DEFORMABLE SHAPE PRIORS IN CHAN-VESE SEGMENTATION OF IMAGE SEQUENCES

Ketut Fundana, Niels Chr. Overgaard and Anders Heyden

Applied Mathematics Group
School of Technology and Society
Malmö University, Sweden
{ketut.fundana, nco, heyden}@ts.mah.se

ABSTRACT

In this paper we propose a new method for variational segmentation of image sequences containing nonrigid, moving objects. The method is based on the Chan-Vese model augmented with a novel frame-to-frame interaction term, which allow us to update the segmentation result from one image frame to the next using the previous segmentation result as a shape prior. The interaction term is constructed to be pose-invariant and to allow moderate deformations in shape. It can handle the appearance of occlusions which otherwise can make segmentation fail. The performance of the model is illustrated with experiments on synthetic and real image sequences.

***Index Terms*—** Segmentation, image sequences, variational active contours, shape priors, level set methods.

1. INTRODUCTION

We address the problem of segmentation in image sequences using region-based active contours and level set methods. Segmentation is a fundamental and difficult process in computer vision whose purpose is to divide a given image into one or several meaningful regions or objects. When applied to image sequences the process becomes more complex because the objects to be segmented may now be moving from frame to frame, change shape, and encounter various occluding objects along the way. This puts additional constraints on the segmentation process.

Segmentation models based on variational formulations of active contours have been applied successfully to many problems. Such methods may be either boundary-based, such as geodesic active contours [1], or region-based, such as Chan-Vese models [2], or combinations thereof. However, active contour-based methods may fail due to noise, clutter and occlusion. In order to make the segmentation process robust against these effects, it has been proposed to incorporate shape priors into the segmentation process. In recent years, many researchers have successfully introduced shape priors into segmentation methods such as in [3, 4, 5, 6, 7].

When segmenting nonrigid moving objects in image sequences, appropriate segmentation methods, which can deal with motion and shape deformations, should be used. The application of active contour methods for segmentation in image sequences gives promising results as in [8, 9]. These methods use variants of the classical Chan-Vese model as the basis for segmentation. In [8], for instance, it is proposed to simply use the result from one image as an initializer in the segmentation of the next.

In this paper we propose and analyze a novel variational segmentation method for image sequences, based on minimizing an energy functional containing the standard Chan-Vese functional, as one part, and an interaction term which penalizes the deviations from the previous shape, as a second part. The interaction term is defined using a transformed distance map to the previous contour, where different transformation groups, such as Euclidean, similarity or affine, can be used depending on the particular application.

2. THEORETICAL BACKGROUND

2.1. Region-Based Segmentation

We begin with a brief review of the classical Chan-Vese segmentation model [2]. A gray-scale image is considered to be a real valued function $I : D \to \mathbf{R}$ defined on the *image domain* $D \subset \mathbf{R}^2$, usually a rectangle. A point $\mathbf{x} \in D$ is often referred to as a pixel, and the function value $I = I(\mathbf{x})$ as the *pixel value*, or the *gray-scale value*. The Chan-Vese model is an active contour model in which the idea is to find a contour Γ (a finite union of disjoint, simple, closed curves) such that the image I is optimally approximated by a single gray-scale value μ_{int} on $\text{int}(\Gamma)$, the *inside* of Γ, and by another gray-scale value μ_{ext} on $\text{ext}(\Gamma)$, the *outside* of Γ. The optimal contour Γ^* and the corresponding pair of optimal gray-scale values $\boldsymbol{\mu}^* = (\mu^*_{\text{int}}, \mu^*_{\text{ext}})$ are defined as the solution of the variational problem,

$$E_{CV}(\boldsymbol{\mu}^*, \Gamma^*) = \min_{\boldsymbol{\mu},\Gamma} E_{CV}(\boldsymbol{\mu}, \Gamma), \tag{1}$$

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where E_{CV} is the well-known Chan-Vese functional,

$$E_{CV}(\boldsymbol{\mu},\Gamma)=\alpha|\Gamma|+\beta\Big\{\frac{1}{2}\int_{\mathrm{int}(\Gamma)}(I(\mathbf{x})-\mu_{\mathrm{int}})^2\,d\mathbf{x}+ \\ +\frac{1}{2}\int_{\mathrm{ext}(\Gamma)}(I(\mathbf{x})-\mu_{\mathrm{ext}})^2\,d\mathbf{x}\Big\}. \quad (2)$$

Here $|\Gamma|$ is the arc length of the contour, and $\alpha,\beta>0$ are weight parameters.

For any fixed contour Γ, it can be shown that the best choice of the gray-scale values $\boldsymbol{\mu}=(\mu_{\mathrm{int}},\mu_{\mathrm{ext}})$ are the mean values of the pixel values inside and the outside Γ, respectively:

$$\mu_{\mathrm{int}}=\mu_{\mathrm{int}}(\Gamma)=\frac{1}{|\mathrm{int}(\Gamma)|}\int_{\mathrm{int}(\Gamma)}I(\mathbf{x})\,d\mathbf{x}, \quad (3)$$

$$\mu_{\mathrm{ext}}=\mu_{\mathrm{ext}}(\Gamma)=\frac{1}{|\mathrm{ext}(\Gamma)|}\int_{\mathrm{ext}(\Gamma)}I(\mathbf{x})\,d\mathbf{x}. \quad (4)$$

Here the symbol $|.|$ denotes the area. If we introduce the so-called "reduced" Chan-Vese functional

$$E^R_{CV}(\Gamma):=E_{CV}(\boldsymbol{\mu}(\Gamma),\Gamma), \quad (5)$$

then the optimal contour Γ^* can be found by solving the simpler minimization problem

$$E^R_{CV}(\Gamma^*)=\min_{\Gamma}E^R_{CV}(\Gamma). \quad (6)$$

Once Γ^* is found we have $\boldsymbol{\mu}^*=\boldsymbol{\mu}(\Gamma^*)$. The minimization problem in (6) is solved using a gradient descent procedure in the level set framework, as described in the next section.

2.2. Gradient Descent using the Level Set Method

If a contour Γ is represented as the zero level set of a function $\phi:\mathbf{R}^2\to\mathbf{R}$ as $\Gamma=\{\mathbf{x}\in\mathbf{R}^2\,;\,\phi(\mathbf{x})=0\}$, then the sets $\mathrm{int}(\Gamma)=\{\mathbf{x}\,;\,\phi(\mathbf{x})<0\}$ and $\mathrm{ext}(\Gamma)=\{\mathbf{x}\,;\,\phi(\mathbf{x})\geq 0\}$ are the inside and the outside of Γ, respectively. Geometric quantities such as the outward unit normal $\mathbf{n}$ and the curvature κ can be expressed in terms of ϕ as $\mathbf{n}=\nabla\phi/|\nabla\phi|$ and $\kappa=\nabla\cdot\big(\nabla\phi/|\nabla\phi|\big)$. The function ϕ is usually called the *level set function* for Γ, cf. e.g. [10].

A curve evolution, i.e. a mapping $t\mapsto\Gamma(t)$, can be represented by a time dependent level set function $\phi:\mathbf{R}^2\times\mathbf{R}\to\mathbf{R}$ as $\Gamma(t)=\{\mathbf{x}\in\mathbf{R}^2\,;\,\phi(\mathbf{x},t)=0\}$. The normal velocity of $t\mapsto\Gamma(t)$ is the scalar function $d\Gamma/dt$ defined by

$$\frac{d}{dt}\Gamma(t)(\mathbf{x}):=-\frac{\partial\phi(\mathbf{x},t)/\partial t}{|\nabla\phi(\mathbf{x},t)|}\qquad(\mathbf{x}\in\Gamma(t)). \quad (7)$$

The gradient descent flow for the problem of minimizing a functional $E(\Gamma)$ is the solution to the initial value problem:

$$\frac{d}{dt}\Gamma(t)=-\nabla E(\Gamma(t)),\qquad\Gamma(0)=\Gamma_0, \quad (8)$$

where Γ_0 is an initial contour specified by the user. Here $\nabla E(\Gamma)$ is the so-called L^2-gradient (or *shape gradient*) of the energy functional $E(\Gamma)$, cf. e.g. [11] for definitions of these notions. In the case of E^R_{CV} the L^2-gradient is in (5) is

$$\nabla E^R_{CV}(\Gamma)=\alpha\kappa+\beta\big[\frac{1}{2}(I-\mu_{\mathrm{int}}(\Gamma))^2-\frac{1}{2}(I-\mu_{\mathrm{ext}}(\Gamma))^2\big]. \quad (9)$$

Combined with the definition of gradient descent evolutions (8) and the formula for the normal velocity (7) this gives the gradient descent procedure in the level set framework:

$$\frac{\partial\phi}{\partial t}=\Big(\alpha\kappa+\beta\big[\frac{1}{2}(I-\mu_{\mathrm{int}}(\Gamma))^2-\frac{1}{2}(I-\mu_{\mathrm{ext}}(\Gamma))^2\big]\Big)|\nabla\phi|,$$

where $\phi(\mathbf{x},0)=\phi_0(\mathbf{x})$ represents the initial contour Γ_0, and $\mu_{\mathrm{int}}(\Gamma)$ and $\mu_{\mathrm{ext}}(\Gamma)$ are given by (3) and (4), respectively.

3. SEGMENTATION OF IMAGE SEQUENCES

3.1. A Variational Updating-Model

In this section we are going to present the basic principles behind our variational model for updating segmentation results from one frame to the next in an image sequence.

Let $I_j:D\to\mathbf{R}$, $j=1,\dots,N$, be a succession of frames from a given image sequence. Also, for some integer k, $1\leq k\leq N$, suppose that all the frames $I_1,I_2,\dots,I_{k-1}$ have already been segmented, such that the corresponding contours $\Gamma_1,\Gamma_2,\dots,\Gamma_{k-1}$ are available. In order to take advantage of the prior knowledge obtained from earlier frames in the segmentation of I_k, we propose the following method: If $k=1$, i.e. if no previous frames have actually been segmented, then we just use the classical Chan-Vese model, as presented in Sect. 2. If $k>1$, then the segmentation of I_k is given by the contour Γ_k which minimizes an *augmented* Chan-Vese functional of the form,

$$E^A_{CV}(\Gamma_{k-1},\Gamma):=E^R_{CV}(\Gamma)+\gamma E_I(\Gamma_{k-1},\Gamma), \quad (10)$$

where E^R_{CV} is the reduced Chan-Vese functional defined in (5), $E_I=E_I(\Gamma_{k-1},\Gamma)$ is an *interaction term*, which penalizes deviations of the current active contour Γ from the previous one, Γ_{k-1}, and $\gamma>0$ is a coupling constant which determines the strength of the interaction. The precise definition of E_I is described below.

3.2. The Interaction Term

The interaction $E_I(\Gamma_0,\Gamma)$ between a fixed contour Γ_0 and an active contour Γ, used in (10), may be chosen in several different ways. Two common choices are the so-called pseudo-distances, cf. [5], and the area of the symmetric difference of the sets $\mathrm{int}(\Gamma)$ and $\mathrm{int}(\Gamma_0)$, cf. [3]. Here we propose a new pose-invariant interaction term.

To describe this interaction term, let $\phi_0 : D \to \mathbf{R}$ denote the *signed distance function* associated with the contour Γ_0, that is, the function:

$$\phi_0(\mathbf{x}) = \begin{cases} \operatorname{dist}(\mathbf{x}, \Gamma_0) & \text{for } \mathbf{x} \in \operatorname{ext}(\Gamma_0), \\ -\operatorname{dist}(\mathbf{x}, \Gamma_0) & \text{for } \mathbf{x} \in \operatorname{int}(\Gamma_0). \end{cases} \quad (11)$$

Then the interaction $E_I = E_I(\Gamma_0; \Gamma)$ is defined by the formula,

$$E_I(\Gamma_0, \Gamma) = \min_T \int_{\operatorname{int}(\Gamma)} \phi_0(T^{-1}\mathbf{x})\, d\mathbf{x}, \quad (12)$$

where the minimum is taken over the *group of Euclidean transformations* $T : \mathbf{R}^2 \to \mathbf{R}^2$ which preserves the orientation of the plane, that is, translations and rotations but not reflections. Minimizing over groups of transformations is the standard devise to obtain pose-invariant interactions, see [3] and [5].

For any given contour Γ, let $T = T(\Gamma)$ denote the transformation which minimizes the expression on the right hand side of (12). Since this is an optimization problem $T(\Gamma)$ can be found using gradient descent. For simplicity of presentation, suppose we only consider the group of translations $T_{\mathbf{a}} : \mathbf{x} \mapsto \mathbf{x} + \mathbf{a}$, $\mathbf{a} \in \mathbf{R}^2$, and want to determine the optimal translation vector $\mathbf{a} = \mathbf{a}(\Gamma)$. Then we have to solve the optimization problem

$$\min_{\mathbf{a}} \int_{\operatorname{int}(\Gamma)} \phi_0(\mathbf{x} - \mathbf{a})\, d\mathbf{x}.$$

The optimal translation $\mathbf{a}(\Gamma)$ can then be obtained as the limit, as time t tends to infinity, of the solution to the initial value problem

$$\dot{\mathbf{a}}(t) = \int_{\operatorname{int}(\Gamma)} \nabla\phi_0(\mathbf{x} - \mathbf{a}(t))\, d\mathbf{x} \ , \qquad \mathbf{a}(0) = 0. \quad (13)$$

Similar gradient descent schemes can be devised for rotations and scalings (in the case of similarity transforms), cf. [3], but will not be written out explicitly here.

3.3. The Gradient Descent Equations

The augmented Chan-Vese functional (10) is minimized using standard gradient descent (8) described in Sect. 2 with ∇E equal to

$$\nabla E_{CV}^{A}(\Gamma_{k-1}, \Gamma) := \nabla E_{CV}^{R}(\Gamma) + \gamma \nabla E_I(\Gamma_{k-1}; \Gamma), \quad (14)$$

and the initial contour $\Gamma(0) = \Gamma_{k-1}$. Here ∇E_{CV}^{R} is the L^2-gradient (9) of the reduced Chan-Vese functional, and ∇E_I the L^2-gradient of the interaction term, which is given by the formula,

$$\nabla E_I(\Gamma_{k-1}, \Gamma; \mathbf{x}) = \phi_{k-1}(T(\Gamma)\mathbf{x}), \qquad (\text{for } \mathbf{x} \in \Gamma), \quad (15)$$

see [12]. Here ϕ_{k-1} is the signed distance function for Γ_{k-1}.

4. EXPERIMENTS

In this section we present the results obtained from experiments using two different image sequences. We use the Chan-Vese model to segment a selected object with approximately uniform intensity and apply the proposed method frame-by-frame. First we compute the optimal translation vector (13) based on the previous contour, we then use this vector to translate the previous contour until it is aligned to the optimal position (15). Then the minimum of the functional (10) is obtained by the gradient descent procedure (14) implemented in the level set framework outline in Sect. 2. This procedure is iterated until it converges. See also [10].

The Chan-Vese method will have problems segmenting an object if occlusions appear in the image which cover the whole or parts of the selected object. In Fig 1, we show the segmentation results for a nonrigid object in a synthetic image sequence, where occlusions occur. The Chan-Vese method fails to segment the selected object when it reaches the occlusion (Top Row). Using the proposed method, we obtain much better results (Bottom Row).

Another experiment is given in Fig. 2, where a walking person is being segmented (available at http://homepages.inf.ed.ac.uk/rbf/CAVIAR/). Here the proposed method prevents the segmentation of the spurious objects, as is clearly shown.

In both experiments the coupling constant γ is varied to see the influence of the interaction term on the segmentation results. The contour is only slightly affected by the prior if γ is small. On the other hand, if γ is too large, the contour will be close to a similarity transformed version of the prior.

5. CONCLUSIONS

We have presented a new method for segmentation of non-rigid objects in image sequences. The proposed method is formulated as variational problem, with one part of the functional corresponding to the Chan-Vese model and another part corresponding to a pose-invariant interaction term as a shape prior based on the previous contour. The optimal transformation as well as the shape deformation are determined by minimization of an energy functional using a gradient descent scheme. Preliminary results are shown and its performance looks promising.

6. REFERENCES

[1] V. Caselles, R. Kimmel, and G. Sapiro, "Geodesic active contours," *International Journal of Computer Vision*, vol. 22, no. 1, pp. 61–79, 1997.

[2] T. Chan and L. Vese, "Active contour without edges," *IEEE Transactions on Image Processing*, vol. 10, no. 2, pp. 266–277, 2001.

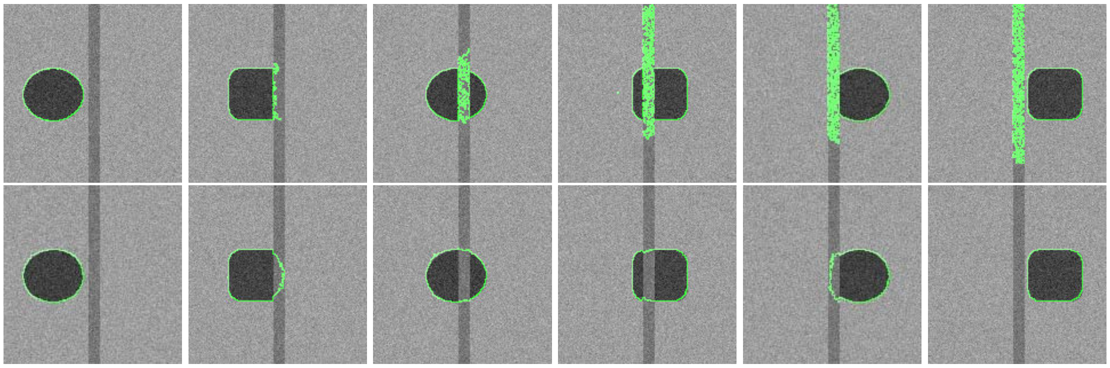

Fig. 1. Segmentation of a nonrigid object in a synthetic image sequences with additive Gaussian noise. Without the interaction term, noise in the occlusion is captured (Top Row). This is avoided when the interaction is included (Bottom Row).

Fig. 2. Segmentation of a person covered by an occlusion in the human walking sequence. Top Row: without interaction term, and Bottom Row: with interaction term

[3] T. Chan and W. Zhu, "Level set based prior segmentation," Technical Report 03-66, Department of Mathematics, UCLA, 2003.

[4] D. Cremers, N. Sochen, and C. Schnörr, "Towards recognition-based variational segmentation using shape priors and dynamic labeling," in *Scale-Space 2003*. 2003, LNCS 2695, pp. 388–400, Springer-Verlag.

[5] D. Cremers and S. Soatto, "A pseudo-distance for shape priors in level set segmentation," in *2nd IEEE Workshop on Variational, Geometric and Level Set Methods in Computer Vision*, 2003.

[6] M. Rousson and N. Paragios, "Shape priors for level set representations," in *ECCV 2002*, A. Heyden and et al, Eds. 2003, LNCS 2351, pp. 78–92, Springer-Verlag.

[7] M.E. Leventon, W.E.L. Grimson, and O. Faugeras, "Statistical shape influence in geodesic active contours," in *CVPR*, 2000.

[8] M. Moelich and T. Chan, "Tracking objects with the chan-vese algorithm," Technical Report 03-14, Department of Mathematics, UCLA, March 2003.

[9] N. Paragios and R. Deriche, "Geodesic active regions and level set methods for motion estimation and tracking," *Computer Vision and Image Understanding*, vol. 97, pp. 259–282, 2005.

[10] S. Osher and R. Fedkiw, *Level Set Methods and Dynamic Implicit Surfaces*, Springer-Verlag, 2003.

[11] J. E. Solem and N. C. Overgaard, "A geometric formulation of gradient descent for variational problems with moving surfaces," in *Scale-Space 2005*. 2005, LNCS 3459, pp. 419–430, Springer-Verlag.

[12] K. Fundana, N.C. Overgaard, and A. Heyden, "Variational segmentation of image sequences using deformable shape priors," in *SCIA 2007*. 2007, LNCS 4522, pp. 31–40, Springer-Verlag.