# OPTIMAL PARTICLE ALLOCATION IN PARTICLE FILTERING FOR MULTIPLE OBJECT TRACKING

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## ABSTRACT

In our previous work [1], we proposed an approach to particle filtering which simultaneously adjusts the proposal variance and number of particles for each frame, in order to minimize the tracking distortion for *single* object tracking. In this paper, we extend our previous work to multiple object video tracking. Under the framework of distributed multiple object tracking, we propose the tracking distortion and use rate distortion theory to derive the optimal particle allocation among *multiple targets* as well as *multiple frames*. We subsequently propose a dynamic proposal variance and optimal particle number allocation algorithm for multi-object tracking. Experimental results show the superior performance of our proposed algorithm to traditional particle allocation methods, i.e. a fixed number of particles for each object in each frame. The proposed algorithm can also be used in decentralized articulated object tracking. To the best of our knowledge, this paper is the first to provide an optimal allocation of a fixed number of particles among multiple objects and frames.

***Index Terms—*** Tracking, resource management

## 1. INTRODUCTION

Over the past decade, particle filters have gained enormous popularity in video tracking. Particles in the particle filtering framework are sampled from a proposal density and assigned weights in order to approximate the posterior density function. It can be shown that if the number of particles is sufficiently large, the sample approximation of the posterior density can be made arbitrarily accurate. Recent technological trends have required the deployment of particle filters for video tracking applications in mobile devices (e.g. handheld video phones). The number of particles used is an essential index of the complexity of the implementation of particle filters. The limited power and scarce computational resources available in embedded computer systems have imposed tremendous constraints on the number of particles used for tracking. In our previous work [1], we proposed an approach to simultaneously adjusts the proposal variance and number of particles for each frame, in order to minimize the tracking distortion for *single* object tracking.

In multiple object tracking (MOT), the tradeoff between tracking quality and tracking resources becomes more severe. A widely accepted approach to address the interaction and data association between objects is a joint state space representation. However, the number of particles it demands grows exponentially in terms of the object tracked. Qu et al. [2] has demonstrated that a distributed Bayesian framework can be used to maintain a linear increase in the number of particles as the number of targets increase.

Even under the distributed framework, we should use particles wisely in order to achieve the best tracking quality given the fixed number of particles. Traditionally, the proposal variance and the number of particles per object per frame are fixed during the entire tracking process. These parameters are set based on trial-and-error experiments prior to tracking. However, this approach does not consider the different characteristics of each object in each frame. For example, within one frame, some objects move fast, other objects move slowly. For the same object, it may move fast in some frames and slow down in the next few frames. When computing and power resources are limited, we should use these information to utilize the available resources wisely and attain the best tracking quality possible. MacCormick and Isard [3] presented survival diagnostic and survival rate as quantities to assess the efficacy of particles filters. However, those concepts cannot tell how to allocate particles between partitions in partitioned sampling. In this paper, under the framework of distributed multiple object tracking, we exploit the characteristic behavior of the targets to dynamically vary the proposal variance and allocate the optimal number of particles for each object as well as for each video frame.

The rest of the paper is organized as follows, in Section 2 we propose a novel criteria to measure the efficiency of particle filtering. We derive the optimal particle allocation equation for multi-object tracking in Section 3. The experimental results with comparison to other methods are given in Section 4, followed by the conclusions given in Section 5.

## 2. TRACKING DISTORTION

In order to introduce the definition and expression of tracking distortion, we start from the one-dimensional particle filtering for tracking. Let the tracking error $\epsilon^i$ of the $i^{th}$ particle be defined as the difference between the true state $S$ and the sampled state $X^i$, i.e. $\epsilon^i = S - X^i$. We assume that the tracking errors $\epsilon^i$ are i.i.d. and symmetrically distributed over the interval $(-\epsilon_{max}, \epsilon_{max})$ with interval density $\beta(\cdot)$. We subsequently assume without loss of generality that the tracking errors $\epsilon^i$ are zero mean random variables (i.e. $E[\epsilon^i] = 0$) with variance $\sigma_\epsilon^2$ (i.e. $var(\epsilon^i) = \sigma_\epsilon^2$). Notice that if the tracking errors $\epsilon^i$ have a nonzero mean, we could shift the center to its mean value. The total tracking error is therefore the difference between $S$ and estimated state $\hat{X}$, i.e. $Y = S - \hat{X}$. If the number of particles $n$ is sufficiently large relative to the error bound $\epsilon_{max}$, it can be shown that $Y$ is a zero-mean random variable (i.e. $E(Y) = 0$) and $var(Y) = \frac{\sigma_\epsilon^2 \zeta_\epsilon}{n}$, where $\zeta_\epsilon = E(\frac{2\epsilon_{max} \int_{-\epsilon_{max}}^{\epsilon_{max}} w^2(\epsilon)\beta(\epsilon)d\epsilon}{(\int_{-\epsilon_{max}}^{\epsilon_{max}} w(\epsilon)\beta(\epsilon)d\epsilon)^2})$ is a constant and $w(\cdot)$ is the un-normalized weight function. Therefore, we define tracking distortion $D$ as the variance of the total tracking error $Y$.

$$D = var(Y) = \frac{\sigma_\epsilon^2 \zeta_\epsilon}{n}, \text{for the scaler state.} \quad (1)$$

In real video tracking, the elements $S$, $X^i$ and $\epsilon^i$ are vectors. For each component of the tracking error $Y^l$, we have $var(Y^l) = \frac{\sigma_{\epsilon^l}^2 \zeta_{\epsilon^l}}{n}$, $l = 1, 2, \ldots, N$. We assume that the error bound $\epsilon_{max}$, the error interval density $\beta(\cdot)$ and the weight function $w(\cdot)$ are the same for all components. Therefore, $\zeta_j = \zeta_{\epsilon_j^l}, \forall l$ for frame $j$. The tracking distortion $D_j$ of the $j^{th}$ frame is defined as the square root of the summation of the squared variance of each component,

$$D_j = (\sum_{l=1}^{N} var^2(Y^l))^{1/2} = \frac{\sigma_j^2 \zeta_j}{n_j}, \text{for the vector state,} \quad (2)$$

where $\sigma_j^2 = \sqrt{\sum_{l=1}^{N} (\sigma_{\epsilon_j^l}^2)^2}$.

Equation (2) corresponds to the result of the convergence of the variance of the particle filter estimator in [4]. From (1) and (2), we observe that for fixed $\sigma_j^2 \zeta_j$, the tracking distortion $D_j$ decreases as the number of particles increases. As the number of particles $n$ tends infinity, the tracking distortion $D_j$ approaches zero. This observation is consistent with the theory of Bayesian tracking.

## 3. OPTIMAL PARTICLE ALLOCATION FOR MULTIPLE OBJECT TRACKING

In this section, we will use rate distortion theory to derive the optimal particle allocation equations for multi-object tracking. Under the framework of distributed multiple object tracking (DMOT) [2], each object is tracked by a distributed tracker. We define the total distortion *per frame* can be expressed as the average of the distortion of the objects in that frame. The total distortion *over a video sequence* is the average of the distortion in all frames of this video sequence.

Since we want to attain the best tracking quality possible by minimizing the tracking distortion, we consider a constraint on the average number of particles $n$ used over $J$ frames and $K$ objects. We seek to determine the optimal number of particles $n_{j,k}$ for the $k^{th}$ object in the $j^{th}$ frame by allocating the total of $nJK$ particles among $J$ frames and $K$ objects such that the total distortion $D_T$ is minimized, i.e.

$$D_T = \frac{1}{JK} \sum_{j=1}^{J} \sum_{k=1}^{K} \frac{\sigma_{j,k}^2 \zeta_{j,k}}{n_{j,k}} \quad (3)$$

such that

$$\sum_{j=1}^{J} \sum_{k=1}^{K} n_{j,k} = nJK. \quad (4)$$

We solve this constrained optimization problem by forming the Lagrangian $P$ given by

$$P = \sum_{j=1}^{J} \sum_{k=1}^{K} \frac{\sigma_{j,k}^2 \zeta_{j,k}}{n_{j,k}} + \lambda(\sum_{j=1}^{J} \sum_{k=1}^{K} n_{j,k} - nJK). \quad (5)$$

By setting $\partial P / \partial n_{j,k} = 0$, we obtain

$$n_{j,k} = n \frac{\sqrt{\sigma_{j,k}^2 \zeta_{j,k}} JK}{\sum_{j'=1}^{J} \sum_{k'=1}^{K} \sqrt{\sigma_{j',k'}^2 \zeta_{j',k'}}}. \quad (6)$$

The parameter $\zeta_{j,k}$ is difficult to compute. However, under the assumptions that $\epsilon_{max}$, $\beta(\cdot)$ and $w(\cdot)$ of each object of adjacent frames are approximately the same, $\zeta_{j,k}$ will be independent of frame $j$ and object $k$. Therefore, we observe that (6) is given by

$$n_{j,k} = n \frac{\sqrt{\sigma_{j,k}^2} JK}{\sum_{j'=1}^{J} \sum_{k'=1}^{K} \sqrt{\sigma_{j',k'}^2}}. \quad (7)$$

Although Eq. (7) is valid under certain assumptions, it provides a good approximation which captures the relationship between $n_{j,k}$ and $\sigma_{j,k}^2$. The assumptions imposed allow us to use this equation in practical algorithms. From (7), we observe given the average number of particles $n$ used over $J$ frames among $K$ objects, the number of particles $n_{j,k}$ allocated to the $k^{th}$ object in the $j^{th}$ frame is determined by the error variance $\sigma_{j,k}^2$. A object with a large error variance should be allocated more particles; whereas a object with a smaller error variance should be assigned fewer particles. For single object tracking, i.e. $K = 1$, (7) reduces to particle allocation among frames as in [1]. We finally obtain the optimal distortion of the $k^{th}$ object in the $j^{th}$ frame as $D_{j,k} = \frac{\sqrt{\sigma_{j,k}^2 \zeta_{j,k}}}{nJK} \sum_{j'=1}^{J} \sum_{k'=1}^{K} \sqrt{\sigma_{j',k'}^2 \zeta_{j',k'}}$.

### 3.1. Error Variance and Proposal Variance

In particle filtering, the particles are generally sampled using a sampling scheme given by $X_j^i = f(X_{j-1}^i) + v_j$, where

$f(X^i_{j-1})$ can be any estimation of the mean of the new sample and $v_j$ is given by a Gaussian distribution $v_j \sim \mathcal{N}(0, \mathbf{R}_G)$, where $\mathbf{R}_G = diag(\varphi_1^2, \varphi_2^2, \ldots, \varphi_N^2)$. The variance of the $l^{th}$ component of $v_j$ is called proposal variance $\varphi_l^2$. We observe that for each component of the tracker for each object in the $j^{th}$ frame

$$\sigma_l^2 = var(\epsilon_j^{i,l}) = var(S_j^l - X_j^{i,l}) = var(v_j^l) = \varphi_l^2, \quad (8)$$

for $l = 1, 2, \ldots, N$. Therefore, the tracking error variance $\sigma_l^2$ is equal to the proposal variance $\varphi_l^2$ used in particle filtering.

### 3.2. Dynamic Proposal Variance

In traditional implementations of the distributed multi-object tracking, the proposal variance $\varphi_{j,k}^2$ is fixed for all objects in all frames, i.e. $\varphi_{j,k}^2 = \varphi^2$. From (7), we observe that in this case the optimal number of particles $n_{j,k}$ is uniform for all objects in all frames, i.e. $n_{j,k} = n$. The proposal variance is selected manually prior to tracking for different video sequences. However, the current method of using a fixed proposal variance for all objects and all frames fails to exploit the different characteristics of each object in each frame to improve the sampling scheme. For example, we may wish to sample fast moving objects using a proposal density function with a larger variance. We therefore introduce the dynamic proposal variance given by

$$v_{j,k} \sim \mathrm{N}(0, \mathbf{R}_{\Delta \widehat{X}_{j,k}}). \quad (9)$$

This scheme implies that the variance of the proposal density $\varphi_{j,k}^2$ is changing with the estimated object motion $\Delta \widehat{X}_{j,k}$. The optimal number of particles $n_{j,k}$ will be allocated to each object in each frame according to the proposal variance.

We will rely on the motion vectors to determine the variance of the position components in the proposal density function. Given the motion vector $(\Delta x, \Delta y)$, the variance should ensure that the position of the object in the next frame lies in the search region of the current frame. Therefore, we obtain

$$\varphi_x^2 = c\sqrt{2}\Delta x, \varphi_y^2 = c\sqrt{2}\Delta y, \quad (10)$$

where $c$ is a constant and $(x, y)$ is the center of the object. In practice, we let $c = 1 \sim 2$ for the sampling scheme $X^i_{j,k} = X^i_{j-1,k} + v_{j,k}$, and we choose $c = 0.1 \sim 0.2$ for the sampling scheme $X^i_{j,k} = X^i_{j-1,k} + \Delta \widehat{X}_{j,k} + v_{j,k}$. We could also adjust the variance of other dimensions of the proposal density, e.g. zooming and rotation, based on the principles presented above.

### 3.3. Optimal Particle Allocation (OPA) Algorithm

Let us assume that we process $J$ frames at a time. When the time elapsed during $J$ frames is only a small fraction of a second, we can consider the proposed approach for real-time tracking systems. The dynamic proposal variance and optimal particle allocation (OPA) algorithm for multiple object tracking is illustrated as:

1. Use block matching (or any other motion detection scheme) to estimate $\Delta \widehat{X}_{j,k}$ for each object in each frame. When there is occlusion, we use the motion just before occlusion as an estimation of current motion.
2. Choose the proposal variance $\varphi_{j,k}^2$ according to $\Delta \widehat{X}_{j,k}$.
3. Use (7) to determine the optimal particle number allocated to each object in each frame.
4. Do distributed multi-object tracking based on the number of particles allocated.
5. Repeat steps (1)-(4) for each group of $J$ frames throughout the entire video sequence.

## 4. EXPERIMENTAL RESULTS

In multi-object tracking, we are more interested in particle allocation among objects in each frame, i.e. $J = 1$. To demonstrate the improved performance of the proposed OPA algorithm, all of the comparative experiments are performed under the distributed multi-object tracking framework with the sampling scheme $X^i_{j,k} = X^i_{j-1,k} + v_{j,k}$ and $J = 1$. For the implementation details of the DMOT algorithm, we refer the reader to [2]. Each object is modelled by a four dimensional ellipse with different color for labelling. We use edge and color characteristics as cues for computing the local particle likelihood. We use magnetic and inertia weights to deal with occlusion. The least-square estimation of the motion vector is given by the average of the observed motion vectors.

We compared our approach with two other variance choosing and particle allocation algorithms on both synthetic and real video sequences: (a) Fixed Proposal Variance, Fixed Particle Allocation (FPV). This is the traditional implementation of DMOT. In practice, the variance is set before tracking to an arbitrary value. In the following experiments, we set the variance to the average value of the dynamic variances obtained by our method. (b) Dynamic Proposal Variance, Fixed Particle Allocation (DPV). This is a modification of FPV where the variance of the proposal density function is dynamically adjusted to reduce the distortion error. (c) Dynamic Proposal Variance, Optimal Particle Allocation (OPA). This is the proposed algorithm.

The synthetic sequence `Tennisball` has three tennis balls moving in a challenging clutter environment with a resolution of $320 \times 240$. At each time, only one tennis ball moves fast, the other two moves slowly. The average number of particles per object per frame $n$ is 10. Tracking results of the different algorithms are shown in Fig. 1. Our proposed OPA algorithm improves the tracking quality dramatically while requiring about the same CPU time (see Table 1).

The `Hall` video clip contains one girl and one boy, who walk alternatively. The resolution is $128 \times 96$ and the frame rate is 20 frames per second. The average number of particles per object per frame $n$ is 20. Fig. 2 illustrates the tracking results of the different algorithms. Our proposed method tracks well while others fail. Fig. 3 shows the actual number of particles each person used.

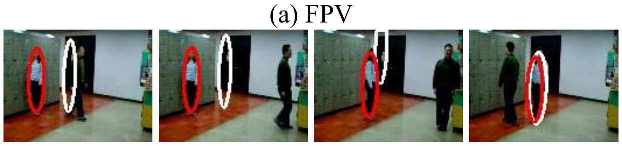
(a) FPV

(b) DPV

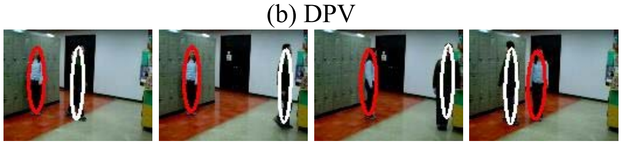
(c) OPA

**Fig. 1**. Tracking results of the synthetic video `Tennisball`: (a) FPV, (b) DPV and (c) OPA. (Each row depicts frames 64, 90, 98, 140.)

(a) FPV



(b) DPV



(c) OPA

**Fig. 2**. Tracking results of the real video `Hall`: (a) FPV, (b) DPV and (c) OPA. (Each row depicts frames 65, 90, 274, 419.)

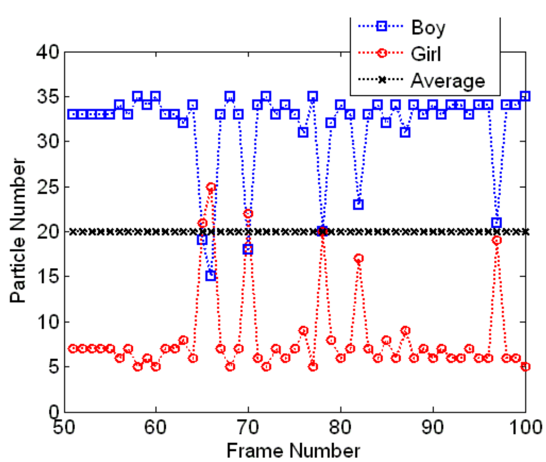



**Fig. 3**. Particle allocation for the `Hall` video sequence.

**Table 1**. Normalized CPU Time Per Frame On The `Tennisball` Sequence

| FPV | DPV | OPA |
|---|---|---|
| 1 | 1.0054 | 1.0054 |

We have implemented all of the algorithms independently in Matlab 7.0 without code optimization on a 2.8 GHz Pentium IV PC. Compared with FPV, the proposed OPA algorithm must spend some CPU time for variance choosing and particle allocation. However, since generating particles and weighting likelihoods are the main factors which impact the entire system's computational cost, the extra CPU time required by the proposed OPA algorithm is negligible. From Table 1, we can see the normalized CPU time required per frame of the three algorithms is about the same. The data has been averaged over 5 iterations on the `Tennisball` sequence.

## 5. CONCLUSIONS

In this paper, we presented a new approach for multi-object tracking which minimizes the total tracking distortion by simultaneously adjusting the proposal variance and the number of particles for each object in each frame based on the motion activity of the tracked object. We derived a theoretical framework based on rate distortion theory to determine the optimal number of particles allocated for each object in each frame. The motivation of our approach is to maximize the performance of particle filters in applications that have limited computational and power resources. The proposed algorithm can also be used in decentralized articulated object tracking. Our proposed optimal particle allocation (OPA) algorithm has the following advantages: (1) it can minimize the total tracking distortion while using the same number of particles; (2) given a fixed power, it can achieve the best tracking quality; (3) for the same tracking quality, it uses the least CPU time and power.

## 6. REFERENCES

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