

# Detection of Extrasystoles in Heart Rate Sequences Based on Short-term Specific Random Elements in Random Sequences Recognition Theory

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**Abstract** - Detection extrasystoles in heart rate sequences is investigated. Decision making is based on short-term specific random elements in random sequences recognition theory. Three types of extrasystoles are detected: extrasystoles with a noncompensatory post-extrasystolic pause, extrasystoles with a compensatory post-extrasystolic pause, and interpolated extrasystoles. The decision opens the possibility to develop wearable, green – energy saving equipment for long term heart rate variability monitoring of ubiquitous, obstrusive people. The experimental results are presented.

**Key words** - heart rate variability, extrasystoles, mHealth, recognition, random sequences, short-term random events.

## I. INTRODUCTION

FAST improving mHealth technologies, wearable, wireless equipment enable us to long-term constantly monitor ubiquitous, unobtrusive personal heart rate variability [1-5]. We can extract valuable information from such data on frequently occurring heart rate disturbances- extrasystoles [6-13]. One has just manage to recognize extrasystoles in the RR sequences observed. Unfortunately, RR sequences of ubiquitous, unobtrusive personal heart rate variability are nonstationary random sequences with complex structure. It is a puzzle to detect in them short-term events – short-term heart rate disturbances – extrasystoles - emerging at random time moments. The work is complicated. We present here theory and a constructive method realizable by computers to solve this problem: Detection of extrasystoles in heart rate sequences, based on short-term specific random elements in random sequences recognition theory. By invoking the facilities rendered by this method, we can expeditiously inform doctors, therapists, nurses, and the families of persons about the health state of a ubiquitous, unobtrusive person, improve home rehabilitation procedures as well as health preventive measures, and achieve economic and societal issues. All that opens a possibility to develop a new type of health services and health service activity support.

## II. STATEMENT OF THE PROBLEM

Let us consider a random sequence

$$Y(i) = X(i) + S(i) \quad (i = 1, \dots). \quad (1)$$

The component  $X(i) (i = 1, \dots)$  in it is a random sequence described by the Gauss law with unknown parameters.

The second component is represented by an expression

$$S(i) = \begin{cases} C(i), & (i = i1, i2) \\ 0, & (i \neq i1, i2) \end{cases}, \quad (2)$$

where  $C(i) (i = i1, i2)$  short term specific random elements are the elements of a single sequence of random amplitude that emerge at random time moments  $(i = i1, i2)$ .

We observe the sample

$$y(i) = x(i) + s(i), \quad (i = 1, \dots, N) \quad (3)$$

of a random sequence  $Y(i) = X(i) + S(i), (i = 1, \dots)$ .

In (3),  $x(i) (i = 1, \dots, N)$  is a sample of the random sequence  $X(i) (i = 1, \dots)$  and

$$s(i) = \begin{cases} c(i), & (i = i1, i2) \\ 0, & (i \neq i1, i2), \end{cases}$$

is a sample of the random sequence  $S(i)$ .

We need to determine the argument values  $(i = i1, i2)$  of the appearance of the short-term random specific elements  $y(i) = x(i) + s(i), s(i) \neq 0, (i = i1, i2)$ .

## III. SOLVING OF THE PROBLEM

Consider a situation where

$$Y(i) = X(i) + S(i), \quad (i = 1, \dots),$$

$$S(i) = \begin{cases} C(i), & (i = i1, i2) \\ 0, & (i \neq i1, i2). \end{cases} \quad (4)$$

Define a sequence of random variables

$$U(i) = [X(i) - M]/D, \quad (i = 1, 2, \dots). \quad (5)$$

$M$  and  $D$  are unknown.

Instead of  $M$  and  $D$  we use their estimates  $m$  and  $d$ .

Afterwards, we describe the sample  $u(i)$  of the random sequence  $U(i)$  by a random sequence

$$u(i) = [x(i) - m]/d \quad (i = 1, \dots, N), \quad (6)$$

It is distributed by Tompson's law with  $N-2$  degrees of freedom [14].

We shall look for the two short-time random specific elements  $y(i1) = x(i1) + c(i1) \wedge y(i2) = x(i2) + c(i2)$  in the following manner.

Suppose that there are no components  $c(i1) \wedge c(i2)$  in the sample  $y(i)$  ( $i = 1, \dots, N$ ), i.e.  $c(i1) = 0 \wedge c(i2) = 0$ . Then  $y(i) = x(i)$  ( $i = 1, \dots, N$ ).

Consider the event

$B : U(i) \geq u(l)$ , where

$$u(l) = \max_{1 \leq i \leq N \wedge i \neq k} |u(i)|, \quad u(k) = \max_{1 \leq i \leq N} |u(i)|.$$

The probability of event  $B$

$$p(B) = \int_{u(l)}^{\infty} f(N-2, t) dt. \quad (7)$$

If, after the  $N$  test, the event  $B$  was occurred two times, then the estimation of the probability  $p(B)$  of event  $B$  is

$$\tilde{p}(B) = 2/n. \quad (8)$$

The hypothesis  $H$  is tested with the reliability level  $\alpha$

$$H : \tilde{p}(B) \leq p(B) \quad (9)$$

with the alternative  $A : \tilde{p}(B) > p(B)$ .

Calculate a confidence interval

$$[p\alpha(B, \alpha) \leq \tilde{p}(B) \leq p\nu(B, \alpha)]. \quad (10)$$

Verification of the hypothesis  $H$  is replaced by that of inequality

$$p\nu(B, \alpha) \leq p(B). \quad (11)$$

Let us calculate  $p\nu(B, \alpha)$  with the confidence level  $\alpha$ , by solving the integral equation

$$\int_0^{p\nu(B, \alpha)} Be(x, 3, N-2) dx = \frac{1+\alpha}{2}, \text{ where}$$

$$Be(x, 3, N-2) = \frac{\Gamma(N+1)}{\Gamma(3)\Gamma(N-2)} x^2 (1-x)^{N-3}, \quad (12)$$

$$0 \leq x \leq 1, \quad 0 < N < \infty.$$

If the data do not contradict the hypothesis  $H$ , then the assumption that there are no short-time random specific elements in the observed sequence  $y(i)$  ( $i = 1, \dots, N$ ) is true with the confidence level  $\alpha$ . It means that there are no short-term random specific elements.

If the data contradict the hypothesis  $H$ , then we can state, with the reliability level  $\alpha$ , that  $y(k) = x(k) + c(k) \wedge y(l) = x(l) + c(l)$  are two short-term random specific elements.

To answer the question whether the elements  $y(k)$  and  $y(l)$  make extrasystoles with a noncompensatory post-extrasystolic pause, we check the condition  $Se$  and the hypothesis  $Hen1, Hen2, Hen3$ . If the condition  $Se$  is satisfied and the data  $y(li)$  ( $i = 1, \dots, N$ ) are compatible with the hypotheses  $Hen1, Hen2, Hen3$ , then  $y(k)$  is an

extrasystole and  $y(l)$  is a noncompensatory post-extrasystolic pause.

Let us verify whether the elements  $y(k)$  and  $y(l)$  satisfy the condition  $Se$ .

Denote  $i1 = \min(k, l); i2 = \max(k, l)$ . If  $i2 = i1 + 1$ , then the elements  $y(k) \wedge y(l)$  are adjacent.

Verify the hypothesis  $Hen$  whether the elements  $y(i1) \wedge y(i2)$  are an extrasystole with a noncompensatory post-extrasystolic pause:

$Hen : y(i1) \wedge (x(i1) \wedge y(i2)) \wedge x(i2) \wedge y(i1) + y(i2) \wedge (x(i1) + x(i2))$  with an alternative

$$Aen : y(i1) \geq x(i1) \vee y(i2) \leq x(i2) \vee y(i1) + y(i2) \geq x(i1) + x(i2).$$

Define the sequence as

$$z(i) = \begin{cases} 0, & i = i1 \\ 0, & i = i2 \\ 1, & i \neq i1, i2 \quad (i = 1, \dots, n). \end{cases}$$

Calculate

$$K = \sum_{i=1}^N z(i), \quad m = \frac{1}{K} \sum [z(i) y(i)],$$

$$d = \frac{1}{K-1} \sum_{i=1}^N [(z(i)(y(i)-m)]^2, \quad Kk(1) = \sum_{i=1}^{N-1} z(i)(z(i+1))$$

$$k(1) = \frac{1}{Kk(1)-1} \sum_{i=1}^{N-1} [z(i)(y(i)-m)][z(i+1)(y(i+1)-m)] \quad (13)$$

Next, consider the first-order autoregression equation

$$(X(i)-M) + A(1)(X(i-1)-M) = BV(i), \quad (14)$$

where  $Ev(t) = 0, EV^2(t) = 1$ .  $M, A(1), B$  are the coefficients of equation (16), and  $m, a(1), b$  are their estimates.

$$m = \frac{1}{K} \sum [z(i) y(i)], \quad a(1) = -\frac{k(1)}{d}, \quad b = \sqrt{d(1-a(1)a(1))}.$$

We calculate a forecast  $xp(i1)$  of  $x(i1)$ :

$$xp(i1) = m - a(1)(y(i1-1) - m) \quad (15)$$

and its variance estimate

$$d(xp(i1)) = b^2. \quad (16)$$

We calculate a forecast  $xp(i2)$  of  $x(i2)$ :

$$xp(i2) = m - a(1)(y(i2+1) - m) \quad (17)$$

and its variance estimate:

$$d(xp(i2)) = b^2 \quad (18)$$

We calculate a forecast of  $x(i1) + x(i2)$ :

$$xp(i1) + xp(i2) = [m - a(1)(y(i1-1) - m)] + [m - a(1)(y(i2+1) - m)] \quad (19)$$

and their variance estimate:

$$d[xp(i1) + xp(i2)] = 2b^2 \quad (20)$$

Let us now test the hypothesis  $Hen1 : y(i1) \wedge (x(i1))$ .

Define a random value

$$u(i1) = |(y(i1) - xp(i1)) / \sqrt{d(xp(i1))}|, \quad (21)$$

Next, we define the event  $R1 : U(i) \geq u(i1)$ .

Then we calculate the probability of event  $R1$

$$p(R1) = P\{U(i) \geq u(i)\} = 1 - T(N-2)\{u(i)\}. \quad (22)$$

Now let us calculate the probability estimate of event  $R1$  :

$$\tilde{p}(R1) = 1/N, \quad (23)$$

Afterwards we to test the hypothesis

$$Hen1: \tilde{p}(R1) < p(R1). \quad (24)$$

If the data are compatible with the hypothesis  $Hen1$ , then the assumption  $y(i1) = x(i1)$  is true with the reliability level  $\alpha$  and  $y(i1)$  can not be an extrasystole.

Now let us test the hypothesis:  $Hen2: y(i2) \geq x(i2)$ .

Define a random value

$$u(i2) = |(y(i2) - xp(i2)) / \sqrt{d(xp(i2))}|. \quad (25)$$

Next we define the event  $R2: U(i) \geq u(i2)$ . Then we calculates the probability

$$p(R2) = P\{U(i) \geq u(i2)\} = 1 - T(N-2)\{u(i2)\}. \quad (26)$$

Afterwards we calculate probability estimation of  $R2$  event  $\tilde{p}(R2) = 1/N$ .

Let us test the hypothesis  $Hen2: \tilde{p}(R2) < p(R2)$ .

If the data are compatible with the hypothesis  $Hen2$ , then  $y(i2) = x(i2)$  is true with the reliability level  $\alpha$  and  $y(i2)$  can not be a compensatory pause. If the data contradict the hypothesis  $Hen2$ , then the hypothesis  $Hen3: y(i1) + y(i2) \geq x(i1) + x(i2)$  is verified.

Calculate

$$u(i1) + u(i2) = \frac{[y(i1) + y(i2)] - [xp(i1) + xp(i2)]}{d[xp(i1) + xp(i2)]}. \quad (27)$$

We define the event  $R3: U(i1) + U(i2) \geq u(i1) + u(i2)$ .

Calculate the probability  $p(R3) = 1 - T(N-2)[u(i1) + u(i2)]$ .

Next we calculate the probability estimation of event  $R3$

$$\tilde{p}(R3) = 1/N. \quad (28)$$

Now we test the hypothesis  $Hen3: \tilde{p}(R3) < p(R3)$ .

If the data are compatible with the hypothesis  $Hen3$ , then there cannot be any extrasystole with a noncompensatory post-extrasystolic pause.

If the data contradicts the hypothesis  $Hen3$ , then the assumption  $y(i1) + y(i2) = x(i1) + x(i2)$  with the reliability level  $\alpha$  is not true, therefore  $y(i1)$  is the extrasystole with a noncompensatory post-extrasystolic pause  $y(i2)$ .

To answer the question whether the elements  $y(k)$  and  $y(l)$  make an extrasystole with a compensatory pause, we check the condition  $Se$  and hypotheses  $Hep1, Hep2, Hep3$ .

$$Hep1: y(i1) \geq x(i1); Hep2: y(i2) \geq x(i2);$$

$$Hep3: y(i1) + y(i2) = x(i1) + x(i2).$$

To answer the question whether the elements  $y(k)$  and  $y(l)$  make an interpolated extrasystole, we verify the check condition  $Se$  and hypotheses  $Hei1, Hei2, Hei3$ .

$$Hei1: y(i1) \geq x(i1); Hei2: y(i2) \geq x(i1);$$

$$Hei3: y(i1) + y(i2) = x(i1).$$

#### IV. EXPERIMENTAL INVESTIGATION

In our experiments we have used RR sequences of the 134 people from 15 to 80 years old, present at home, at work, hospital, polyclinic, or sportsmen examination centre. Duration of the sequence records was 1076 hours. The records contain 4.410.139 RR intervals. To illustrate the extrasystole detection situation in a non-stationary sequence of RR intervals, Fig. 1 shows the record of an RR interval sequence of a ubiquitous man in the lapse of 18 hours. In a segment of this record marked by the sign  $\uparrow$ , an extrasystole is detected and colored in red, as shown in Fig. 2. With a view to estimate the accuracy of extrasystole detection, experts selected 1243 RR sequences duration 50-500 RR intervals. In the RR sequence of each selected interval there is one extrasystole. The following results of extrasystole recognition have been obtained. In 426 RR sequences with an extrasystole with a non-compensatory post-extrasystolic pause 93,43 % of extrasystoles have been recognized and 6,57 % not recognized. In 704 RR sequences with an extrasystole with a compensatory post-extrasystolic pause 93,75 % of extrasystoles have been recognized and 6,25 % not recognized. In 113 RR sequences with an interpolated extrasystole 95,58 % of extrasystoles have been detected and 4,42 % not recognized. The state of the recognized extrasystoles is illustrated in Fig.'s 3-5, while that of not recognized extrasystoles is shown in Fig.'s 6-8.

#### V. CONCLUSIONS

The theory and the constructive method presented provide an opportunity to detect extrasystoles: extrasystoles with a noncompensatory post-extrasystolic pause, extrasystoles with a compensatory post-extrasystolic pause, and interpolated extrasystoles in the background of heart rate sequences.

The probability that decisions about the existence of an extrasystoles in the background of a heart rate sequence, while it doesn't exist in reality, will be made, is posed and controlled on the reliability level  $\alpha$  of hypothesis verification.

To estimate the probability of extrasystoles missed, need statistical characteristics of extrasystoles. This shortcoming can be eliminated. To this end, it is reasonable to invoke the theory and method for detection of extrasystoles, described in this paper. They provide with a possibility to accumulate missing information by analyzing the properties of extrasystoles and use it in the estimation of probability of heart rate extrasystoles.

The theory and a constructive method, presented in the paper, render possibilities to improve, home rehabilitation procedures and health preventive measures to develop a new type of health services and health service activity support, achieve economic and societal issues.

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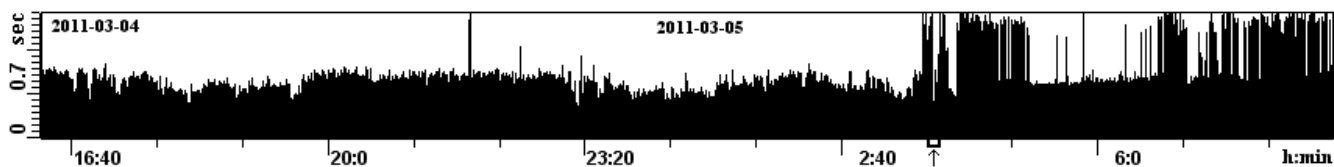


Fig. 1. Ubiquitous, unobtrusive person 18-hour RR interval sequence.

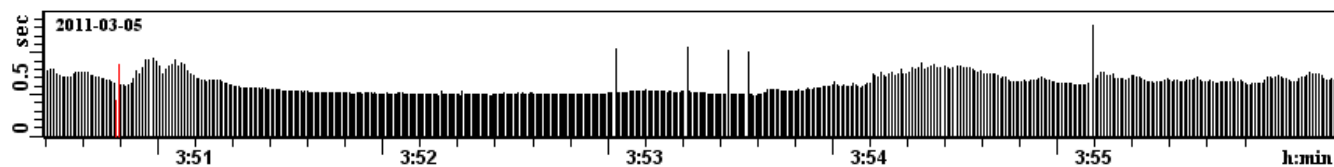


Fig. 2. Part of RR interval sequence, presented in Fig.1, marked by xxx. In this RR interval sequence were found extrasystole with a compensatory post-extrasystolic pause (emphasis mark – red colour).

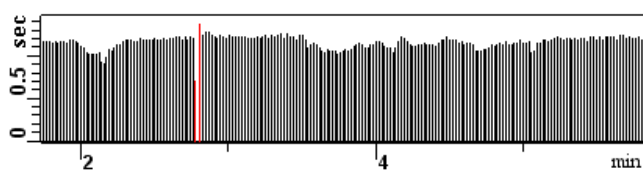


Fig.3. Extrasystole with a noncompensatory post-extrasystolic pause recognized (emphasis mark – red colour).

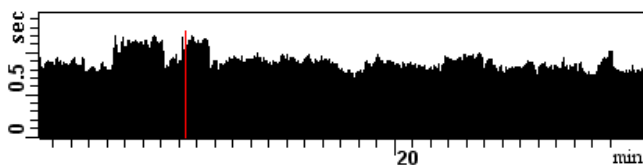


Fig.4. Extrasystole with a compensatory post-extrasystolic pause recognized (emphasis mark – red colour).

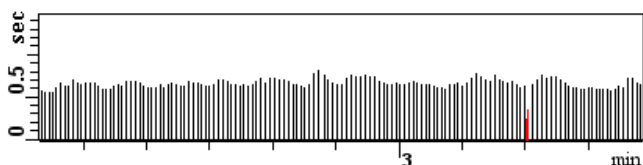


Fig.5. Interpolated extrasystole recognized (emphasis mark – red colour).

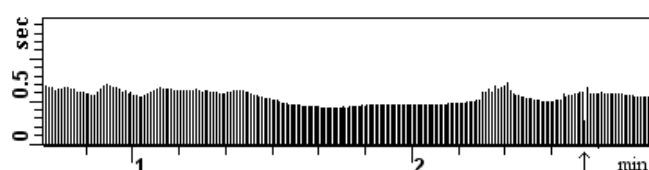


Fig. 6. Extrasystole with a noncompensatory post-extrasystolic pause (emphasis mark ↑) unknown.

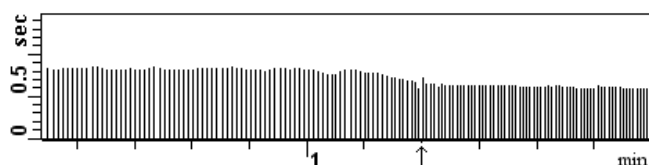


Fig.7. Extrasystole with a compensatory post-extrasystolic pause (emphasis mark ↑) unknown.

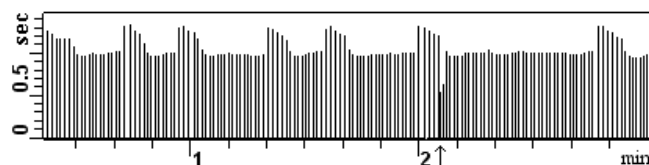


Fig. 8. Interpolated extrasystole (emphasis mark ↑) unknown.

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