Cyclic Filter Bank Implementations of Symmetric Extension for Subband/Wavelet Image Compression

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Abstract – Symmetric extension is typically employed in subband/wavelet image coders to improve compression performance, particularly at the image boundaries. This paper introduces improvements to the symmetric extension filter bank. The filter bank implementation employs a cyclic frequency domain representation and is able to accommodate IIR approximations that effectively have perfect stopband suppression. Enhancements are also introduced at the multirate system level. The new implementation offers greater flexibility in choice of filters and is shown to result in better compression performance.

I. Introduction

In this paper, we present an implementation of the symmetric extension method [1][2] for subband/wavelet image compression that results in improved performance. The new implementation employs cyclic filter banks, which were considered in [4]. These filter banks provide a sampled frequency domain representation for analysis and synthesis and can be designed to have perfect reconstruction. Within this framework, we develop cyclic frequency domain realizations of time-domain symmetric extension for image compression. In the past, not all the possible time-domain symmetric extensions have been implemented using the cyclic filter bank. And the necessary conditions for implementing time-domain symmetric extension in the frequency domain using cyclic filter bank have not been carefully studied.

In this work, the analysis and synthesis filters employed are still linear phase filters, such as the 9/7 filters [7], but are implemented in the cyclic frequency domain. The new framework accommodates both FIR filters and IIR approximations in a natural way, all with perfect reconstruction. Moreover, this framework represents a generalization of the symmetric extension method [1][2] in that it can accommodate IIR filters with both rational and irrational transfer functions—a convenient property not present in other formulations—that allows one to realize a variety of useful wavelet packet decompositions. Under the constraint of symmetric extension, these filters can obtain perfect stopband attenuation with exact reconstruction and, in wavelet vernacular, represent infinite support orthogonal bases.

In addition, we present enhancements to the design from a system level. In particular, we employ spectral reversal correction and a *transitional band normalization* approach to designing the constituent filters of the symmetric extension wavelet packet transform. The compression performance of the new method is evaluated using the SPIHT algorithm [7] and is shown to outperform the popular decompositions based on the biorthogonal 9/7 filters and the 28/28 filters [8], as well as the 8×40 GenLOT [10].

II. Cyclic Domain Symmetric Extension

Without loss of generality, a finite length signal can be treated as one period of an infinite duration periodic signal. If such a periodic signal $x(n)$ with period N is filtered by an arbitrary filter $h(n)$, then the output $y(n)$ is also periodic with period N. DFTs can be used to represent this convolution, that is

$$Y(k) = H(\omega)\Big|_{\omega=\frac{2\pi k}{N}} \cdot X(k), \qquad (1)$$

for k=0, 1, 2, …, N-1, where $X(k)$ and $Y(k)$ are the N-point DFTs of $x(n)$ and $y(n)$ respectively and $H(\omega)$ is the discrete-time Fourier transform of $h(n)$. Cyclic filter banks are based on this perspective. Further details may be found in [4]-[6]. Here we use cyclic filter banks as the framework for the symmetric extension method.

The symmetric extension method exploits symmetry in the analysis and synthesis filters in conjunction with reflecting the image about its boundary points. To the first point, lowpass symmetric filters can have even length (the Half Sample or HS form) or odd length (the Whole Sample or WS form). The corresponding highpass filters are HS antisymmetric and WS symmetric respectively. To the second point, there are several ways to create symmetry about the image boundary. The two basic extensions employed in the symmetric extension method are whole sample (WS) and half sample (HS) extensions [2][3]. For WS-extensions, a sequence (*a, b, c, d*) is extended about the last sample point to form (*a, b, c, d, c, b, a*). To realize an HS-extension, the last sample point is repeated. Thus for the example sequence above, the HS-extension would be (*a, b, c, d, d, c, b, a*).

To employ the symmetric extension method in a maximally decimated filter bank, sample symmetry must be present both after filtering and after downsampling. Careful attention should be paid to the nature of the signal symmetry, as it can change after downsampling. As a case in point, an HS-symmetric or antisymmetric signal is no longer symmetric after downsampling [2][3]. However, a WS-symmetric signal *is* symmetric after downsampling. In the next subsections, we take a closer look at the symmetry issues in the context of analysis-synthesis.

A. Whole Sample Symmetry

To start, consider the case of an even length signal. If we perform a WS-symmetric extension on both sides, then after filtering, the signal will still be WS-symmetric on both sides. After downsampling, one side will be WS-symmetric; the other side will be HS-symmetric. It is common that for multi-level decompositions, mixed symmetric WS-HS bands of this type will need to be successively split several times. And, since the same WS-symmetric filter is typically applied again for the subsequent decompositions, the HS-symmetric side of the signal needs to be reverted back to WS symmetry to ensure downsampling can be performed for the next split. Such a conversion is easy to do in the time domain. All that is required is to remove one sample. For example, if the sequence were (*a, b, c, c, b, a*), one removes the HS reflection sample to obtain (*a, b, c, b, a*). In the popular subband/wavelet coders, this change of symmetry operation (e.g. for 512×512 images using the 9/7 filters) is typically performed in between each successive level of decomposition.

In contrast to other symmetric extension implementations, here filtering is performed in the cyclic frequency domain via equation (1). By employing the properties of the DFT, downsampling can be performed directly on the filtered spectrum. However, in the case the size-limited change of symmetry mentioned above from WS to HS, direct conversion in the cyclic frequency domain is not computationally efficient. In fact, it is often easier to transform (IDFT) the sampled spectrum to the time domain first, modify the symmetry from HS to WS, and then transform back to the cyclic frequency domain, all of which is not attractive from a computation perspective. Thus, this type of change in symmetry is best avoided.

Symmetry conditions after decimation are different for odd length signals, in that no change of symmetry is needed. The geometry is such that after downsampling the resulting subband is WS-symmetric on both sides, and further decompositions can be performed easily in the same way all in the cyclic frequency domain. Thus, change of symmetry during multilevel decompositions can be avoided with a dyadic decomposition structure and by using odd-length signals and WS-symmetric filters. We hasten to point out that in this case the length of the high-frequency band is shorter than that of the low-frequency band by 1. Consequently for multilevel *uniform* band decompositions, the change of symmetry problem remains for the high-frequency bands.

B. Half Sample Symmetry

Even-length biorthogonal wavelet filters have HS symmetry. For convenient downsampling, we wish the output of the filtering to be WS-symmetric, as mentioned earlier. Thus, HS-symmetric extension on the input signal is required for HS-symmetric filters. In this case, the output of the lowpass HS-symmetric filter is WS-symmetric. The output of the corresponding highpass HS-antisymmetric filter is WS-antisymmetric.

The decimation structure for HS-symmetric & antisymmetric filters is different from that for WS-symmetric filters. It can be shown that as long as the original signal is of even length, after decimation, the low-frequency band is HS-symmetric and the high-frequency band is HS-antisymmetric. Both bands in this case are ready for further decomposition, in that no change of symmetry is required. Recall for WS-symmetric filters, there is at least one subband that needs a change of symmetry. This property makes HS-symmetric/antisymmetric filters more convenient than WS-symmetric filters when symmetric extension is performed in the cyclic frequency domain.

C. Spectral Reversal Correction

When a subband decomposition is performed, the spectrum of the high frequency subband is reversed. This is a consequence of the spectral aliasing associated with the downsampling operation. To achieve higher performance for wavelet packet symmetric extension (which we implement with HS symmetric filters on even length signals), we incorporate spectral reversal *correction* for the high frequency bands.

Using spectrum reversal correction, both low and high frequency subbands are HS-symmetric and are automatically positioned for the next level decomposition. The subband tree structure underlying the decomposition can be expressed as a cascade of one-level decompositions. That is, assume that $C_x^p(n)$ ($n = 0, 1, 2, ..., N-1$) are the x band wavelet coefficients at the pth ($p = 0, 1, 2, ...$) level, where N is even.

(1) $S(k) = \mathrm{DCT}\{C_x^p(n)\}$, $\quad k = 0, 1, ..., (N-1)$

(2) $\begin{cases} U_l(k) = |H(\pi k/N)|\, S(k) \\ U_h(k) = |G(\pi k/N)|\, S(k) \end{cases}$, $\quad k = 0, 1, ..., (N-1)$

where $H(\omega)$ and $G(\omega)$ are the discrete-time Fourier transforms of the lowpass and highpass filters respectively, and $U_l(k)$ and $U_h(k)$ are the output magnitude spectra from the lowpass and highpass filters respectively.

(3) $\begin{cases} S_l(k) = U_l(k) - U_l(N-k) \\ S_h(k) = \alpha_k [U_h(N/2+k) + U_h(N/2-k)] \end{cases}$,

where $\alpha_k = \begin{cases} 1/\sqrt{2} & k = 0 \\ 1 & k \neq 0 \end{cases}$.

Observe that $U_l(N) = U_h(N) = 0$.

(4) $\begin{cases} C_{xl}^{p+1}(n) = \mathrm{IDCT}\{S_l(k)\} \\ C_{xh}^{p+1}(n) = \mathrm{IDCT}\{S_h(k)\} \end{cases}$, $\quad k = 0, 1, ..., (\frac{N}{2}-1)$

$C_x^p(n)$ is decomposed into $C_{xl}^{p+1}(n)$ and $C_{xh}^{p+1}(n)$. It turns out that when cascading, the IDCT in step 4 and the DCT in step 1 for the next level decomposition cancel out, which reduces complexity. The process for reconstructing from subbands is a straightforward dual operation and thus is omitted here.

D. The Effect from Change of Symmetry

Recall that if the decomposition structure is not dyadic, change of symmetry is unavoidable for WS-symmetric filters. For HS-symmetric filters such symmetry changes are not needed. Therefore, computationally WS-symmetric filters are not particularly efficient for realizing symmetric extension in the cyclic frequency domain. Interestingly, it is possible to convert WS-symmetric filters to HS-symmetric filters with identical magnitude frequency responses. The only issue is that the resulting HS-symmetric filters become IIR. Fortunately, this does not pose a problem in our case because we implement the cyclic frequency domain representation of the filter.

This conversion has been tested on the 9/7 filters for uniform the subband decomposition structure. By conversion of the 9/7 filters to the HS form, the change of symmetry is avoided (both converted and unconverted 9/7 filters have exactly the same frequency response magnitude). Our experimental results show that the converted 9/7 filters lead to higher compression performance than the originals.

III. Condensed Wavelet Packet Symmetric Extension

The proposed cyclic frequency domain formulation of symmetric extension provides greater flexibility in constructing linear-phase orthogonal wavelet packet decompositions. The lowpass and highpass analysis/synthesis wavelet filters are related by the equations [12]: $G(\omega) = e^{j\alpha(\omega)} H(\omega + \pi)$, where $H(\omega)$ and $G(\omega)$ are the lowpass and highpass filters respectively, $|H(\omega)|^2 + |G(\omega)|^2 = 1$ and $\inf_{|\omega| \le \pi/2} |H(\omega)| > 0$. To illustrate how this flexibility can be applied to implementation, consider the Meyer wavelet of the form

$$H(\omega) = \begin{cases} 1 & 0 \le \omega \le \pi/3 \\ \cos\left[\frac{\pi}{2} \upsilon_n \left(\frac{3}{\pi}\omega - 1 \right)\right] & \pi/3 < \omega \le 2\pi/3 \\ 0 & 2\pi/3 < \omega \le \pi \end{cases} \qquad (2)$$

where

$$\upsilon_n(t) = \frac{\int_0^t u^n (1-u)^n \, du}{\int_0^1 u^n (1-u)^n \, du}, \qquad 0 \le u \le 1, \quad n = 0, 1, 2, \ldots \qquad (3)$$

To improve coding performance, we consider the analysis filters as a system and normalize the constituent transition bands within the multilevel decompositions. This was the preferred approach for implementing tree-structured subband filters for audio and speech coding applications [9]. Instead of employing the same filter repeatedly for all levels of decomposition (as has been the wavelet convention), different filters are employed at different levels where the filter transition bands are made progressively wider for successive levels down the tree [9]. With wider transition widths, the effective filter lengths can be made progressively shorter, resulting in more compact wavelet bases. This approach is now illustrated for a three-level decomposition using the Meyer wavelet.

The filter for the first split is chosen to have a transition region of $\pi/4$:

$$H_0(\omega) = \begin{cases} 1 & 0 \le \omega \le 3\pi/8 \\ \cos\left[\frac{\pi}{2} \upsilon_3 \left(\frac{4}{\pi}\omega - \frac{3}{2} \right)\right] & 3\pi/8 < \omega \le 5\pi/8 \\ 0 & 5\pi/8 < \omega \le \pi \end{cases} \qquad (4)$$

For the second level the transition region is specified as $\pi/2$:

$$H_1(\omega) = \begin{cases} 1 & 0 \le \omega \le \pi/4 \\ \cos\left[\frac{\pi}{2} \upsilon_3 \left(\frac{2}{\pi}\omega - \frac{1}{2} \right)\right] & \pi/4 < \omega \le 3\pi/4 \\ 0 & 3\pi/4 < \omega \le \pi \end{cases} \qquad (5)$$

The third and last level has a transition region of π:

$$H_2(\omega) = \cos\left[\frac{\pi}{2} \upsilon_3 \left(\frac{\omega}{\pi} \right)\right] \qquad 0 \le \omega \le \pi \qquad (6)$$

Because the transition regions are doubled for each decimation stage in the tree, the *actual transition regions* when mapped to the original rate are identical. Since $H_0(\omega)$, $H_1(\omega)$ and $H_2(\omega)$ are orthogonal analysis-synthesis filters, perfect reconstruction is preserved in this process. Inspection of the resulting wavelets shows that the bases are effectively more compact or condense compared to normal dilation wavelet bases, hence the name "condensed wavelets."

IV. Compression Performance of Condensed Wavelet Packet Symmetric Extension

To assess the relative performance of the proposed approach, a 3-level uniform condensed wavelet packet decomposition was evaluated on the images: Barbara, Goldhill, Lena, Café, Bike, and Woman using the SPIHT [7] algorithm. An additional 3-level dyadic decomposition of the lowest frequency band was performed as part of the decomposition, which is known to be a good subband partitioning for natural images. This particular tree structure is called the 3+3 structure in [8].

The results of these comparisons are shown in Table 1. In all cases, symmetric extension was used in the systems we compared. In particular, the condensed wavelet packet decomposition (denoted CWP) is compared with a 6-level dyadic decomposition with 9/7 filters; a 3+3 wavelet packet transform using the 28/28 filters; a 3+3 wavelet packet transform using 9/7 filters; and the 8×40 GenLOT from [10]. The data we present here for the GenLOT was taken directly from reference [10]. In selecting the filters for the comparison, we chose the 28/28 filters because they have the highest compression performance for the 3+3 decomposition structure among all the FIR biorthogonal wavelet filters tested by the authors.

As can be seen by inspecting Table 1, the condensed wavelet packet symmetric extension method generally outperforms the others for all coding rates. The most dramatic gain is a 2.2 dB improvement over the 6-level dyadic 9/7 filter decomposition for the image Barbara. At the same time, unlike other wavelet packet transforms, it still keeps 0.2 to 0.6 dB gain over the 6-level dyadic 9/7 filter decomposition for the remaining images, which has less oscillatory characteristics.

	9/7 (dyadic)	CWP (3+3)	28/28 (3+3)	9/7 (3+3)	GenLOT
Bpp	PSNR				
Lena (512×512)					
0.125	31.10	**31.19**	31.14	31.08	*31.16
0.25	34.13	**34.35**	34.24	34.06	*34.23
0.5	37.24	**37.50**	37.33	37.09	*37.32
1	40.47	**40.65**	40.46	40.22	*40.43
2	45.25	**45.39**	45.18	44.83	---
Barbara (512×512)					
0.125	24.86	26.30	26.30	25.83	***26.37**
0.25	27.59	**29.54**	29.41	28.64	*29.53
0.5	31.40	**33.60**	33.27	32.38	*33.47
1	36.44	**38.41**	37.96	37.11	*38.08
2	42.75	**44.27**	43.95	43.02	---
Goldhill (512×512)					
0.125	28.49	**28.63**	28.56	28.55	*28.60
0.25	30.57	**30.83**	30.70	30.67	*30.79
0.5	33.14	**33.45**	33.27	33.13	*33.36
1	36.58	**36.89**	36.65	36.45	*36.80
2	42.10	**42.38**	42.13	41.81	---
Café (512×512)					
0.125	20.61	**20.85**	20.79	20.72	---
0.25	22.95	**23.23**	23.05	22.94	---
0.5	26.43	**26.73**	26.39	26.23	---
1	31.68	**31.76**	31.19	30.99	---
2	**38.92**	38.68	38.08	37.87	---
Bike (512×512)					
0.125	25.74	**26.32**	26.11	25.95	---
0.25	29.01	**29.54**	29.13	28.95	---
0.5	32.94	**33.36**	32.78	32.57	---
1	37.68	**37.91**	37.24	37.02	---
2	43.92	**43.98**	43.31	43.02	---
Woman (512×512)					
0.125	27.32	**27.62**	27.58	27.35	---
0.25	29.95	**30.26**	30.15	29.87	---
0.5	33.59	**33.95**	33.73	33.40	---
1	38.31	**38.59**	38.32	38.01	---
2	44.13	**44.39**	44.13	43.82	---

Table 1. Coding results for comparison. Bit rate (bpp)/PSNR(dB). *Data quoted from [10]

In addition to the improved compression performance, the condensed wavelet packet symmetric extension algorithm is attractive computationally. If the image dimension is $N=2^m$, the total computational cost for the 3-level uniform condensed wavelet transform is about $2m+1.3$ multiplications and $6m-4$ additions per data point. Thus, if $N=512$, then 19.3 multiplications and 50 additions would be required. Dividing the image into smaller blocks reduces the computational complexity, leading to a transform that is competitive with the implementation of 9/7 filters. Table 2 compares the computational complexity of the 128×128 block size CWP with the other conditions used in Table 1. Much of the CWP complexity may be attributed to the N-point DCTs. Using fast DCT methods e.g. [11], additional reduction in complexity can be realized. Further reduction in computational complexity at this point remains a work in process.

Transform	No. of mult.	No. of add.
CWP (3+3)	13.3	33
9/7 (6-level dyadic)	12	18.7
9/7 (3+3)	27.2	42.3
28/28 (3+3)	84.6	163.1
8×40 GenLOT	36	60

Table 2. Comparison of computational complexity of the 128×128 block-size condensed wavelet packet (CWP) transform with other types of transform

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