IMAGE DENOISING WITH DIRECTIONAL BASES

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ABSTRACT

Directional information is an important component of both natural and synthetic images, and it is exploited in many image processing applications. Directional basis analysis is used to capture significant structural information. This paper presents an empirical study of image denoising with directional bases. We consider two distinct approaches. One involves the Multi-resolution Fourier Transform (MFT) facilitated with a multi-directional selective filter. The other is based on statistics, Independent Component Analysis (ICA) that adaptively decomposes an image into a set of directional bases. We then present a combined approach that benefits from the computational efficiency of the MFT and the data adaptiveness of ICA. Experimental results are compared with those from other recent directional transforms such as the Curvelet and Directional cosine transform.

1. INTRODUCTION

Removal of noise from noisy images to obtain the unknown original image is often referred to as *denoising*. Gaussian additive white noise has a frequency spectrum that is continuous and uniform over a specified frequency band. It is spatially uncorrelated, and the noise for each pixel is independent and identically distributed (iid).

$$\bar{f} = f + \xi \tag{1}$$

where $\bar{f}$ is the noisy image, f is the original image and ξ is i.i.d. noise. Images are assumed to be linear-shift invariant, and linear methods such as the Wiener filter and the Kalman filter are often employed for denoising. Linear denoising methods are simple and inexpensive to implement, however they tend to blur the edge structure of the image, structure that is very important to the human visual system. The Markov random field, as well as various extensions, has been utilised to model the contextual information embedded in image formation. Partial differential equation (PDE)-based techniques have also attracted much attention recently, in which image details are preserved by adding an edge detection term. The single value decomposition method decomposes the column space of the observation matrix into a dominant and a subordinate part, revealing which of its subspaces can be attributed to the noise-free signal and which can be attributed to the noise. It is often assumed that these two subspaces are orthogonal to each other, which implies that signal and noise are independent. In a similar manner, a blind source separation or the Independent Component Analysis (ICA) decomposes signals assuming the following relation between components [1].

$$P(A \wedge B) = P(A) \cdot P(B) \tag{2}$$

where A and B are the independent components of signal. Given sufficiently large number of components, a few of the components can be pure noise components.

The wavelet transform can decompose the original signal into a smooth part (lowpass) and a detailed part (highpass). For most signals, energy is mainly distributed in the smooth subband, and energy in the detail subband is clustered to a few large wavelet coefficients, corresponding to the edge structure of the original signal. Donoho and his colleagues [2] pioneered a wavelet denoising scheme by using soft thresholding and hard thresholding. This approach, with the orthonormal wavelet, thresholds the wavelet transform coefficients within the detail subband. It is well known that Donoho's method offers the advantages of smoothness and adaptation. However, as Coifman and Donoho pointed out, this algorithm exhibits visual artefacts: Gibbs phenomena in the neighbourhood of discontinuities. However, the fundamental limitation of the orthonormal wavelet transform is the limited directional subband regardless of scale. In response, Starck and colleagues [3] proposed the Curvelet transform that extracts directional features in multi-scale using the Ridgelet. Later, other similar transforms followed such as the Contourlet which also consists of directional filter banks [4]. These transforms have proven effective in denoising. Compared with these new techniques, a much older technique with inherent directional feature recognition can achieve similar performance with a directional filter, that is the Multiresolution Fourier Transform (MFT). We present the ICA-MFT combined algorithm as well as the MFT with a multi-directional filter.

The paper is organised as follows. The next section starts with an introduction to ICA in the context of the denoising task. In section 3, denoising using the MFT with a Gaussian mask is explored and a combined approach is described in section 4. Section 5 introduces the MFT facilitated with a multi-directional filter based on Radon analysis. Section 6 reports an experimental comparison with other state-of-the-art techniques [3].

2. ICA BASED APPROACH

Independent Component Analysis (ICA) has been frequently applied to computational neuroscience and the modelling of simple and complex cells in the human primary visual cortex (V1) [1, 5] which is responsible for directional feature identification. Recently, an ICA based denoising method has been developed by Hyvarinen and his colleagues [6]. The basic motivation behind this method is that the ICA components of many signals are often very sparse so one can remove noise in the ICA domain.

$$x = As \tag{3}$$

where x, A and s are the observed data, a linear mixing matrix and the source (latent) data respectively, which are independent and nongaussian. Unlike the wavelet-based denoising methods, an ICA based method uses a representation that is estimated solely by the statistical properties of the available data. The estimation of the ICA data model can be reduced to the search for uncorrelated directions in which the components are as nongaussian as possible and as a result the independent components have a sparse (supergaussian) distribution as possible. Hyvarinen developed a sparse code based noise shrinkage method similar to the wavelet shrinkage method [6].

$$x = As + v \tag{4}$$

where v is a Gaussian noise vector and x the noisy signal. An approximate version of s, $\hat{s}$ can be obtained, applying shrinkage on $\hat{A}^{-1}x$. The components, $\hat{A}^{-1}x$ (neuron from a physiological viewpoint) with small activities are assumed as noise and shrunken, retaining only a few components with large activities [6]. This, however differs in the following aspects. The shrinkage nonlinearities are estimated separately for each component, as opposed to a single fixed model in wavelet shrinkage. Also maximum-likelihood estimation is used in the nonlinearity estimation instead of minimax estimation. This method assumes training to estimate the orthogonal basis with noise-free data that has similar statistical properties. However, we attempted denoising without prior-training with empirically optimized settings.

3. MFT BASED APPROACH

The ability to capture the directional patterns which exist at various locations, scale and orientation is a recent research trend in the image processing community. For example, the curvelet [3] represents a curve as a superposition of functions of various lengths and width controlling orientation across various scales. Various implementations have been proposed such as the curvelet, contourlet, brushlet, etc. The same ability can be found in the Multi-resolution Fourier Transform (MFT) [7]. The MFT has been proposed as a combination of STFT and the wavelet. With the windowing function $g(t)$, the transform of a function $f \in L^2(R)$ at position u, frequency ξ and scale s is defined as below.

$$Mf(u, \xi, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(t) g(s(t-u)) e^{-i\xi t} dt \tag{5}$$

The Laplacian pyramid is used to decompose the image according to frequency which shows isotropic behavior. At each scale, the windowed Fourier transform is applied with the same window. The high frequency directional patterns can be observed in the Fourier local spectrum. This is where an elliptical shape of Gaussian filtering is suggested in [8] as follows.

$$G(x) = \frac{1}{2\pi} \left(\frac{-x^T C^{-1} x}{2} \right) \tag{6}$$

where C is a covariance matrix, which can be obtained from the inertia tensor of the spectrum. The frequency window is effectively concentrated on a narrow oriented band. Experimental denoising results are presented in section 6.

4. ICA-MFT COMBINED APPROACH

The methods in the previous two sections approach the denoising problem in a different way. The ICA-based method uses purely statistical properties of the available data, adapting to the data. As a result, it requires larger computation even without a training process as the specified number of bases increases. The second approach takes advantage of the Fourier spectrum that exhibits a directional energy pattern. The nonlinearity of the Gaussian function formed by the inertia tensor from the spectrum is used for soft-thesholding. It, however, is not sufficient to represent a multi-directional pattern with a single Gaussian model. A Gaussian mixture model could be employed [9], but estimation of the mixture model increases the computational burden and nonlinear estimation can suffer from the local minima problem. We combine the two methods, effectively achieving a semi-adaptive wave packet basis, in a way that an algorithm can be computationally efficient and also adaptive to data, in two steps.

1. Perform ICA on each subband of the MFT with a limited number of components specified, for instance a third as many as the sufficient number of bases. This ratio needs to be determined adaptively and accurately so that no component with a meaningful pattern is discarded, where the discarded components are assumed to represent pure noise. This reduces the computation significantly.

2. Apply a 2D Gaussian filter obtained from the Fourier spectrum of the bases found in step.1. As most of the components are localised in orientation as well as frequency, a much narrower and oriented Gaussian model that fits the data can be obtained from the spectrum.

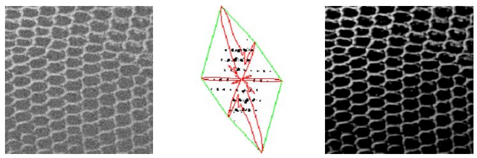

Fig. 1. MFT-Slice Denoising (noisy image, a multi-directional filter on spectrum and denoised image

5. MFT-SLICE APPROACH

The last approach benefits most from the fact that the ICA decomposes a block of a specific frequency band comprising multi-directional features into a set of bases localized in orientation (similar to ridgelet basis), so that the elliptical shape of the Gaussian filter obtained from the local spectrum fits the Fourier spectrum of the basis well. In this way, the inherent denoising capability of ICA discussed earlier combines with that of the MFT. Alternatively, however, we can replace ICA by providing a multi-directional filter using Fourier slice analysis. The Fourier slice analysis involves the computation of projection $r(\theta)$ for $0 \leq \theta < \pi$.

$$r(\theta) = c_i \cdot \Sigma\Sigma|F(x,y)|\delta(xcos\theta + ysin\theta) \qquad (7)$$

where c_i indicates a normalization constant at MFT scale i, and F is the Fourier spectrum. The shape S of the multi-directional filter consists of a set of points as below.

$$S = \{[r(\theta)cos\theta, r(\theta)sin\theta]^T\} \qquad (8)$$

The resulting contour represents the energy distribution of the significant directional pattern and is illustrated in Fig.1.The shape is used instead of the elliptical shape for the Gaussian filter for hard-thresholding, i.e. coefficients outside the shape are zeroed. The novelty of the approach is that instead of performing shrinkage on the transformed coefficients, we clean up the basis functions to allow a better reconstruction.

6. EXPERIMENTAL RESULTS

To illustrate the effectiveness of the proposed algorithms, comparative results with recent directional wavelet transforms are presented at various noise levels. The MFT was implemented on the Laplacian pyramid with 3 scale levels and a 50% overlapping cos^2 window of size 16×16. The MFT-Slice approach is facilitated with a window of size 32×32 for better directional analysis. We evaluated ICA, ICA-MFT and MFT-Slice against other directional basis transforms: Curvelet [3], Directional Cosine Transform (DDCT) [8], MFT--Gaussian Filtering (MFT) [8] and the Translation Invariant Wavelet Packet (TIWP). The test images `Lena` and `Jaguar` are shown in Fig.2 and Fig.3 with comparative results on a noisy image of SNR 15dB. `Lena` has a region of fur/feathers on her hat that creates multi-directional patterns, while the rest of the image is either homogeneous or directional. Clearly most of the methods with directional bases preserve the texture while TIWP produces quite a blurred image. The MFT,

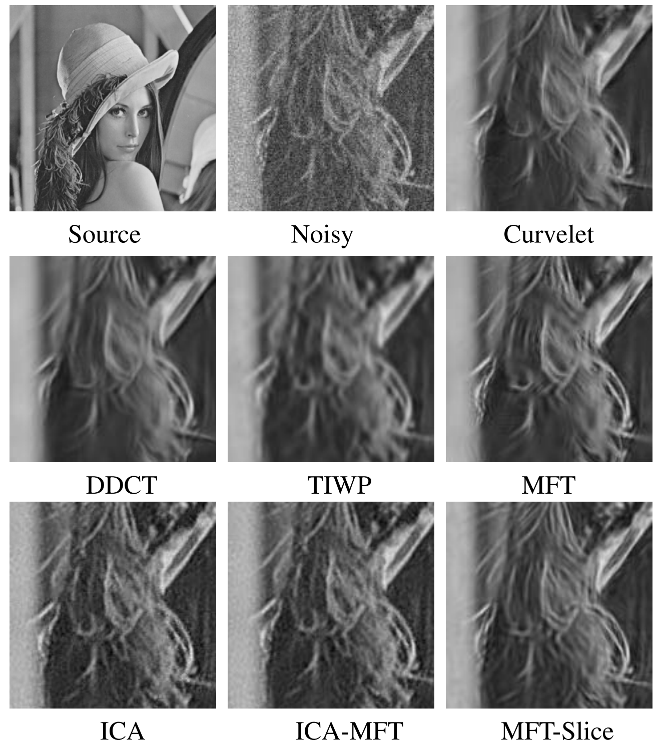

Fig. 2. Comparative evaluation : `Lena`

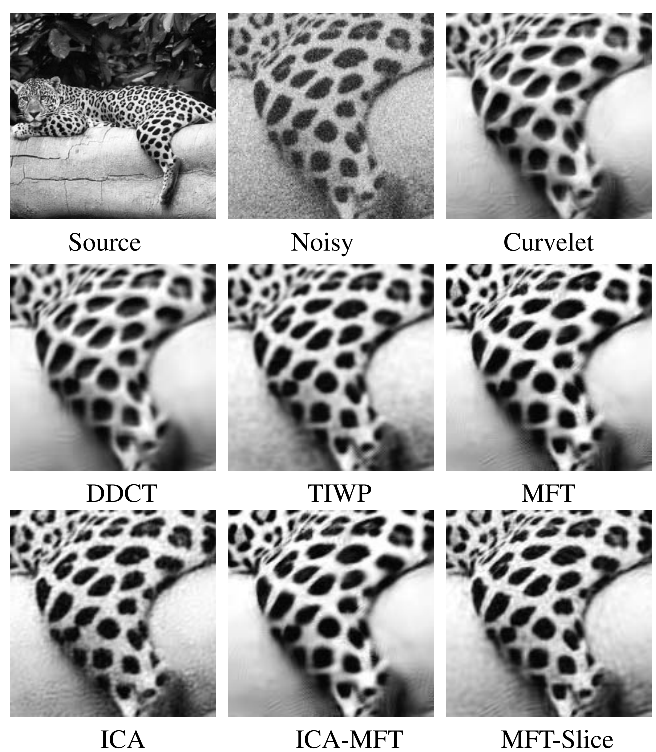

Fig. 3. Comparative evaluation : `Jaguar`

Table 1. SNR Results for different noise levels

	Noise(dB)	Curvelet	DDCT	TIWP	MFT	ICA	ICA-MFT	MFT-Slice
`lena`	5	17.55	17.44	18.91	18.97	18.74	**19.50**	19.43
	10	18.67	18.56	21.05	21.61	19.51	21.32	**21.82**
	15	19.40	19.37	23.49	24.42	21.48	22.65	**24.55**
	20	19.83	19.80	26.21	26.80	22.98	23.01	**27.67**
`jaguar`	5	12.19	12.18	15.41	**15.45**	12.24	12.66	13.36
	10	12.88	12.84	**18.11**	17.90	12.66	13.71	17.72
	15	13.24	13.26	20.94	19.89	13.28	15.23	**21.04**
	20	13.43	13.42	**23.85**	21.07	14.38	14.59	23.54

ICA-MFT and MFT-Slice show better results. `Jaguar` features a blob pattern on the Jaguar skin. The Curvelet and DDCT suffer from Gibb's phenomenon while both ICA-based approaches show rather blurred images. As the blob texture of the Jaguar requires various directional bases for reconstruction, most of the allocated bases of ICA are exhausted for orientation, and this leaves no basis to hold noise. The MFT and MFT-Slice generally show better results with much lower computational cost than the other methods. The MFT-Slice preserves the multi-directional pattern and removes noise better than the MFT as shown in both figures. The results of experiments at various noise levels are presented numerically in Table.1 (whole image). In `Lena`, MFT-Slice shows good SNR results and preserves edge structures as shown in Fig.2 and Fig.3. In `Jaguar`, despite TIWP providing good SNR results , the edge structures are quite blurred.

7. CONCLUSION

In this paper, we have briefly introduced two combined approaches to image denoising involving directional information. The results compare well with other proposed directional wavelet bases. It should be noted that the approach of combining different analysis methods is applicable not only to the problem of noise removal, but also it provides a new avenue to directional image analysis and numerical harmonic analysis as a whole. The MFT, originally proposed as a general image analysis tool has been around for more than a decade, with successful applications in feature extraction, motion estimation and texture analysis. By employing the idea of directional frequency filtering, the MFT finds a connection with the recent curvelet transform but with much lower complexity. The previously proposed MFT based anisotropic image denoising is limited fundamentally by its single-direction feature hypothesis, which assumes there is only one feature present in a local window. We tackled this limitation by two possible solutions. One is to use a source separation method, ICA, to decompose the signal into adaptive bases, in which the basis functions are expected to be a single directional component which can be dealt with effectively by the original MFT directional filter. The second is to introduce a multi-directional filter in the local Fourier spectrum by performing a Fourier slice integration of the magnitudes. This results in an adaptively shaped frequency mask which allows multiple components without prior knowledge of the number of components. The denoising experiments presented are intended as an example to show the power of the combined analysis although further improvement is possible by a frequency mask consisting on 1D Gaussian on every fourier slice instead of the hard-thresholding mask. On-going research is focused on employing the combined approach in other vision-related tasks such as segmentation and coding.

8. REFERENCES

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