A NEW NONLINEAR DIFFUSION METHOD TO IMPROVE IMAGE QUALITY

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ABSTRACT

We propose a nonlinear diffusion method based on the gradient vector field construction to remove noises in image while preserving fine details. The blocky effect and over-smoothing, as usually seen in images processed by diffusion operators, are greatly reduced by our method. Results obtained from various images, including synthetic and magnetic resonance imaging (MRI), are used to demonstrate the performance of our new method. Comparing it with other diffusion methods, we find it obtains better performance in terms of removing noises without destroying detail features of images.

Index Terms— Noise removal, diffusion, PDE

1. INTRODUCTION

Nonlinear diffusion methods have proven to be very useful in many applications of image processings, from enhancing medical images [1] to improving image analyses [2, 3]. Denote the observed image by u, a diffusion term in the form of $g(|\nabla u|^2)$ is often used with g being a nonnegative function. The nonlinear diffusion scheme of Perona and Malik [4] has been used extensively in multiscale description of images, image segmentations, edge detections, and image enhancements. Their method can keep the edges relatively unchanged and its edge detector outperforms the linear Canny edge detector. The work was then extended to 3D images by Gerig et al. [1] and to vector-valued images by Sapiro and Ringach [5] to process 3D magnetic resonance imaging (MRI) data and multispectral MRI images. The Perona-Malik method, though effective in removing noise, has the drawback of creating blocky effects [6]. This artifacts, in fact, are due to the trade-off between noise removal and edge preservation embedded in the method. Other methods have been proposed to address this problem, one of them was Weickert [7], in which directional diffusion methods were shown to generate better results in enhancing line-like structures. Other techniques such as the multiscale anisotropic diffusion equation [8] have also been developed to circumvent this problem. Rudin-Osher-Fatemi [9] developed another nonlinear diffusion method by minimizing the total variation (TV) of the images under certain conditions. This approach introduces nonlinear diffusion filters and has been used as a regularization method for many other applications where one seeks to identify discontinuous functions [10, 11].

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2. NONLINEAR DIFFUSION AND GRADIENT VECTOR FIELD CONSTRUCTION

2.1. Nonlinear Diffusion

The basic idea behind these diffusion methods originated from a well known physical heat transfer process which equilibrates concentration differences without creating or destroying mass. This process can be modeled by partial differential equations, and their solutions describe the heat transfer at any particular time. Let the image domain be an open rectangle $\Omega = (0, a_1) \times (0, a_2)$, $\Gamma \equiv \partial\Omega$ be its boundary and the observed image $I(x)$ be represented by a bounded function $I : \Omega \longrightarrow \mathbb{R}$. Then an evoluting version $u(x, t)$ of $I(x)$ with a scale time parameter $t \geq 0$ is obtained as the solution of the following diffusion equations

$$\begin{array}{rcl} u_t & = & div \cdot (D(\nabla u)\nabla u) \\ u(x,0) & = & I(x) \\ < D(\nabla u)\nabla u, n > & = & 0 \end{array} \tag{1}$$

where I is the initial condition under the reflecting boundary conditions. In particular, the classical Perona and Malik diffusion equation [4] is

$$u_t = div \cdot (g(|\nabla u|)\nabla u), \tag{2}$$

$$u(x, t = 0) = I. \tag{3}$$

Here the diffusion coefficient $g(\cdot)$ (also called *flux term*) is a nonnegative function of the magnitude of the image gradient $\nabla u = \sqrt{u_x^2 + u_y^2}$ and generally set to be

$$g(|\nabla u|^2) = \frac{1}{\sqrt{1 + |\nabla u|^2/\lambda^2}} \quad (4)$$

where λ is a trade-off term that can be set depending on the estimation of the noise level. In (4), $|\nabla u|$ acts like a fuzzy edge detector since pixels that have large $|\nabla u|$ values are more likely to be an edge. The role of $g(|\nabla u|)$ is to adaptively control the smoothing effect. In order to avoid destroying the fine details of the image, this adaptive smoothing control must achieve two goals, (1) smooth homogeneous areas selectively, i.e., less smoothings in areas with strong image features such as edges and boundaries, and more smoothings in the other areas; and (2) preserve edges by controlling the direction of smoothing, i.e., minimal smoothings in the direction across the image features, and maximal smoothings in the direction along the image features.

In fact, we can see how the Perona-Malik diffusion accomplishes the above requirements if the smoothing equation is expressed in a new $[\eta, \xi]$ coordinates. Let η be the direction of the gradient and ξ be the direction perpendicular to the gradient, i.e., the direction of a level set. Assume that $g(|\nabla u|)$ has the form of (4). Then the evolution equation (2) can be expressed in terms of $u_{\eta\eta}$ and $u_{\xi\xi}$ as

$$u_t = \frac{1}{\sqrt{1 + (\frac{|\nabla u|^2}{\lambda})^2}} \left(\frac{1}{1 + (\frac{|\nabla u|^2}{\lambda})^2} u_{\eta\eta} + u_{\xi\xi} \right) \quad (5)$$

where $u_{\eta\eta}$ and $u_{\xi\xi}$ denote the second order derivative of u with respect to η and ξ, respectively. This demonstrates how the Perona-Malik equation accomplishes the two goals: (1) when $|\nabla u|$ is large, the first term $\frac{1}{\sqrt{1+\left(\frac{|\nabla u|^2}{\lambda}\right)^2}}$ allows less smoothings, and when $|\nabla u|$ is small, there is more smoothings. Thus it adaptively controls the degree of smoothings; (2) two factors are assigned to the two different smoothing directions, namely, η and ξ, to adaptively smooth the image in the direction along and across edges.

2.2. Gradient Vector Field Construction

In this paper, we propose a nonlinear diffusion method that has better performance in the sense that it can reduce noises and blocking effects while still preserve the edges and boundaries. The method is motivated by the gradient vector field (GVF) mechanism introduced in [12], which is proposed as a method to increase the capture range of the standard snake ("active contour") method and allow the snake to deform to convexity. GVF snake achieves the goals by introducing a new external force $\mathbf{v}$ based on the gradient vector and obtained from the following equations based on Euler equations

$$\begin{aligned} u_t &= \mu \nabla^2 u - (f_x^2 + f_y^2)(u - f_x) \\ v_t &= \mu \nabla^2 v - (f_x^2 + f_y^2)(v - f_y), \end{aligned} \quad (6)$$

where f is the edge force, which usually depends on $|\nabla I|^2$. Note that the first term in (6), $\nabla^2 u$, has an isotropic smoothing effect on the field $\mathbf{v}$, which is desirable for homogeneous areas, but not for edges and boundaries as they may be over-smoothed. The second term in (6) forces the field $\mathbf{v}$ to be close to the edge map, ∇f, in order to preserve edges. Because the gradient information from the object boundaries is propagated throughout the image by the diffusion process, the GVF model has a much larger "capture range".

3. EDGE ENHANCED NONLINEAR DIFFUSION

We now compare our diffusion method with that of the Perona-Malik (PM) diffusion. It is known that the PM diffusion process could create blocky effects in the resultant images. Under the framework of regularized nonlinear diffusion, our method aims to remove the noises and preserve edges, while reducing the blocky effects. Let $u(x, y)$ be the true noise-free image and $I(x, y)$ be the observed image with noise $n(x, y)$ for $x, y \in \Omega$

$$I(x, y) = u(x, y) + n(x, y). \quad (7)$$

For simplicity, we assume that the variance of noise is known as $\sigma^2 \approx \int_\Omega (u - I)^2 d\Omega$. In practice we can estimate the variance of the noise by some methods [13]. When the region of image under consideration is smooth, then $|\nabla f|^2$ is small and (6) can be written as

$$\mathbf{u}_t \approx \mu \nabla^2 \mathbf{u}, \quad \mathbf{v}_t \approx \mu \nabla^2 \mathbf{v}. \quad (8a)$$

For a large $|\nabla f|^2$ usually found around the edges, (6) becomes approximately

$$\mathbf{u}_t \approx (f_x, f_y), \quad \mathbf{v}_t \approx (f_x, f_y). \quad (8b)$$

Such construction achieves the goal of adaptively adjusting the weight of smoothings depending on the image features. In our method, we have an additional edge preserving term,

$$\begin{aligned} u_t &= \mu \mathrm{div} \cdot (g(|\nabla G_{\sigma_0} * u|)\nabla u) - |\nabla u|^2 (u - I), \\ u(x, y, t = 0) &= I \end{aligned} \quad (9)$$

where G is a 2D Gaussian kernel such that

$$G_{\sigma_0}(x, y) = C\sigma_0^{-1} \exp(-(x^2 + y^2)/4\sigma_0) \quad (10)$$

and μ of (9) represents the tradeoff between smoothings and edge preservation. By replacing the gradient $|\nabla u|$ in (2) by $|\nabla G_{\sigma_0} * u|$, we avoid instability and inconsistence [14]. This diffusion process achieves two objectives: (1) when it is near an edge, the second term will play a dominant role and thus adaptively preserves the edges. At the same time, the diffusion coefficient $g(\cdot)$ is also used to reduce smoothness and enhance edges when the edges are nearby; (2) when it is in a homogeneous region it will reduce the noise and smooth the image. Furthermore, the force term $(u - I)$ ensures the algorithm have a nontrivial steady state, therefore eliminating the need to choose a stop time. From this observation we can go on updating the value of μ dynamically to reach a steady state. By multiplying (9) by $(u - I)$ and integrating by parts over Ω, we then have

$$\mu = \frac{|\nabla u|^2 \sigma^2}{\int g(|\nabla G_{\sigma_0} * u|) \nabla u \mathrm{div} \cdot (u - I) d\Omega}. \tag{11}$$

4. EXPERIMENT RESULTS

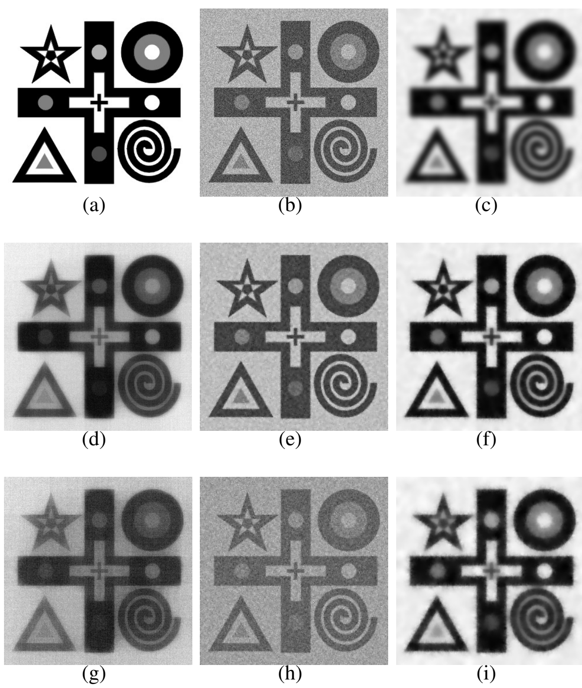

Fig. 1. (a) Original image, (b) degraded by additive Gaussian noise (SNR = 8.14), (c-f) results of the Perona-Malik method, the ROF method, the FPDE method, and our method, respectively. Visually, our method removes much of the noises without blurring the edges. (g-i) are the results of the ROF method, the FPDE method, and our method on the highly degraded image (a), (SNR = 3.73).

Our diffusion equations (9) are solved numerically using an iterative approach. We employed the efficient AOS scheme to calculate the nonlinear diffused image and then updated the process by adding the edge preserving term. We summarize the steps of our method in Table 1 where α is a predetermined threshold to stop the iteration. We apply our method to a synthetic image,shown in Fig. 1(a), which is degraded by additive Gaussian noise, Fig. 1(b). The signal to noise ratio (SNR) is 8.14 dB. The noisy image is processed by the Perona-Malik method, ROF, FPDE, and our method. The results are shown in Fig. 1(c-f), respectively. The differences between our edge enhanced diffusion and the other three are fairly clear. Our method preserves and enhances the edges while efficiently reducing the noise and avoiding the blocky effects. In particular, the two small circles inside the cross have sharper boundaries and are closer to true image in our method than images derived from the others. Similar improvement is also observed from the triangle at the lower left corner. Moreover, when we decrease the SNR, our method show obvious improvements over the other methods. Fig. 1(g-i) are the diffused results obtained on degraded images with SNR of 3.73db by using ROF, FPDE, and our method. They further demonstrates that our method is more robust to noises and adaptive to image structures.

Table 1. Pseudo code of our proposed algorithm

For $i = 1$ to maximum iteration number,

1. Discrete ∇u_i as:

   $$[\frac{u_i(m+1,n) - u_i(m,n)}{\Delta x}, \frac{u_i(m,n+1) - u_i(m,n)}{\Delta y}]$$

   and the diffusivity coefficients $g(|\nabla G_{\sigma_0} * u_i|)$
2. Update

   $$\mu = \frac{|\nabla u_i|^2 \sigma^2}{\int g(|\nabla G_{\sigma_0} * u|) \nabla u_i div \cdot (u_i - I) d\Omega}$$
3. Update the nonlinear diffused image by AOS method

   $$u_{i+1} = \mathrm{AOS}(u_i, g)$$
4. Update the image as $u_{i+1} = \mu u_{i+1} - |\nabla u_i|^2 (u_i - I)$
5. STOP if $\|u_i - u_{i+1}\| < \alpha$
6. $i = i + 1$.

As a second example, we apply the proposed method on a rat brain MRI image, Fig. 2 The raw image, Fig. 2(a), was obtained from a 4.7T small animal MRI scanner. We first use a contrast-limited adaptive histogram equalization (CLAHE) to enhance image contrasts for better visual comparison. The raw image is corrupted by noises and contains complex structures. Fig. 2(b) and (c) show the results given by the Perona-Malik method and the FPDE approach. Though the image af-

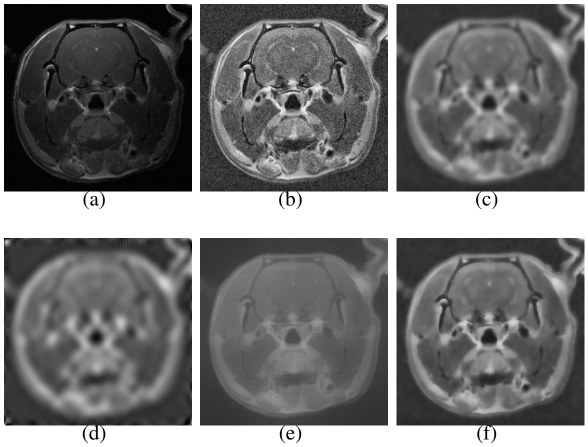

Fig. 2. MRI of rat brain (a) original, (b) image after contrast enhancement by CLAHE, (c) the Perona-Malik diffusion method, (d) the FPDE method, (e) the ROF method, and (f) our method.

ter the FPDE method is doing well in the central part, it leaves small cluster points in other areas. The ROF method produces a clean image as shown in Fig. 2(d). However, it does not generate clear structure information and has a reduced contrast. In comparison, our approach produces a result with reduced noise and well-preserved fine structure, Fig. 2(e).

5. CONCLUSION

We developed an edge enhancing noise removal diffusion method in this paper. Our method achieves good results in removing image noise while preserving and enhancing image details such as edges and contours without smoothing out important structure information and is less sensitive to noise level. By comparing our method with the Perona-Malik method and related methods, an fourth-order partial different equation diffusion, and the well-know regularized TV method by Rudin-Osher-Fatemi, we found the new method demonstrate better performance on both test images and real MR images.

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