, decision analysis and expert systems, and other fields, and promoted the developments of pattern recognition, artificial Intelligence, etc. In the field of AI (Artificial Intelligence),Vague Sets Theories have achieved better results than traditional fuzzy theories, so the theories is arousing the concern of many foreign and domestic scholars[7,8].

Vague Sets voting model embodied three aspects as follows: favor, oppose or abstain. And in a practical issue, we have to fully consider these three aspects of information. This paper provides an effective mean for disposing information under a vague environment. Generally speaking, we analyze the favoring part and opposing part first, and try to discover some knowledge. Secondly, according to the knowledge, we partition the abstaining part. At last, we try to found the useful knowledge buried in the abstaining part.[1]

II. CONCEPTS AND DENFINITIONS OF VAGUE SETS

The domain $X=\{x_1,x_2,\cdots,x_n\}$ to be the object of the discussion, the Vague Sets on X is fixed by the membership function t_A and a false membership function f_A, $t_A: X\to[0,1]$; $f_A: X\to[0,1]$, and t_A (x_i) is the lower bound of support membership derived from the support of evidence, f_A (x_i)is the lower bound of opposing membership derived from the opposing of evidence x_i. And we have $t_A(x_i)+f_A(x_i)\le 1$. The membership of Vague A is an interval $[t_A(x_i),1-f_A(x_i)]$, within [0,1], which is to describe the Vague value how x_i belong to the A, and recorded as: $v_A(x_i)$.

To Vague sets *A:*

If x is discrete,

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then $A=\sum_{i=1}^{i=n}[t_A(x_i),1-f_A(x_i)]/x_i, x_i\in X$

If x is continuous,

then $A=\int_x [t_A(x),1-f_A(x)]/x, x\in X$.

Example1:

If $v_A(x)=[t_A(x),1-f_A(x)]=[0.5,0.8]$], then $t_A(x)=0.5$, $f_A(x)$=1-0.8=0.2, $\pi_A(x)$=1-$t_A(x)$ - $f_A(x)$ =0.3.

This can be interpreted as follows: the extent of object x belonging to A is 0.5; the extent of x not belonging to A is 0.2; the extent of uncertainty is 0.3. In a voting model: 10 people attend voting, and the result is: 5 voted for something, 2 voted against something, meanwhile three abstained.

III. TRANSFORM VAGUE SETS TO FUZZY SETS

$\forall x\in X, \pi_A(x)=1-t_A(x)-f_A(x)$ is the vague degree of the Vague Sets A, which shows how vague the Vague Sets is. Greater the value $\pi_A(x)$ is, more people abstain during the voting. When $t_A(x_i)+f_A(x_i)=1(\forall x_i\in X)$, vague sets A is a Fuzzy Sets. Two Principles for transform vague sets $A(u)$ to fuzzy sets $\mu_{A^F}(u)$: Should meet the following criteria [9]

Principle1: $t_A\le\mu_{A^F}(u)\le 1-f_A$.

Principle2: if $t_A=1-f_A$, then $\mu_{A^F}=t_A=1-f_A$.

Setting A on the field X of Vague Sets, $\forall x\in X$, $v_A(x)=[t_A(x),1-f_A(x)]$, the model voting for, against and abstained some percentage of people considered to abstain in the vote will appear again for, against and three abstentions, to abstain from voting on the thinning crowd $\pi_A(x)$ into three parts. If more general use refinement α and β the conduct $\pi_A^{(1)}(x)$ of which ($\alpha,\beta\in[0,1]$), and $0\le\alpha+\beta\le 1$), then x attached to the situation in the second ballot after conversion $(t_A^{(2)}(x), f_A^{(2)}(x), \pi_A^{(2)}(x))$.

$$t_A^{(2)}(x)=t_A^{(1)}(x)+\alpha\ t_A^{(1)}(x) \tag{1}$$

$$f_A^{(2)}(x)=f_A^{(1)}(x)+\beta\ \pi_A^{(1)}(x) \tag{2}$$

$$\pi_A^{(2)}(x)=(1\text{-}\alpha\text{ -}\beta\ )\pi_A^{(1)}(x) \tag{3}$$

The process is the first step in transforming Vague Sets, which is $(t_A^{(2)}(x), f_A^{(2)}(x), \pi_A^{(2)}(x))$, and Vague Sets $v_A(x)$ is the outcome of the second ballot. It is still a part of the first step into abstained information.

Then the first time into the mode of a second transformation, both by the results of the third vote ($(t_A^{(3)}(x), f_A^{(3)}(x), \pi_A^{(3)}(x))$,of which are:

$$t_A^{(3)}(x)=t_A^{(2)}(x)+\alpha\pi_A^{(2)}(x)=t_A^{(1)}(x)+\alpha\pi_A^{(1)}(x)(1-\alpha-\beta) \tag{4}$$

$$f_A^{(3)}(x)=f_A^{(2)}(x)+\beta\pi_A^{(2)}(x)=f_A^{(1)}(x)+\beta\pi_A^{(1)}(x)(1-\alpha-\beta) \tag{5}$$

$$\pi_A^{(3)}(x)=(1\text{-}\alpha\text{ -}\beta\ )\pi_A^{(2)}(x)=(1-\alpha-\beta)^2\pi_A^{(1)}(x) \tag{6}$$

So go after n-1 steps of transformation, we can get the last conversion of the value of voting $(t_A^{(n)}(x), f_A^{(n)}(x), \pi_A^{(n)}(x))$,

$$t_A^{(n)}(x)=t_A^{(1)}(x)+\alpha\pi_A^{(1)}(x)(1+(1\text{-}\alpha\text{ - }\beta)+(1-\alpha-\beta)^2 + \cdots + (1-\alpha-\beta)^{n-2}) \tag{7}$$

$$f_A^{(n)}(x)=f_A^{(1)}(x)+\beta\pi_A^{(1)}(x)(1+(1\text{-}\alpha\text{ - }\beta)+(1-\alpha-\beta)^2 + \cdots + (1-\alpha-\beta)^{n-2}) \tag{8}$$

$$\pi_A^{(n)}(x)=(1-\alpha-\beta)^{n-1}\pi_A^{(1)}(x) \tag{9}$$

When $n\to\infty$, the limit state of the transformation of Vague Sets is ($t_A^{(\infty)}(x), f_A^{(\infty)}(x), \pi_A^{(\infty)}(x)$), of which:

$$t_A^{(\infty)}(x)=t_A^{(1)}(x)+\alpha\pi_A^{(1)}(x)\sum_{n=2}^{\infty}(1-\alpha-\beta)^{n-2} = t_A^{(1)}(x)+\frac{\alpha}{\beta+\alpha}\pi_A^{(1)}(x) \tag{10}$$

$$f_A^{(\infty)}(x)=f_A^{(1)}(x)+\beta\pi_A^{(1)}(x)\sum_{n=2}^{\infty}(1-\alpha-\beta)^{n-2} = f_A^{(1)}(x)+\frac{\beta}{\beta+\alpha}\pi_A^{(1)}(x) \tag{11}$$

$$\pi_A^{(\infty)}(x)=\lim_{n\to\infty}(1-\alpha-\beta)^{n-1}\pi_A^{(1)}(x)=0 \tag{12}$$

At this time $t_A^{(\infty)}(x)+f_A^{(\infty)}(x)=1$, and meet the two criteria of transformation of Vague Sets $A(u)$ into a Fuzzy Sets $\mu_{A^F}(u)$, and the limits state of the transformation of Vague Sets $A(u)$ is a Fuzzy Sets, thus achieving the fuzzy approximation of a Vague Sets

IV. VAGUE APPROXIMATION IDENTIFIED

A. Common Method

Vague Sets is against the current vague approximation research, and information $\pi_A(x)$ on the distribution of uncertainty in the transformation process α and β the identification, in accordance with the basic initial vote in favour of $t_A^{(1)}(x)$ and $f_A^{(1)}(x)$ against the adoption of a single proportion of each of the linear relationship between the distribution or reconcile, there are three regular methods:

1) The Proportion Method

when $\alpha=t_A^{(1)}(x)$, $\beta=f_A^{(1)}(x)$,

$$t_A^{(\infty)}(x)=\frac{t_A(x)}{t_A(x)+f_A(x)} \tag{13}$$

$$f_A^{(\infty)}(x)=\frac{f_A(x)}{t_A(x)+f_A(x)} \tag{14}$$

And that is $A=\sum_{i=1}^{i=n}[t_A(x_i),1-f_A(x_i)]/x_i$ and the domain

$B=\sum_{i=1}^{i=n}[t_B(y_i),1-f_B(y_i)]/y_i$ of the two Vague Sets, after fuzzy approximation $\mu_{A^F}=\sum_{i=1}^{i=n}\frac{t(x_i)}{t(x_i)+f(x_i)}/x_i$ and $\mu_{B^F}=\sum_{i=1}^{i=n}\frac{t(y_i)}{t(y_i)+f(y_i)}/y_i$ intothe Fuzzy Sets.

2) The Sharing Law

When $\alpha=\beta=\frac{1}{2}$,

$$t_A^{(\infty)}(x)=\frac{1+t_A-f_A}{2} \tag{15}$$

$$f_A^{(\infty)}(x)=\frac{1-t_A+f_A}{2} \tag{16}$$

3) The Linear Proportion Method

When $\alpha=\frac{t_A(x)}{t_A(x)+f_A(x)}, \beta=\frac{f_A(x)}{t_A(x)+f_A(x)}$

$$t_A^{(\infty)}(x)=\frac{t_A(x)}{t_A(x)+f_A(x)} \tag{17}$$

$$f_A^{(\infty)}(x)=\frac{f_A(x)}{t_A(x)+f_A(x)} \tag{18}$$

B. The Vague Approximation under Gauss Distribution

Actually, the voting process is of uncertainty, randomness and belongingness, but the result of the initial vote does play a key role in the next voting. Through comprehensively analyzing from the against, for and abstained aspects, the result of the crowd who once voted abstained will be equate to the ratio of the initially for to against or the ratio of the against to the for, that is using t_A/f_A, or f_A/t_A as variables, and the parameters obey the Gauss distribution[10] $\mu=1$ and $\delta^2=1$, $X\sim N(\mu,\delta^2)$ or $X\sim\Phi(\frac{x-\mu}{\sigma})$ to remember.

Distribution Function:

$$F(x)=\frac{1}{\sqrt{2\pi\delta^2}}\int_{-\infty}^{x}\exp(-(t-u)^2/2\delta^2)dt(-\infty<x<+\infty) \tag{19}$$

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x-1}e^{-t^2/2}dt(-\infty<x<+\infty) \tag{20}$$

Gauss distribution of voting model

When $t_A\geq f_A$

$$\alpha=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{t_A}{f_A}-1}e^{-t^2/2}dt\pm\Delta\alpha \tag{21}$$

$$\beta=1-\alpha=1-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{t_A}{f_A}-1}e^{-t^2/2}dt\mp\Delta\alpha \tag{22}$$

when $t_A<f_A$

$$\beta=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{f_A}{t_A}-1}e^{-t^2/2}dt\pm\Delta\beta \tag{23}$$

$$\alpha=1-\beta=1-\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{f_A}{t_A}-1}e^{-t^2/2}dt\mp\Delta\beta \tag{24}$$

$(\Delta\alpha,\Delta\beta)$ is the statistical error probability, which has small values generally.

The number of the people who vote for initially is equate to who vote against, that is $t_A=f_A$, $\alpha=\beta=\Phi(0)=0.5=50\%$, the crowd who voted abstained will vote once again, the support rate for the final candidates will be around 50 percent. When $\lim_{t_A\to 0}f_A/t_A=+\infty$, $\beta=1$ the initial result of voting which is for is tending to 0, the final result of voting which is against may be 0. Actually, if the initial rate of the voting for is too low, the candidate is not qualified to enter the second round. At this time, the candidate should be changed.

When $n\to\infty$, the limit state of the transformed Vague Sets $v_A(x)$ is $(t_A^{(\infty)}(x),f_A^{(\infty)}(x),\pi_A^{(\infty)}(x))$,of which:

$$t_A^{(\infty)}(x)=t_A^{(1)}(x)+\alpha\pi_A^{(1)}(x)\sum_{n=2}^{\infty}(1-\alpha-\beta)^{n-2}=t_A^{(1)}(x)+\frac{\alpha}{\beta+\alpha}\pi_A^{(1)}(x)$$

$$f_A^{(\infty)}(x)=f_A^{(1)}(x)+\beta\pi_A^{(1)}(x)\sum_{n=2}^{\infty}(1-\alpha-\beta)^{n-2}=f_A^{(1)}(x)+\frac{\beta}{\beta+\alpha}\pi_A^{(1)}(x)$$

$$\pi_A^{(\infty)}(x)=\lim_{n\to\infty}(1-\alpha-\beta)^{n-1}t_A^{(\infty)}(x)=0$$

When $t_A\geq f_A$

$$1-f_A^{(\infty)}(x)=t_A^{(\infty)}(x)=t_A^{(1)}(x)+(\Phi(\frac{t_A}{f_A}-1)\pm\Delta\alpha)\pi_A^{(1)}(x) \tag{25}$$

When $t_A<f_A$

$$1-f_A^{(\infty)}(x)=t_A^{(\infty)}(x)=t_A^{(1)}(x)+(\Phi(\frac{f_A}{t_A}-1)\pm\Delta\beta)\pi_A^{(1)}(x) \tag{26}$$

Example 2: In a voting, there are 150 voters taking part in it. Three candidates A, B and C did not have the rate of support over half respectively in the initial voting. Therefore, another voting should be taken. In the initial voting, A had 60 tickets for himself, 45 against himself, B had 45 tickets for himself, 30 against himself and C only have 10 tickets for himself, 0 against.

Use Vague Sets to represent the result of the voting mentioned above: $V_A=[0.4, 0.7]$ $V_B=[0.3, 0.8]$ $V_C=[0.1, 1]$. Obviously, in the initial voting the rate that is for A is 40%, but still not a majority, the lowest rate that is for C is only 10% and B has a 30% supporting rate, which requires a second round of voting, and requires to manipulate the Vague Set mentioned above with fuzzy approximation and predict the final outcome.

Although nobody voted against C, the rate of C that is for C is too low in the initial voting. Therefore, in the second round, we have only to vote for A and B. Used three linear transformations to do the fuzzy approximation.

1) The Proportion Method

$$t_A(A)=57.14\%;t_A(B)=60.00\%$$

2) The Sharing Law

$$t_A(A)=55.00\%;t_A(B)=55.00\%$$

3) The Linear Proportion Method

$$t_A(A) = 57.14\%; t_A(B) = 60.00\%$$

Through the three linear fuzzy approximation, 1) and 3) in the second B will win the election, the support rate of 60.00%; 2) $t_A(A) = t_A(B) = 55.00\%$, this is unable to determine last the results at this time so the third vote needed.

The fuzzy approximation under the Gauss distribution

$$t_A(A) = 0.4 + 0.3 * \Phi(1/3) = 0.5888 = 58.88\%$$

$$t_A(B) = 0.3 + 0.5 * \Phi(1/2) = 0.6458 = 64.58\%$$

Through Gauss distribution in the fuzzy approach that we can predict in the second vote B will win and support rate is about64.58%.

Through the case we can see under different ways of transformation Vague Sets to Fuzzy Sets the final result is different. Linear transformation was too monotonous, but under Gauss distribution Vague sets to Fuzzy Sets fuzzy approaching, the last results are more reasonable and more practical with the facts.

V. CONCLUSION

In this paper, we have a discussion on the approximation method for transform Vague Sets to Fuzzy Sets in-depth. Considering the randomicity, uncertainty, and conformity of voting, we apply Gauss distribution to Vague Sets approximation, and examples presented above prove that the method is reasonable. After all, the process of transformation is constant probability approximation in itself, which is inadequate. But it does have a good award: regardless of how many times of voting to determine the final candidates, the support rate is still consistent with the final Gauss distribution.

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