Subspace Extension to Phase Correlation Approach for Fast Image Registration

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ABSTRACT

A novel extension of phase correlation to subspace correlation is proposed, in which 2-D translation is decomposed into two 1-D motions thus only 1-D Fourier transform is used to estimate the corresponding motion. In each subspace, the first two highest peaks from 1-D correlation are linearly interpolated for subpixel accuracy. Experimental results have shown both the robustness and accuracy of our method.

Index terms: Fourier transform, phase correlation, image registration, subpixel alignment, subspace projection.

1. INTRODUCTION

Phase correlation is a well-known technique of broad applications in motion estimation, image registration, object recognition and even video shot detection [1-4]. The baseline method utilizes the Fourier shift theorem, according to which shifts in the spatial domain correspond to linear phase changes in the frequency domain [5]. Phase correlation can be further extended to estimate changes of rotation and scale using the Fourier-Mellin transform and pseudo-polar Fourier transform [6-8]. Furthermore, phase correlation can be also used for affine motion estimation [9]. However, registration of shifts between images with high accuracy remains a fundamental task which can be further improved upon.

Let $r(x,y)$ be a reference image and $g(x,y)$ be a test image satisfying $r(x,y) = g(x+x_0, y+y_0)$, and their corresponding Fourier transforms are denoted as $G(u,v)$ and $R(u,v)$. Then, we have

$$R(u,v) = G(u,v)e^{j2\pi(ux_0+vy_0)} \quad (1)$$

This can be re-written as

$$P(u,v) = \frac{R(u,v)G^*(u,v)}{\left|R(u,v)G^*(u,v)\right|} = e^{j2\pi(ux_0+vy_0)} \quad (2)$$

where * is the complex conjugate, $j = \sqrt{-1}$, and $P(u,v)$ is the cross power spectrum of the two images.

If we apply the invert Fourier transform F^{-1} to $P(u,v)$, a Dirac function centered at (x_0, y_0) is obtained as below

$$F^{-1}\left(P(u,v)\right) = F^{-1}\left(e^{j2\pi(ux_0+vy_0)}\right) = \delta(x_0, y_0) \quad (3)$$

It is also worth noting the following. Firstly the Fourier transform uses finite size of data, which makes the Dirac function to be a unit impulse despite the periodicity assumption [10]. Secondly the peak value can be substantially less than unity due to non-overlapping regions between the two images. Finally, sub-pixel accuracy is desired because the true motion vector (x_0, y_0) has nothing to do with the underlying discrete image acquisition grid.

Subpixel registration is also very relevant to medical imaging, such as magnetic resonance imaging (MRI), in which non-integer offsets between images commonly occur at the image acquisition stage. Since MRI data is sampled in the spatial Fourier domain, registration by phase correlation is a natural way in such a context [1].

After phase correlation, motion vectors are identified as significant peaks in $\delta(x_0, y_0)$. In order to obtain accurate correlation results, the corresponding candidate peaks should be high enough. As a result, some pre-processing steps like windowing or filtering of the data are adopted to overcome aliasing noise. Nevertheless, 2-D Fourier transform used in these approaches seems still very computational expensive, especially for the case with a large volume of data, such as MRI analysis.

The motivation here is to present a fast and robust solution to phase-correlation based image registration. A projection-based subspace scheme is proposed for 1-D phase correlation, and an improved scheme for subpixel registration using linear interpolation is also presented.

2. THE APPROACH

The concepts of subspace and projection are not new in phase correlation, such as the work in [1] and [12]. However, in both cases they are used to identify subpixel displacement on the basis of 2-D phase correlation, which is different from our approach as explained below.

2.1. Subspace phase correlation

For $r(x,y) = g(x+x_0, y+y_0)$, 2-D motion (displacement) between the images can be denoted as $M_{xy}(r,g) = (x_0, y_0)$, which can be further decomposed to two 1-D motion as

$$M_x(r,g) = (x_0, 0) \tag{4-1}$$

$$M_y(r,g) = (0, y_0) \tag{4-2}$$

At the same time, 2-D phase correlation can also be decomposed as two 1-D phase correlation operations in order to estimate the horizontal and vertical shifts x_0 and y_0 respectively.

Let $G_x^{'}(u,y)$ and $R_x^{'}(u,y)$ be the 1-D Fourier transform to each column of $g(x,y)$ and $r(x,y)$, i.e. $G_x^{'}(u,y) = F_x(g(x,y))$, $R_x^{'}(u,y) = F_x(r(x,y))$. The projected spectrums of $G_x^{'}(u,y)$ and $R_x^{'}(u,y)$ onto the u-axis, $G_x(u)$ and $R_x(u)$, can then be defined as follows:

$$G_x(u) = \int_y G_x^{'}(u,y)dy \tag{5-1}$$

$$R_x(u) = \int_y R_x^{'}(u,y)dy \tag{5-2}$$

Then, 1-D phase correlation between $G_x(u)$ and $R_x(u)$ can be obtained as

$$P_x(u) = \frac{R_x(u)G_x^*(u)}{\left|R_x(u)G_x^*(u)\right|} = e^{j2\pi u x_0} \tag{6}$$

and the shift along the x-axis, x_0 is estimated by

$$p_x(x) = F^{-1}(P_x(u)) = F^{-1}\left(e^{j2\pi u x_0}\right) = \delta_x(x_0) \tag{7}$$

Similarly, the shift along the y-axis y_0 can be estimated by

$$\begin{aligned} p_y(y) &= F^{-1}(P_y(v)) \\ &= F^{-1}\left(\frac{R_y(v)G_y^*(v)}{\left|R_y(v)G_y^*(v)\right|}\right) = \delta_y(y_0) \end{aligned} \tag{8}$$

where $R_y(v)$ and $G_y(v)$ are projected spectrums defined by 1-D Fourier transform $F_y(\cdot)$ to each row of the images as

$$G_y(v) = \int_x F_y(g(x,y))dx \tag{9-1}$$

$$R_y(v) = \int_x F_y(r(x,y))dx \tag{9-2}$$

Let us denote by $g_x(x)$, $g_y(y)$, $r_x(x)$ and $r_y(y)$ the 1-D signal projections of the 2-D images $g(x,y)$ and $r(x,y)$, i.e. $g_x(x) = \int_y g(x,y)dy$, $g_y(y) = \int_x g(x,y)dx$, $r_x(x) = \int_y r(x,y)dy$ and $r_y(y) = \int_x r(x,y)dx$. It is easy to find that actually only one 1-D Fourier transform is required to obtain either $G_x(u)$, $R_x(u)$ or $R_y(v)$ and $G_y(v)$, owing to the linearity of the Fourier transform.

$$G_x(u) = F_x\left(\int_y g(x,y)dy\right) = F_x(g_x(x)) \tag{10-1}$$

$$G_y(v) = F_y(g_y(y)) \tag{10-2}$$

$$R_x(u) = F_x(r_x(x)) \tag{11-1}$$

$$R_y(v) = F_y(r_y(y)) \tag{11-2}$$

2.2. Subpixel registration

Usually, an initial 2-D displacement (x_0, y_0) is obtained by separately checking the highest peak in x and y directions. If (x'_0, y'_0) as the subpixel offset, then x'_0 will be estimated either between x_0 and $x_0 - 1$ or between x_0 and $x_0 + 1$ depending upon whether $p_x(x_0 - 1)$ or $p_x(x_0 + 1)$ has the higher absolute height. Similarly, y'_0 is estimated either between y_0 and $y_0 - 1$ or between y_0 and $y_0 + 1$ depending upon whether $p_y(y_0 - 1)$ or $p_y(y_0 + 1)$ has the higher absolute height. As x'_0 and y'_0 are determined independently, we only present the way how x'_0 is decided.

As $p_x(x_0)$ and $p_y(y_0)$ are the highest peaks respectively in $p_x(x)$ and $p_y(y)$, the above means that the first two highest peaks (the main peak and its highest side peak) are selected for subpixel registration. However, if the heights of two side peaks are close enough, initial integer offset is remained. This is measured by $d(x_0-1,x_0+1)<\eta$, which is further defined by

$$d(x_0-1,x_0+1)=\left|\frac{p_x(x_0-1)-p_x(x_0+1)}{p_x(x_0-1)+p_x(x_0+1)}\right| \quad (12)$$

where $\eta=0.15$ is a threshold used to identify two side peaks of similar height. It is worth noting that if one of the two side peaks is positive and the other is negative, the corresponding measurement $d(x_0-1,x_0+1)$ becomes larger than 1 and the following subpixel interpolation scheme is applied.

$$x'_0=\begin{cases}\dfrac{x_0\,|p_x(x_0)|+(x_0-1)\,|p_x(x_0-1)|}{|p_x(x_0)|+|p_x(x_0-1)|} \\ \quad if \quad |p_x(x_0-1)|>|p_x(x_0+1)|; \quad or \\ \dfrac{x_0\,|p_x(x_0)|+(x_0+1)\,|p_x(x_0+1)|}{|p_x(x_0)|+|p_x(x_0+1)|} \\ \quad if \quad |p_x(x_0+1)|>|p_x(x_0-1)|.\end{cases} \quad (13)$$

3. RESULTS AND DISCUSSIONS

The MRI data set used in our experiments is from Hoge and contains five MRI images of a grapefruit [1]. The true offsets between each pair of images are known and subsequently used as ground truth maps for performance evaluation. The first MRI image is shown in Figure 1.

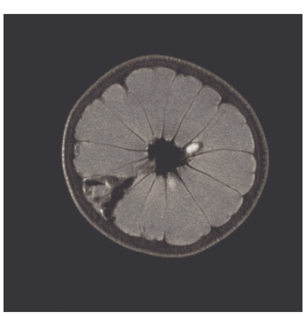

Figure 1. One test image of size 256*256 (Courtesy of W. S. Hoge).

3.1. Accuracy analysis

In this group of experiments, the performance of subpixel registration using our method applied to noise-free original images is compared against the techniques of Hoge [1], and Foroosh et al [11]. Pair-wise registration results in x and y directions are illustrated in Table I and II, respectively.

Table I. Pairwise registration results in x-direction

Image Pairs	Physical Offset	Foroosh et al [11]	Hoge [1]	Our method
(1,2)	**-2.40**	**-2.23**	**-2.03**	**-2.18**
(1,3)	**-4.80**	**-4.07**	**-4.13**	**-4.33**
(1,4)	**-7.20**	**-6.59**	**-6.81**	**-6.65**
(1,5)	**-7.20**	**-6.59**	**-6.82**	**-6.61**
(2,3)	**-2.40**	**-2.10**	**-2.10**	**-2.29**
(2,4)	**-4.80**	**-4.55**	**-4.28**	**-4.55**
(2,5)	**-4.80**	**-4.55**	**-4.78**	**-4.57**
(3,4)	**-2.40**	**-2.00**	**-2.17**	**-2.40**
(3,5)	**-2.40**	**-2.00**	**-2.18**	**-2.41**
(4,5)	**0.00**	**-0.26**	**0.01**	**0.00**
Error analysis	**mean**	**-0.34572**	**-0.30861**	**-0.24066**
	Std-dev	**0.28075**	**0.20681**	**0.22808**
	MSE	**0.19046**	**0.13373**	**0.10474**

Table II. Pairwise registration results in y-direction

Image Pairs	Physical offset	Foroosh et al [11]	Hoge [1]	our method
(1,2)	**-4.00**	**-4.23**	**-4.01**	**-4.00**
(1,3)	**-8.00**	**-8.24**	**-8.01**	**-8.00**
(1,4)	**-4.32**	**-4.41**	**-4.17**	**-4.37**
(1,5)	**-12.00**	**-12.26**	**-12.02**	**-12.00**
(2,3)	**-4.00**	**-3.60**	**-3.99**	**-3.69**
(2,4)	**-0.32**	**-0.39**	**-0.15**	**-0.38**
(2,5)	**-8.00**	**-8.24**	**-8.00**	**-8.00**
(3,4)	**3.68**	**3.61**	**3.84**	**3.57**
(3,5)	**-4.00**	**-3.56**	**-4.51**	**-4.00**
(4,5)	**-7.68**	**-7.92**	**-7.85**	**-7.62**
Error analysis	**mean**	**0.06127**	**0.02243**	**-0.01462**
	Std-dev	**0.26525**	**0.19999**	**0.11185**
	MSE	**0.0671**	**0.0365**	**0.01147**

According to the ground truth (physical offset), an error vector between physical offset and estimated displacements is obtained for each approach in x and y directions, respectively. Let δ_x and δ_y denote the corresponding error vectors, i.e. $\delta_x(i)=x(i)-\hat{x}(i)$ and $\delta_y(i)=y(i)-\hat{y}(i)$, where $x(i)$ and $y(i)$ are the i^{th} physical offset, and $\hat{x}(i)$ and $\hat{y}(i)$ are their estimates. For each error vector, generally it is modelled as a normal distribution and its mean μ and standard deviation

σ are used to measure the accuracy of the estimation. However, one error vector may have a small mean error but a big standard deviation, or a big mean error but a small standard deviation. As a result, mean square error (MSE) between the estimates and ground truth is used as an overall measurement.

In Table I and II, the mean, standard deviation and range errors in x and y directions are given for comparison purposes. It is easy to see that the overall accuracy along the y-axis is better than that along x-axis, which is possibly due to the difference in generating displacements in different directions (see [1] for detail). In both x and y directions the approach from [11] generates maximum error values.

In x direction, Hoge's method yields minimum standard deviation but moderate mean error. On the other hand, our approach using subspace (1-D) phase correlation produces minimum (absolute) mean error but also causes a slightly higher standard deviation. In the y direction, Hoge's method performs worse both in terms of mean error and standard deviation. Considering the MSE, the proposed approach seems the best solution in these experiments. Moreover, the results confirm the fact that 2-D displacements can be successfully estimated using subspace phase correlation.

3.2. Computational complexity analysis

In both 2-D phase correlation and subspace correlation, fast Fourier transform (FFT) forms the main computational load. In some approaches, additional processing is required such as windowing, partial differencing or even singular value decomposition [1]. If the original images are of $N \times N$, then the computing complexity of FFT in 2-D and subspace correlation is $O(N^2 \log_2 N)$ and $O(N \log_2 N)$, respectively. To the best of our knowledge, our subspace scheme is the fastest and lowest–complexity scheme among those reported in the literature while producing high-accuracy registration results.

4. CONCLUSIONS

A novel extension to the phase correlation image registration approach is presented, in which subspace phase correlation is introduced along with linear interpolation for subpixel accuracy. Experimental results on real MRI images showed accuracy and robustness of our approach.

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