DENOISING VIA NONLINEAR IMAGE DECOMPOSITION FOR A DIGITAL COLOR CAMERA

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ABSTRACT

To remove signal-dependent noise of a digital camera, we present a denoising approach via nonlinear image-decomposition. In the approach, at the first decomposition stage, multiplicative image-decomposition is performed, and a noisy image is represented as a product of its two components so that its structural component corresponding to a cartoon image approximation may not be corrupted by the noise and its texture component may collect almost all the noise. At the successive denoising stage, the structural component is used instead of the unknown true signal value, to adapt the soft-thresholding-type denoising manipulation of the texture component to the signal dependency of the noise. At the final image-synthesis stage, one combines the separated structure component with the denoised texture component to reproduce a denoised image. The approach selectively removes the signal-dependent noise without not only blurring sharp edges but also destroying visually important textures.

***Index Terms*—** Image decomposition, variational problem, signal-dependent noise, soft-thresholding, adaptive denoising

1. INTRODUCTION

In a digital color camera, several factors cause signal-dependency of additive noise that contaminates its output images [1 ~ 3]. The variance of the signal-dependent noise not only depends on signal intensity, but also differs among the color channels. It is very difficult to remove the signal-dependent noise selectively. This paper presents a new denoising approach that removes the actual signal-dependent noise selectively from camera's output images.

As the denoising approach, there are two major successful approaches. The first approach is to process wavelet coefficients of the image [4 ~ 8]. The second is the variational approach [9 ~ 12]. Recently, some researchers have studied a related but somewhat different variational problem of nonlinear image-decomposition, and have proposed its variational models [13 ~ 17]. This image-decomposition problem is to decompose an image into its multiple components: a structural component, a texture component and a residual component. The structural component corresponds to large objects in the image, and the texture component corresponds to fine image details showing periodicity and oscillation. Among various nonlinear image-decomposition models [13 ~ 17], there are successful nonlinear variational models based on the TV, which is considered promising as a pre-process for the post denoising.

To solve the problem of removing the signal-dependent noise, this paper presents a new denoising approach via the nonlinear image-decomposition. At the first decomposition stage of this nonlinear decomposition-and-denoising approach, a noisy image is decomposed into its components so that the structural component corresponding to a cartoon image approximation may not be corrupted by the noise and the texture and the residual components may collect almost all the noise. As the type of the nonlinear image-decomposition, there are two possible types: additive decomposition and multiplicative decomposition. As for the nonlinear image-decomposition model described in detail later, the multiplicative decomposition is superior to the additive decomposition in that the residual component is much smaller [18], and hence this paper chooses the multiplicative decomposition. At the successive denoising stage, at each pixel, the structural component is utilized instead of the unknown true signal, to adapt the soft-thresholding-type denoising manipulation of the texture and the residual components to the signal-dependency of the noise. At the final image-synthesis stage, the structure component is combined with the denoised texture and residual components, and a denoised image is reproduced. The nonlinear decomposition-and-denoising approach selectively removes the signal-dependent noise without not only blurring sharp edges but also destroying visually important textures.

The nonlinear image-decomposition model, refereed to as the BV-L1 variational model [17], is considered proper for the pre-process of the nonlinear decomposition-and-denoising approach. This model decomposes an image into its component belonging to BV and its component in L1. The BV and the L1 components correspond to the geometrical structure and the structural texture, respectively. The residual is not necessarily small, and should not be neglected. The BV-L1 model is adequate particularly to the much noise case; in such a case, the signal-dependent noise looks like the salt-and-pepper noise, and it shows the property of the impulsive noise. In such a case, the BV-L1 model removes the noise from the BV component almost perfectly. On the other hand, the L1 component and the residual are contaminated with the noise, and the effects of the noise are selectively removed from the L1 component and the residual by utilizing the BV component.

2. SIGNAL DEPENDENCY OF NOISE IN A DIGITAL COLOR CAMERA

In a digital color camera, several factors cause signal-dependency of additive noise that contaminates its output images [1 ~ 3]. The variance of the signal-dependent noise not only depends on signal intensity, but also differs among the three color channels. The signal-dependent noise is modeled as the additive noise model:

$$F = S + 1 + w(S) \cdot N \qquad (1)$$

, F: Noisy observation $(1 \le F \le 256)$, S: Signal $(0 \le S \le 255)$,

N: Gaussian noise with zero mean unit variance,

w: Standard deviation of noise,

where the function w defines the signal-dependency of the additive noise and determines its standard deviation. The function w is different among the three color channels. Fig.1 shows typical examples of the measured noise's signal-dependency functions, w's, of a certain digital color camera, for the three primary color channels and the various ISO-sensitivity values. Fig.2(a) shows a noisy test color image produced by adding to the original noise-free color image of Fig.2(g) signal-dependent noise artificially generated according to the signal-dependent noise model of Fig.1. As shown in Fig.1 and Fig.2(a), the camera's noise is more noticeable in dark regions than in bright regions, and in the high ISO-sensitivity case the signal-dependent noise looks like the salt-and-pepper noise and shows the property of the impulsive noise.

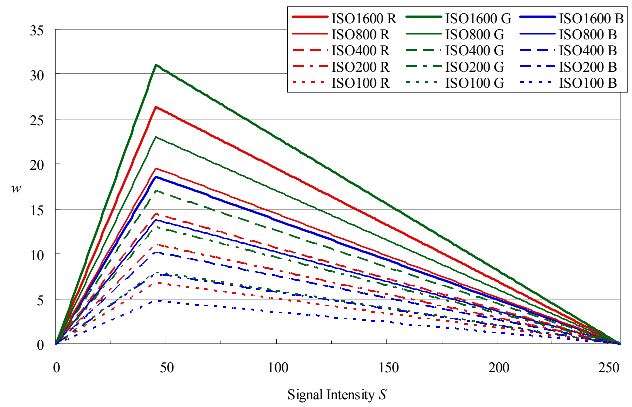

Fig.1. Measured noise's signal-dependency functions, w's, of a certain digital color camera for the three primary color channels and the various ISO-sensitivity values.

3. NONLINEAR DECOMPOSITION-AND-DENOISING APPROACH

The nonlinear decomposition-and-denoising approach selectively removes the signal-dependent noise of a digital color camera while preserving not only sharp edges but also visually important image textures. This chapter describes the concept and the outline of the approach. The approach is composed of the first nonlinear image-decomposition stage, the second nonlinear denoising stage, and the final image-synthesis stage.

3.1. Nonlinear Image-Decomposition Stage

At the first nonlinear image-decomposition stage, a noisy image is decomposed into its two components: a structure component and a texture component, so that the separated structural component corresponding to a cartoon approximation of the image may hardly be contaminated by the noise and the separated texture component may collect most of the noise. As for the nonlinear image-decomposition model, we employ the BV-L1 variational model [17], which is one of the most successful variational models.

As the type of the image-decomposition, there are two possible types: the additive decomposition and the multiplicative decomposition. The additive image-decomposition is defined by

$$F-1 \approx U+V \qquad (2)$$

$$, F: \text{Input image}\,(1 \le F \le 256),\; U: \text{Structural component},\; V: \text{Texture component},$$

whereas the multiplicative image-decomposition is defined by

$$F \approx U \times V \quad , \quad 1 \le U \le 256\,, \quad V > 0 \qquad (3)$$

The BV-L1 variational model is originally proposed for the additive decomposition, but it is applicable to the multiplicative decomposition. With the log transformation, the problem of the multiplicative decomposition is converted into the problem of the additive decomposition, as follows:

$$f \approx u+v \quad ; \; f=\log F\,,\; u=\log U\,,\; v=\log V\,. \qquad (4)$$

The multiplicative decomposition problem is easily solved in the log domain, with the BV-L1 model. The residual D in the additive decomposition is defined by F - U - V - 1, whereas the residual D in the multiplicative decomposition is defined by F - $U \cdot V$. As for the BV-L1 model, the multiplicative decomposition is superior to the additive decomposition in that the residual D is much smaller, and hence this paper employs the multiplicative decomposition. However, in the BV-L1 model the residual D is not necessarily small, and the denoising manipulation is applied to D.

To decompose a color image, the BV-L1 variational model is independently applied to each primary color channel. For a noise-free color image, its separated log texture components v's of the three color channels show high inter-channel cross-correlations, whereas for a noisy color image its separated log texture components v's show low inter-channel cross-correlations, and particularly in heavily noisy image regions local inter-channel cross-correlations show negative values. The nonlinear denoising stage utilizes this property.

3.1.1. Multiplicative BV-L1 Image-Decomposition Model

This image-decomposition model decomposes the log input f into the log structural geometrical component u and the log structural texture component v and the residual D = exp(f) - exp(u+v). To solve this image-decomposition problem, this model makes some assumptions about the two components u, v, as follows: 1) the log structural geometrical component u lives in the BV space, and 2) the log structural texture component v lives in the L1 space. For the impulsive noise, its TV norm is much larger than its L1 norm, and hence not only the structural textures but also the impulsive noise is collected in v, and the impulsive noise is almost completely removed from u; the BV-L1 model is fairly robust against the impulsive noise. The BV-L1 model [17] is formulated as the following variational problem of decomposing the log input f into the two components u, v.

$$\inf_{u\in \mathrm{BV},\, v\in \mathrm{L1}} \left\{ J(u) + \frac{1}{2\alpha}\cdot \|f-u-v\|_{L^2}^2 + \lambda \cdot \|v\|_{L^1} \right\},\; \alpha>0\,,\; \lambda>0\,. \qquad (5)$$

In the following, u and v are referred to as the BV component and the L1 component, respectively. In addition, the residual D is referred to as the L2 component. In this model, the L2 component D is not necessarily small, and hence should be taken into account. To solve (5), the decomposition algorithm is constructed as the alternate iterative algorithm with the Chambolle's projection algorithm [12] and the soft-thresholding. The algorithm gives the unique minimizer of (5).

3.2. Nonlinear Denoising Stage

3.2.1. Adaptive Soft-Thresholding for the Log Texture Component

The separated log texture component v of each primary color channel is separately manipulated. In addition, for each pixel, the separated structural component U is utilized instead of the unknown true signal value *S*, to adapt the denoising manipulation of *v* to the signal dependency of the noise, and thus the signal-dependent noise is selectively removed from it.

In the separated log texture components of the three color channels, the noise causes small variations, and those variations are uncorrelated among the three color channels. To utilize these properties, we employ an adaptive soft-thresholding technique where for each pixel the soft-threshold is adaptively controlled according to the separated structural component and the locally estimated inter-channel cross-correlation of the log texture components.

The adaptive soft-thresholding technique used for denoising *v* is constructed as described below. If an original noise-free signal *S* is smoothly varying, the multiplicative image-decomposition of a noisy input *F* will be approximately estimated as follows:

$$F = S + 1 + w(S)\cdot N = (S+1)\cdot\left(1 + N\cdot w(S)/(S+1)\right) \approx U\cdot V \quad (6)$$
$$;\quad U \approx S+1,\quad V \approx 1 + N\cdot w(S)/(S+1).$$

In the log domain, the separated log texture component *v* is estimated approximately by

$$v = \log(V) \;\approx\; \log\left(1 + N\cdot w(U-1)/U\right). \qquad (7)$$

In this case, *v* stems from the noise, and is expected to lie within the following range:

$$\log\left(1 - c\cdot w(U-1)/U\right) \le v \le \log\left(1 + c\cdot w(U-1)/U\right),\ \exists\, c > 0. \quad (8)$$

Taking it into account, this paper constructs the adaptive soft-thresholding in the log domain, as follows:

$$v' = \mathrm{AST}^{(1)}\left(v;\tau^{(\pm)}\right) \qquad (9)$$

$$,\ \mathrm{AST}^{(1)}\left(v;\tau^{(\pm)}\right)_{(i,j)} = \begin{cases} v_{i,j} - \tau^{(+)}_{i,j} &, \textit{if}\ v_{i,j} > \tau^{(+)}_{i,j} > 0 \\ 0 &, \textit{if}\ \tau^{(-)}_{i,j} \le v_{i,j} \le \tau^{(+)}_{i,j} \\ v_{i,j} - \tau^{(-)}_{i,j} &, \textit{if}\ v_{i,j} < \tau^{(-)}_{i,j} < 0 \end{cases}$$

$$\tau^{(\pm)}_{i,j} = \begin{cases} \log\left(1 \pm c_{i,j}\cdot w\left(U_{i,j}-1\right)/U_{i,j}\right) &, \textit{if}\ c_{i,j} < U_{i,j}/w\left(U_{i,j}-1\right) \\ \pm\infty &, \textit{if}\ c_{i,j} \ge U_{i,j}/w\left(U_{i,j}-1\right) \end{cases}$$

$$c_{i,j} = c_0\cdot\left(corv_{i,j} - 1.0\right)^2,\qquad c_0 > 0,$$

$corv_{i,j}$: Locally-estimated inter-channel correlation coefficient of the log texture components $v's$.

where *corv* is defined as the minimum value among the three cross-correlation coefficients, locally estimated within a 3 by 3 window for each pair of two primary colors. The multiplier parameter *c* is adaptively controlled according to *corv*. As *corv* gets smaller, the magnitude of the soft-threshold is increased; thus in noisy image regions, the noise's effects are much removed from *v*.

3.2.2. Adaptive Soft-Thresholding for the Residual

The adaptive soft-thresholding technique used for denoising of the residual *D* is constructed as described below. If an original noise-free signal *S* is smoothly varying, the residual *D* will stem from the noise, and it will be approximately estimated to be proportional to the noise:

$$D = F - U\cdot V \;\propto\; w(S)\cdot N. \qquad (10)$$

Hence, *D* is expected to lie within the following range:

$$-r\cdot w(U-1) \le D \le r\cdot w(U-1),\ \exists\, r > 0. \qquad (11)$$

Taking it into account, this paper constructs the adaptive soft-thresholding, as follows:

$$D' = \mathrm{AST}^{(2)}(D;\tau) \qquad (12)$$

$$,\ \mathrm{AST}^{(2)}(D;\tau)_{(i,j)} = \begin{cases} \mathrm{sign}\left(D_{i,j}\right)\cdot\left(\left|D_{i,j}\right| - \tau_{i,j}\right) &, \textit{if}\ \left|D_{i,j}\right| > \tau_{i,j} \\ 0 &, \textit{if}\ \left|D_{i,j}\right| \le \tau_{i,j} \end{cases}$$

$$\tau_{i,j} = r_{i,j}\cdot w\left(U_{i,j}-1\right) \ge 0,\quad r_{i,j} = r_0\cdot\left(corD_{i,j} - 1.0\right)^2,\quad r_0 > 0,$$

$corD_{i,j}$: Locally-estimated inter-channel correlation coefficient of the residuals $D's$.

3.2.3. Post-processing

The adaptive soft-thresholding techniques sometimes produce isolated non-zero points in its denoised log texture *v*' and the denoised residual *D*'. These isolated non-zero points are often perceived as visible granular artifacts in a denoised image. As the post-process of the adaptive soft-thresholding, such isolated non-zero points are removed from *v*' and *D*'.

3.3. Image-Synthesis Stage

At the final stage, the structure component *U* is combined with the denoised log texture component *v*' and the denoised residual *D*', and thus a denoised image is reproduced. The image-synthesis is performed as follows:

$$F' = U \times \exp(v') + D'. \qquad (13)$$

4. EXPERIMENTAL SIMULATIONS

Fig.2 shows the result of the denoising simulation. The noisy test image *F* of Fig.2(a) is produced by adding artificial signal-dependent noise equivalent to the ISO 1600 sensitivity to the original noise-free image *S* of Fig.2(g), and is denoised by the nonlinear decomposition-and-denoising approach with the multiplicative BV-L1 decomposition model. The separated BV component *U* of Fig.2(b) is not corrupted by the noise, and most of the noise is concentrated in the separated L1 component *V* of Fig.2(c). The residual *D* of Fig.2(e) is also contaminated by the noise somewhat. In *V* and *D*, the noise appears as colored noise. As shown in Fig.2(d) and Fig.2(f), the adaptive soft-thresholding and the post-process remove the noise from *V* and *D*. This denoising approach removes the noise considerably, and reproduces the denoised image *F*' of Fig.2(h), where sharp color edges are not much blurred and visually important major image textures are preserved to some extent. The peak SNR's of the primary three color channels are improved by about 6.5 ~ 7.5 [dB].

5. CONCLUSIONS

To remove signal-dependent noise of a digital camera, this paper presents a nonlinear decomposition-and-denoising approach with the multiplicative BV-L1 nonlinear image-decomposition [17]. The approach, at the first stage, decomposes a noisy image into its structural component and its texture component, and then at the second stage utilizes the separated structural component to adapt the soft-thresholding-type denoising manipulation of the texture component and the residual to the signal dependency of the noise, and at the final stage combines the separated structure component with the denoised texture component and the denoised residual, thus to reproduce a denoised image. In the high ISO-sensitivity case, the approach, particularly, performs very successfully, and selectively removes signal-dependent noise without blurring sharp edges and eliminating visually important textures.

6. REFERENCES

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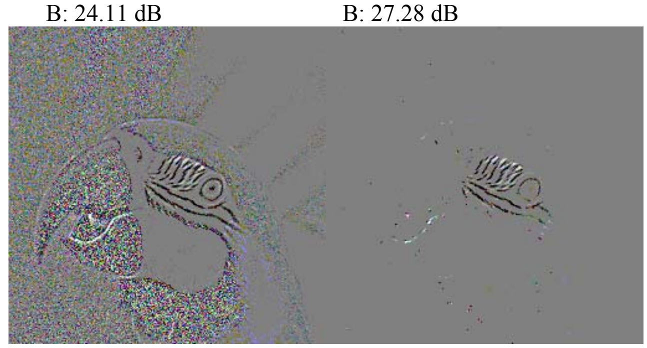
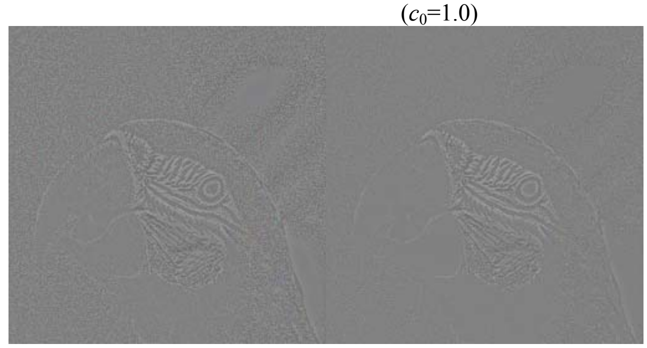
(a) Noisy test image F-1 R: 21.62 dB, G: 20.82 dB, B: 24.11 dB

(b) Separated BV component U-1 R: 27.09 dB, G: 26.45 dB, B: 27.28 dB



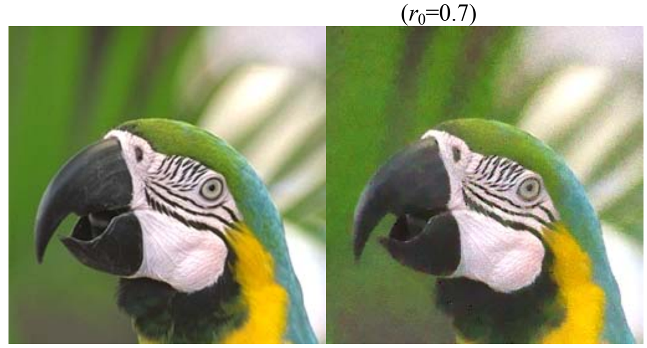
(c) Separated L1 component V

(d) Denoised L1 component V’ (c_0=1.0)



(e) Residual D

(f) Denoised residual D’ (r_0=0.7)



(g) Original noise-free image S

(h) Denoised image F’-1 R: 29.14 dB, G: 27.85 dB, B: 30.58 dB

Fig.2. Denoising result of the noisy test image of the ISO 1600 sensitivity by the nonlinear decomposition-and-denoising approach with the multiplicative BV-L1 image-decomposition model.