A THREE-STEP NONLINEAR LIFTING SCHEME FOR LOSSLESS IMAGE COMPRESSION

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ABSTRACT

We propose an adaptive wavelet transform that results in fewer large detail coefficients while preserving image contours in the approximation subband. The transform is constructed with a three-step nonlinear lifting scheme: a fixed prediction followed by a space-varying update and a non-additive prediction. The interest of the proposed scheme is demonstrated for nearly lossless compression.

***Index Terms*—** Image coding, wavelet transforms, adaptive signal processing, nonlinear system

1. INTRODUCTION

Wavelet-based image compression algorithms are becoming extremely popular, and they have been adopted into the still-image coding standard JPEG2000. Such compression algorithms exploit the ability of wavelet representations to efficiently decorrelate and approximate data with few non-zero wavelet coefficients. However, classical linear wavelets cannot efficiently model higher order singularities, like edges or contours in images. This has motivated various researchers to look for new multiresolution decompositions that can take into account the characteristics of the input signal or image. To a certain extent, this can be achieved by decompositions with respect to fixed but tailor-made bases [1, 2, 3, 4], by nonlinear wavelets [5, 6, 7] or by adaptive subband structures [8, 9, 10, 11, 12, 13].

Some of these decompositions exploit the flexibility of the *lifting* scheme [14] to introduce nonlinearities or different kinds of adaptivity. In [11, 12, 13], we presented a new class of adaptive wavelet decompositions that can capture the directional nature of picture information. This method exploits the properties of seminorms to build lifting structures able to choose between different update filters.

In this paper, we consider a different approach. We build our adaptive wavelet transform by means of a three-step nonlinear lifting scheme: a fixed prediction followed by a space-varying update and a non-additive prediction.

The paper is organized as follows. Section 2 reviews the lifting construction. Section 3 presents the three-step nonlinear lifting scheme and its application to one-dimensional signals. Section 4 gives some examples and simulation results. Finally, concluding remarks are made in Section 5.

2. THE LIFTING SCHEME

The lifting scheme is a very general and highly flexible tool for building new wavelet decompositions from existing ones. Although originally they were developed to design wavelets on complex geometrical surfaces, they are often used as an alternate implementation of classical wavelets. Furthermore, the lifting scheme offers the possibility to replace linear filters by nonlinear ones and to construct adaptive wavelet decompositions.

The main ingredients of the lifting scheme, as illustrated in Fig. 1, are an existing wavelet transform WT, a prediction operator P and an update operator U. The input signal x_0 is first split into an approximation signal x and a detail signal y by a given wavelet transform WT (which may be a simple polyphase decomposition, also called 'lazy wavelet transform'). The prediction operator P acting on x is used to modify y, yielding a new detail signal $y' = y - P(x)$. In practice, the prediction operator P is chosen such that $P(x)$ is an estimate of y and hence the new signal y' is 'smaller' than y. Subsequently, the update operator U acting on y' is used to modify x, resulting in an approximation signal $x' = x + U(y')$. Generally, the update operator is chosen in such a way that the resulting signal x' satisfies a certain constraint such as preserving the average of the input x_0.

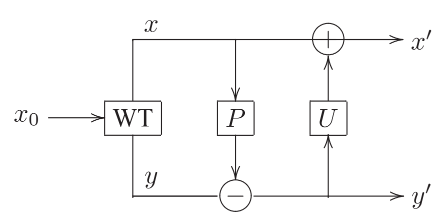


Fig. 1. *Classical lifting scheme*

A general lifting scheme may comprise any sequence of update and prediction lifting steps. In practice, these lifting steps are chosen in such a way that the resulting decomposition is an 'improvement' of the original one. For example, the lifted wavelet may have more vanishing moments than the original one, or it may be better able to decorrelate the signals within a given class etc.

The original signal is reconstructed by reversing the lifting steps and applying the inverse of WT. Hence, perfect reconstruction is guaranteed by the intrinsic structure of this scheme and does not require any particular assumptions on the lifting operators P and U , or about the underlying sampling grid. Moreover, the operators '$+$' and '$-$' used in the above expressions can be replaced by any pair of invertible operators. With the lifting scheme, it becomes possible to build 'any wavelet you like' on 'every geometrical structure you are interested in'. In this paper, we will exploit this fact by adapting both update and prediction operators to the local properties of the signal.

3. A THREE-STEP NONLINEAR LIFTING SCHEME

Consider the lifting scheme illustrated in Fig. 2.

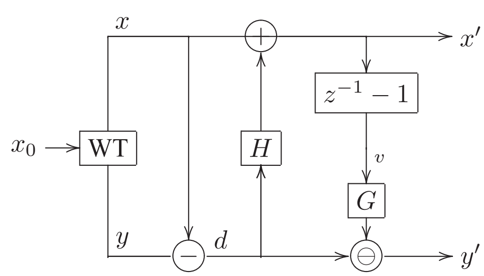


Fig. 2. *Three-stage lifting scheme*

Here, WT is the lazy wavelet transform:$x(n) = x_0(2n)$, $y(n) = x_0(2n-1)$, and H, G are thresholding operators defined for all $u \in \mathbb{R}$ as

$$H(u) = \begin{cases} \frac{1}{2}u & \text{if } |u| < T \\ \alpha\frac{T}{2}\,\text{sign}\,(u) & \text{otherwise,} \end{cases} \tag{1}$$

$$G(u) = \begin{cases} u & \text{if } |u| > T' \\ \alpha' T' & \text{otherwise,} \end{cases} \tag{2}$$

where T, T' are positive threshold values and $\alpha, \alpha' \in \{0, 1\}$ constants which determine the kind of thresholding (hard or soft-thresholding) applied.

The approximation signal x' is given by

$$x'(n) = x(n) + H(d(n))\,, \tag{3}$$

where $d(n) = y(n) - x(n)$ is a first detail signal. The reasoning behind this procedure is the following. In the regions where the underlying signal x_0 is locally smooth, the difference signal d (which can be seen as a local gradient of x_0) will be small and the approximation signal x' will be computed as a linear combination of the polyphase components x and y. On the contrary, near discontinuities the difference signal d takes large values and hence x' is slightly (or not at all) modified in order to prevent smoothing the edges.

As shown in Fig. 2, the detail signal y' is obtained through a non-additive prediction lifting step. Note that the standard subtraction '$-$' has been replaced by a nonlinear operator '$\ominus$' defined as

$$t \ominus u = \frac{t}{\beta + |u|}\,, \quad \beta \geq 1\,,$$

for all $t, u \in \mathbb{R}$. Thus, the detail signal y' is given by

$$y'(n) = \frac{d(n)}{\beta + |G(v(n))|}\,, \tag{4}$$

where $v(n) = x'(n-1) - x'(n)$. Here, $v(n)$ can be viewed as an estimation of $d(n)$. If this estimation is big, we are scaling d by v, so we are kind of compressing the dynamic range of d, leading to a final detail signal close to zero. If this estimation is small, the final detail will be equal to d.

A one-dimensional toy example

Let us consider the case where $\alpha = \alpha' = 0$ (i.e. hard-thresholding) and $\beta = 1$. From (1)-(4) we get

$$x'(n) = \begin{cases} \dfrac{x(n) + y(n)}{2} & \text{if } |y(n) - x(n)| < T \\ x(n) & \text{otherwise,} \end{cases} \tag{5}$$

$$y'(n) = \begin{cases} y(n) - x(n) & \text{if } |x'(n-1) - x'(n)| \leq T' \\ \dfrac{y(n) - x(n)}{1 + |x'(n-1) - x'(n)|} & \text{otherwise.} \end{cases} \tag{6}$$

One can see that the adaptive scheme performs as the Haar wavelet except for those locations where the gradient is large. If $|y(n) - x(n)| \geq T$, the scheme 'recognizes' that there is an edge and does not apply any smoothing in the update step. Since the resulting approximation signal x' retains the discontinuities, large values $|x'(n-1) - x'(n)|$ are likely to indicate large $|y(n) - x(n)|$. In such cases, the scheme scales the detail signal, hence maintaining its magnitude small.

We apply this scheme to the one-dimensional signal shown in the left of Fig. 3. It is a piecewise regular signal of 512 samples with values between 0 and 1. We choose $T = 2T' = 0.2$ and perform a two-level decomposition. The resulting approximation and detail signals are compared to those obtained with the linear Haar and 5/3 wavelet decompositions. Unlike the linear schemes, the considered decomposition does not smooth the discontinuities in the approximation signal and avoids the oscillation effects. In addition, it results in fewer and smaller non-zero details.

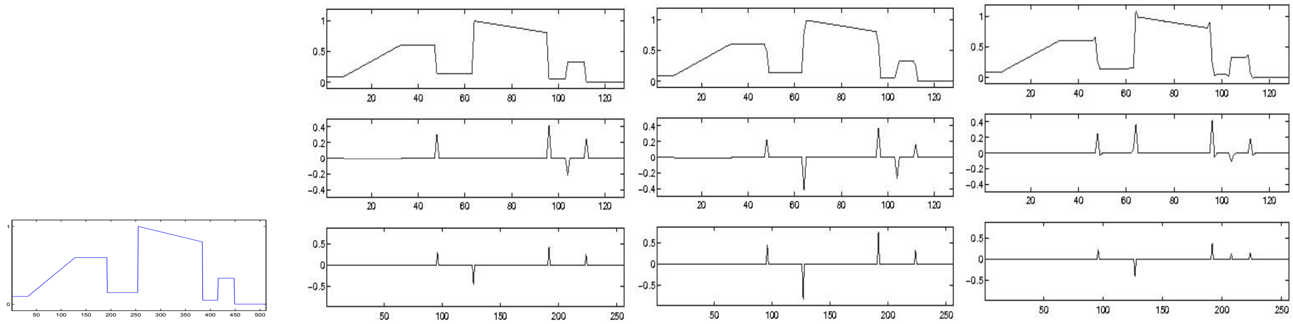


Fig. 3. *From left to right: original signal and nonlinear, Haar and 5/3 wavelet decomposition. The upper rows show the approximation signals and the middle and bottom rows the detail signal.*

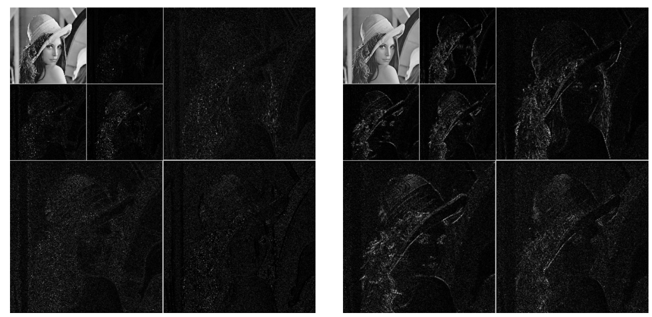
4. SIMULATIONS

In this section, image nearly lossless compression examples using the proposed adaptive lifting scheme are presented. Here, the extension of the adaptive scheme to the two-dimensional case is achieved by applying the one-dimensional filters to the image data in a separable way. The images are first filtered vertically and then horizontally, resulting in a four-band decomposition.

In the following experiment, we fix $\alpha = \alpha' = 1$, $T = 2T' = 30$ and $\beta = 1$.

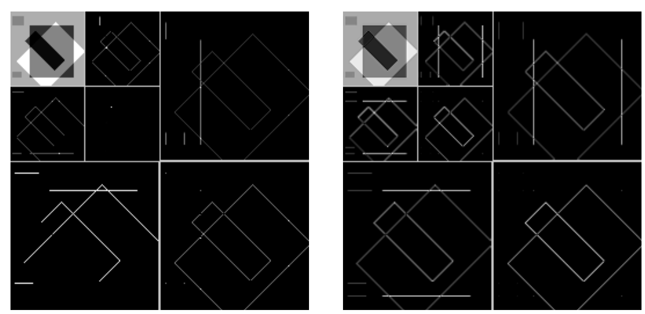
We assess the coding efficiency of our scheme by attaching a nearly lossless coder and computing the actual bitrate. We use the embedded image coding algorithm EZBC proposed in [15]. The same coder is attached to the Haar, 5/3 and 9/7 wavelet schemes. For a fair comparison, the detail bands of the linear schemes are normalized by $1+T'$. Table 1 shows the average bitrate[1] for 4 levels of decomposition when applied to some well-known natural images. In all cases, the nonlinear decomposition achieves lower bitrates.

As an illustration, Fig. 4 and Fig. 5 show the two-level multiresolution decomposition of two test images. Results are displayed for the nonlinear (left) and 5/3 wavelet (right) schemes. The original synthetic image and the approximation images scaled to the original size are shown in Fig. 6. For displaying purposes, the details of each detail band have been independently scaled with a range three standard deviation below and above zero. One can observe that the adaptive scheme avoids blurring the edges in the approximation while obtaining detail images with less detail information and without oscillations around edges.



(a) (b)

Fig. 4. *Multiresolution decompositions of Lena image: (a) Nonlinear and (b) 5/3 wavelet schemes.*



(a) (b)

Fig. 5. *Multiresolution decompositions of a synthetic image: (a) Nonlinear and (b) 5/3 wavelet schemes.*

	Cameraman	Barbara	Lenna	Peppers
Haar	0.533	0.576	0.288	0.262
5/3	0.514	0.457	0.208	0.179
9/7	0.551	0.498	0.262	0.225
Nonlinear	0.324	0.396	0.195	0.144

Table 1. *Nearly lossless coding rates (in bpp) for* 4 *levels of decomposition.*

5. CONCLUSIONS

We have presented a three-step lifting scheme with two nonlinear steps, able to better preserve the sharp transitions in the approximation, while reducing both magnitude and oscilla-

[1] The PSNR obtained in all cases was above 60 dB.

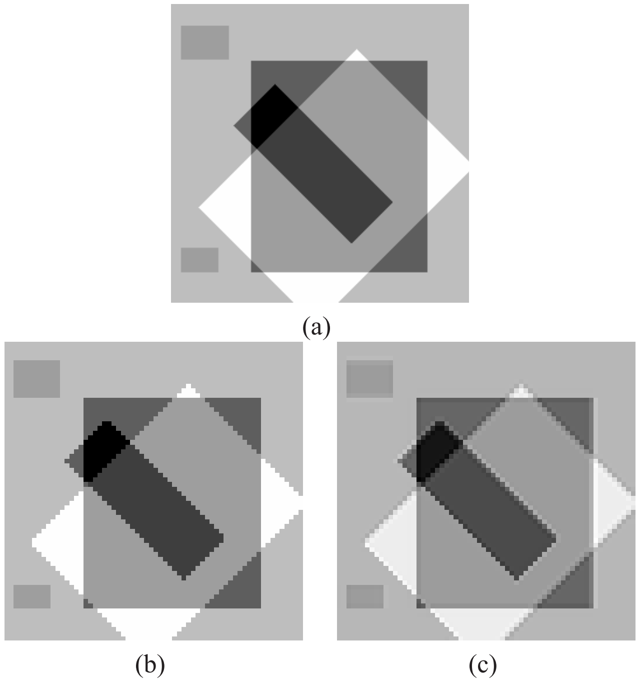


Fig. 6. *(a) Original and approximation image after a two-level decomposition: (b) Nonlinear and (c) 5/3 wavelet schemes.*

tions in the detail coefficients. Thanks to these properties, we have proven the interest of such adaptive decompositions for image approximation and lossless compression. Future work aims at extending this framework for lossy image and video compression.

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