MODELING OF FRONT EVOLUTION WITH GRAPH CUT OPTIMIZATION

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ABSTRACT

In this paper, we present a novel active contour model, in which the traditional gradient descent optimization is replaced by graph cut optimization. The basic idea is to first define an energy function according to curve evolution and then construct a graph with well selected edge weights based on the objective energy function, which is further optimized via graph cut algorithm. In this fashion, our model shares advantages of both level set method and graph cut algorithm, which are "topological" invariance, computational efficiency, and immunity to being stuck in the local minima. The model is validated on synthetic images, applied to two-class segmentation problem, and compared with the traditional active contour to demonstrate effectiveness of the technique. Finally, the method is applied to samples imaged with transmission electron microscopy that demonstrate complex textured patterns corresponding subcellular regions and micro-anatomy.

Index Terms— Image segmentation, Image color analysis, Image texture analysis

1. INTRODUCTION

Images corresponding to natural scenes and certain class of scientific data are often complex requiring methods for automated or semi-automated annotation for subsequent indexing, mining, and comparative analysis. In this paper, we'll couple the active contour models and graph cut optimization method in a complementary fashion to demonstrate a superior performance. One motivation for the development of the method is based on segmentation of the specimen imaged with transmission electron microscopy displaying complex textured patterns corresponding to organelle and various complexes. It is important to delineate and characterize these structures as a function of different experimental variables. Given the image complexities and required reliability, we have opted for a trainable system for partitioning an image into distinct regions.

The active contour model evolves a front toward the desired object boundary based on local and global shape constraints and forces that reside in the image. It is an application of Differential Geometry[11], first introduced as "snakes" within the Lagrangian framework and the "level set" within the Eulerian framework (e.g., implicit representation for active contours) [8, 12]. The level set formulation allows for control over topological changes such as merging or splitting of fronts. The "geodesic active contour model"[19] transformed the image segmentation problem into a geodesic computation in a Riemannian space, according to a metric induced by the image. These methods leverage the gradient information as a constraint for terminating the curve evolution; thus, segmenting the image into distinct regions. These techniques may often have undesirable effects of uncontrolled leakage as a result of perceptual boundaries. Chan and Vese [2] developed an active contour model without edges that solve for an equilibrium state for regions inside and outside of the front. Their method has been extended to textured images [14]. The main limitation is in the initialization, which may not lead to globally consistent labeling.

Graph cut was first proposed by Greig et al.[5] in the context of max-flow/min-cut algorithms (graph cut algorithms) in the context of combinatorial optimization for minimizing an energy function. Graph cut algorithm [21, 15, 7, 9] has emerged as an increasingly useful method for energy minimization in early vision, such as segmentation[13, 10, 20], restoration[21] and stereo reconstruction[17, 18]. The advantage of the graph cut approach is efficient optimization of the energy function. Its disadvantage is in generating noticeable geometric artifacts, known as metrication errors, as a result of the discrete topology of graphs.

The combination of active contour and graph cut was first proposed in[1], where the geodesic active contours and graph cuts were unified. The authors pointed out that with a large enough neighborhood system and specifically selected edge weights, the cost of the cuts on the image grid would approximate to the Euclidean length of the segmented object boundary.

In this paper, we propose an new active contour model which unifies Chan and Vese's active contour model[2] and graph cut algorithm. This model combines advantages of "topologically" free front evolution, globally optimization, and reduces the sensitivity to initialization. The rest of this paper is organized as follows: In 2, the Chan and Vese's model and the graph cut method is summarized. Section3, provides the details of our approach. Section4, effectiveness of the proposed method against the traditional level set approach is demonstrated. Additionally, we show the performance of our method on complex samples that are imaged with transmission electron microscope. Section5 concludes the paper.

2. RELATED WORK

2.1. Active Contour Model

The active contour models are widely used for image segmentation. In the Chan and Vese's model[2], the energy functional $F(c_1, c_2, C)$ is defined as

$$\begin{aligned} F(c_1,c_2,C) &= \mu \cdot Length(C) + v \cdot Area(inside(C)) \\ &+ \lambda_1 \int_{inside(C)} |u_0(x,y) - c_1|^2 dxdy \\ &+ \lambda_2 \int_{outside(C)} |u_0(x,y) - c_2|^2 dxdy \end{aligned} \tag{1}$$

where u_0 corresponds to the image, c_1 and c_2 are the mean foreground and mean background intensity at a specific iteration, and $\mu \geq 0$, $v \geq 0$, $\lambda_1, \lambda_2 \geq 0$ are fixed parameters. The level set formulation of this model is given by considering $C \subset \Omega$ as the zero level set of a Lipschitz function $\phi : \Omega \rightarrow \mathbb{R}$, in which Ω is a

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bounded open subset of $\mathbb{R}^2$. Using the Heaviside function H, and the one-dimensional Dirac measure δ_0, defined by

$$H(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0 \end{cases} \quad \delta_0(z) = \frac{dH(z)}{dz} \tag{2}$$

The curve evolution front, ϕ, can be written as:

$$\frac{\partial \phi}{\partial t} = \delta \left(\mu div(\frac{\nabla \phi}{\mid \nabla \phi \mid}) - v - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right) \tag{3}$$

The level set evolves based on gradient decent method, which makes the active contour models sensitive to initialization.

2.2. Graph Cut Method

Graph cut is a powerful tool for energy minimization. In the context of segmentation, it is a binary labeling approach based on the graph $G = \langle \bar{V}, \bar{E} \rangle$ constructed from the image, where $\bar{V}$ is the set of all nodes and $\bar{E}$ is the set of all arcs connecting adjacent nodes. Usually, the nodes are pixels in the image and arcs are adjacency relationships with four or eight connections between neighboring pixels. Additionally, there are special nodes, referred to as terminals, in the graph structure, where terminals correspond to the set of labels that can be assigned to pixels. In the case of a graph with two terminals, terminals are referred to as the source(S) and the sink(T). Then the labeling problem is to assign an unique label x_p (0 for background and 1 for foreground) for each node $p \in \bar{V}$ and the image cutout is performed by minimizing the Gibbs energy $E(X)$ [16]:

$$E = \sum_{p \in \bar{V}} E_1(x_p) + \sum_{(p,q) \in \bar{E}} E_2(x_p, x_q) \tag{4}$$

Where $E_1(x_p)$ is the likelihood energy, encoding the fitness cost for assigning x_p to p, and $E_2(x_p, x_q)$ is the prior energy, denoting the cost when the labels of adjacent nodes p and q are x_p and x_q respectively. Basically, the optimization algorithms could be classified into two groups: Goldberg-Tarjan style "push-relabel" methods[6] and Ford-Fulkerson style "augmenting paths" [4] . The details of the two methods could be found in[3].

3. APPROACH

We focus on the two-class segmentation problem to show that equation 4 can approximate evolution of a front represented as level set. Let C be the curve, $u^k(p)$ be the k^{th} feature, p_F^k be the probability function of the k^{th} feature of foreground and p_B^k be the probability function of the k^{th} feature of background. The energy function to be minimized is defined as follows:

$$\begin{aligned} E &= \mu \cdot Length(C) + v \cdot Area(inside(C)) \\ &- \sum_{k=1}^{N} \lambda_F^k \int_{Foreground} \log p_F^K(u^k(p))dp \\ &- \sum_{k=1}^{N} \lambda_B^k \int_{Background} \log p_B^K(u^k(p))dp \end{aligned} \tag{5}$$

in which, u,v,λ_F^k and λ_B^k are fixed parameters. Let $\phi(p) > 0$ if $p \in Foreground$, and $\phi(p) < 0$ if $p \in Background$. Then the above energy function can be formulated as,

$$\begin{aligned} E &= \mu \int_\Omega \mid \nabla H(\phi(p)) \mid dp + v \int_\Omega H(\phi(p))dp \\ &- \sum_{k=1}^{N} \lambda_F^k \int_\Omega \log p_F^K(u^k(p)) \cdot H(\phi(p))dp \\ &- \sum_{k=1}^{N} \lambda_B^k \int_\Omega \log p_B^K(u^k(p)) \cdot (1 - H(\phi(p)))dp \end{aligned} \tag{6}$$

The Euler-Lagrange equation for ϕ can be written as:

$$\partial_t \phi = \delta \cdot \left(\sum_{k=1}^{N} \log \frac{p_F^k(u^k)^{\lambda_F^k}}{p_B^k(u^k)^{\lambda_B^k}} + \mu div \frac{\nabla \phi}{\mid \nabla \phi \mid} - v \right) \tag{7}$$

However, gradient descent greatly relies on the initialization and cannot guarantee global convergence, but the graph cut method can. Assume that $x_p = H(\phi(p))$, so that $x_p \in \{0, 1\}$, $x_p = 1$, if $p \in Foreground$ and $x_p = 0$, otherwise. Based on a discrete graph grid, the objective energy function can be written as:

$$\begin{aligned} E &= \mu \cdot Length(C) + v \sum_p x_p \\ &- \sum_{k=1}^{N} \lambda_F^k \sum_p \log p_F^k(u^k(p)) \cdot x_p \\ &- \sum_{k=1}^{N} \lambda_B^k \sum_p \log p_B^k(u^k(p)) \cdot (1 - x_p) \end{aligned} \tag{8}$$

in which, p is treated as the node of the graph. To optimize the active contour model mentioned above via graph cut, another important aspect is to choose the n-link (link between nodes) of the graph model and to approximate the Euclidean length of $C(\mid C \mid_\epsilon)$. For an 8-connected neighborhood system, as shown in Figure1, [1] demonstrates that:

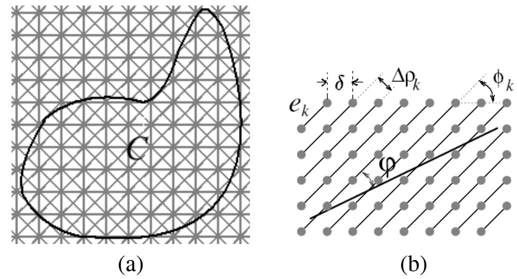

Fig. 1. (a)an 8-neighborhood configuration on a 2D grid, and (b) one family of lines on the grid.

$$\mid C \mid_\epsilon \approx \sum_k n_c(k) \cdot \frac{\delta^2 \cdot \triangle \phi_k}{2 \cdot \mid e_k \mid} \tag{9}$$

where $n_c(k)$ is the number of intersections of the curve C with the k^{th} family of edge-lines, δ is the cell-size of the grid, $\mid e_k \mid$ is the Euclidean length of vector e_k, and $\triangle \phi_k$ is the angular differences between the k^{th} and $(k+1)^{th}$ edge lines: $\triangle \phi_k = \phi_{k+1} - \phi_k$. It is clear that selection of constant edge weights within each family of edge lines as $w_k = \frac{\delta^2 \cdot \triangle \phi_k}{2 \cdot \mid e_k \mid}$ will produce a Euclidean length of C that can be approximated by the cut length in the graph grid. For

Edge	Weight	For
$p \to S$	$v - \sum_k \lambda_F^k \log p_F^k(u^k(p))$	$p \in G$
$p \to T$	$- \sum_k \lambda_B^k \log p_B^k(u^k(p))$	$p \in G$
$e_{\{p,q\}}$	$\frac{\pi}{8T}, T \in \{1, \sqrt{2}\}$	$\{p,q\} \in \mathbb{N}$

Table 1. Edge weights for the graph construction, G is the graph, $\mathbb{N}$ is the neighborhood system.

an 8-neighborhood system, the edge-weights of the graph shown in Table 1.

With the edge weights defined above, we construct a classical two-terminal graph, and apply the graph cut algorithm describe in[20] to solve the optimization problem.

4. EXPERIMENTAL RESULTS

Image	Size	Feature Type	T_1	T_2
Cat	350×209	Color	10.2s	2.3s
Texture	319×158	Texture	3.3s	2.7s
Zebras	481×321	Both	99.6s	80.8s

Table 2. Computational time for the traditional level set (T_1) and our method (T_2).

We have applied our approach to the two-class image segmentation problem, in which both color and texture features could be incorporated. The color features are represented in (RGB) domain, and texture features are extracted by gabor filter bank. Taking the user specified foreground and background samples as input, we establish four (or two, if only color or texture is considered) Gaussian Mixture Models (GMM) to model the distributions of foreground color, background color, foreground texture and background texture, which are represented as GMM_F^1, GMM_B^1, GMM_F^2 and GMM_B^2, respectively. In this way, we have

$$p_F^k = \frac{GMM_F^k}{GMM_F^k + GMM_B^k}; p_B^k = \frac{GMM_B^k}{GMM_F^k + GMM_B^k}; k = 1, 2$$

We consider each pixel in the image as a node and construct the graph according to Table 1.

Figure 2 shows labeling results on real and synthetic images and comparison of these results with the traditional level set method. It is clear that the proposed method reduces fragmentation. Furthermore, Table 2 indicates that the computational complexity of our method is comparable to the traditional level set method. This is because the traditional level set is iterative, while graph cut is not. Figure 3 indicates performance of the method on samples that are imaged with transmission electron microscopy. Note that the images are generally noisy and different components of the micro-anatomy have unique textures. Again, the system has enabled segmentation as a precursor for detailed morphological analysis.

5. CONCLUSION

In this paper, we have proposed a new active contour model which unifies Chan and Vese's model and the graph cut algorithm. In this way, our model shares advantages of both two standard segmentation approaches, which are "topologically" free, computational efficiency, and immunity to local minimum through the energy minimization approach. These advantages are ensured by intrinsic properties of level set method and graph cut algorithm and are further demonstrated by some comparisons of experimental results between our approach and the traditional level set method. Our future research will focus on the extension of this method to integration of multiphase level set with the graph cut optimization.

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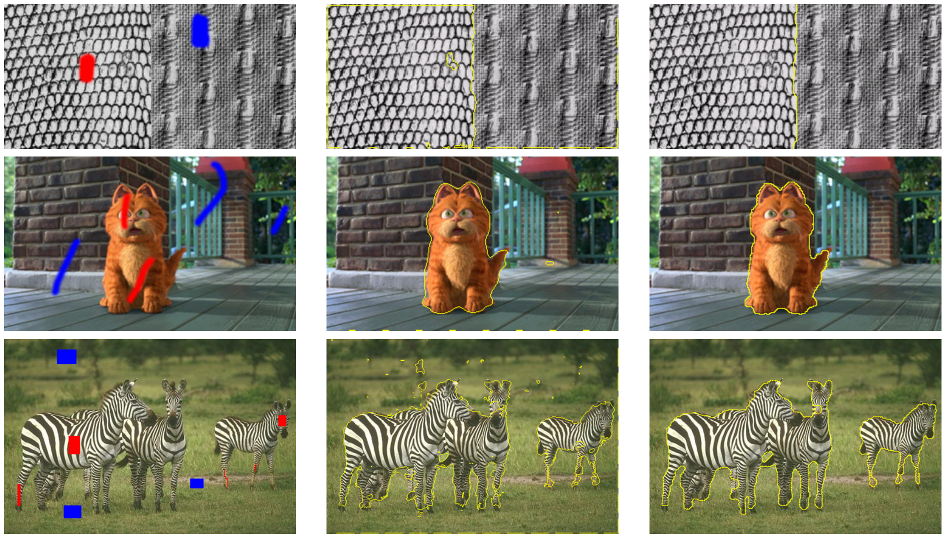
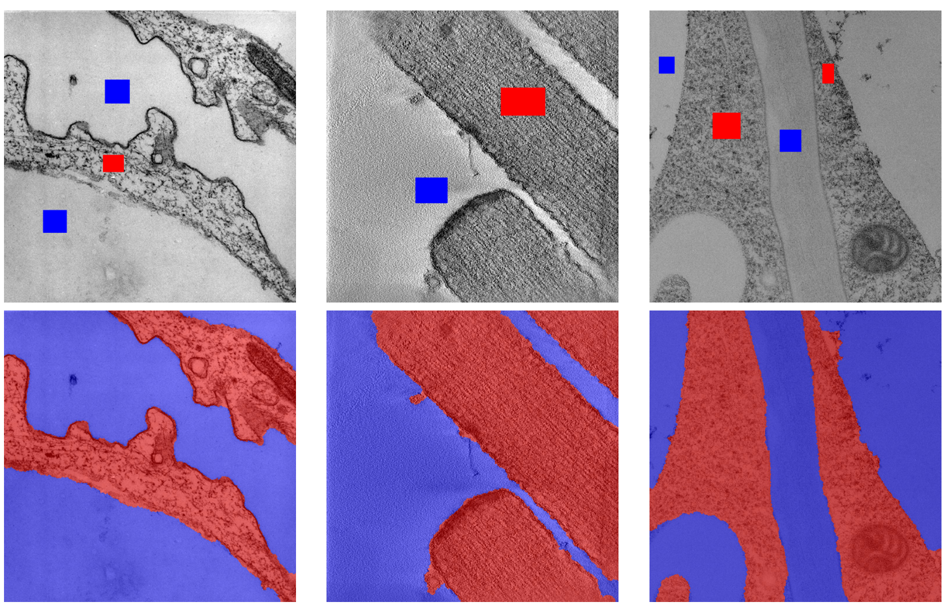


(a) (b) (c)

Fig. 2. Segmentation results for synthetic texture (top row), color animated image (middle row), and natural image with complex color and texture (bottom row): (a) original image with user specified seeds, (b) segmentation with the level set method, and (c) segmentation with the proposed method.



(a) (b) (c)

Fig. 3. Segmentation results for samples imaged through transmission electron microscopy with the trained regions (top row) and automated labeling (bottom row): (a) 70nm thick section through a zebrafish notochord, (b) 1nm thin slice through 3D tomographic volume of frog sensory epithelia hair bundle stereocilia, and (c) 70nm thick section of Arabidopsis hypocotyl tissue.