BLOCK-COORDINATE GAUSS-NEWTON/REGRESSION METHOD FOR IMAGE REGISTRATION WITH EFFICIENT OUTLIER DETECTION

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ABSTRACT

In this paper, the *block-coordinate Gauss-Newton/regression* method is proposed to jointly optimize the spatial registration and the intensity compensation. Here, the intensity compensation is conducted constructing a polynomial regression model, which enables the detection of occluded regions as outliers. Based on the block-coordinate method, we separate the parameter update into two steps for registration and compensation, respectively. Hence, we can perform a joint optimization with low computational complexities, and can apply an appropriate scaling technique to the parameters to be updated for a stable and fast convergence of the algorithm. Excluding outliers, we can successfully align images compensating the intensity differences.

Index Terms— Intensity compensation, regression model, outlier, block-coordinate optimization.

1. INTRODUCTION

Research on the registration of images has been significantly conducted in various fields, such as computer vision and medical imaging. Superposing the multiple captured images into a single image can increase the dynamic range of the pixel levels and improve the signal-to-noise ratio if all images are properly aligned and exposed. When we align differently exposed images, we may use a feature-based registration technique, of which performance is less sensitive to the exposure difference. We can then apply a global photometric registration [2]. However, to reduce the effect of the different exposure on the correlation-based registration, we should account or compensate the exposure difference during registration. To perform an intensity compensation, two images should contain the same scene and be aligned with respect to the overlapped scene. For an accurate registration, contradictively, the images should have the same exposure or illumination conditions. Therefore, devising a joint optimization technique for the spatial registration and exposure compensation is required.

Mann [4] conducted a joint registration based on the comparametric exposure compensation with the affine model. He used the homomorphic warp for the registration, and derived a combined linear equation of the affine function and the warp for a joint optimization. Candocia [1] performed a continuous piecewise-linear fitting to obtain a preferred comparametric function.

In this paper, a joint optimization method, *block-coordinate Gauss-Newton/regression* method is proposed, where the intensity compensation is conducted constructing a polynomial regression model. Based on the block-coordinate method, we update the parameters for registration and intensity compensation sequentially in two separated steps. We can perform a joint optimization with low computational complexities, and can apply an appropriate scaling technique to the parameters to be updated for stable and fast convergence of the algorithm. Furthermore, outliers are detected based on the regression analysis to align images that include occluded parts.

2. INTENSITY COMPENSATION

For an intensity compensation problem, a polynomial regression model is used. To deal with the occlusion problem, we exclude the outliers, which are detected based on a measure of influence and the studentized residuals.

2.1. Polynomial Regression Model

Let $U_1, \ldots, U_m$ and $V_1, \ldots, V_m$ are identically distributed random variables representing the reference and input images, respectively. Consider a polynomial $\eta(v; \boldsymbol{q}) := q_0 + \cdots + q_{t-1}v^{t-1} + q_t v^t$, for $v \in \mathbb{R}$, where a vector $\boldsymbol{q} := (q_0, \ldots, q_t) \in \mathbb{R}^{t+1}$ is a parameter vector. The *polynomial regression model* [5, p. 181] for the intensity compensation is given by

$$U_i = \eta(V_i; \boldsymbol{q}^*) + \varepsilon_i, \text{ for } i = 1, \ldots, m. \tag{1}$$

Here, we impose the Gauss-Markov conditions [5, p. 35] on the random variables ε_i, i.e., $E\{\varepsilon_i\} = 0$, $E\{\varepsilon_i^2\} = \sigma$, and

$E\{\varepsilon_i\varepsilon_j\} = 0$ when $i \neq j$, for $i, j = 1, \ldots, m$[1]. By minimizing the empirical error: $\rho_m(\eta) := m^{-1}\sum_{i=1}^{m}[\eta(V_i;\boldsymbol{q})-U_i]^2$, with respect to $\boldsymbol{q}$, we can obtain a unbiased estimator $\boldsymbol{q}^*$ for $\boldsymbol{q}$ [5, p. 30,(2.9)]. We can then compensate the brightness of the input image using an intensity compensation function $\eta(\cdot;\boldsymbol{q})$.

2.2. Outlier Detection

We employ the studentized residuals, RSTUDENT and the measure of influence, DFFITS_i [5, ch. 8] to detect the outliers, which distort the intensity compensation curve and eventually cause a critical error in the registration. To measure the influence of each sample to the fitting, we calculate DFFITS_i and compare it with a threshold of $2\left[(t+1)/(m-t-1)\right]^{1/2}$. We also compare RSTUDENT values to a predetermined threshold. The threshold is set as 2 since the RSTUDENT has a t-distribution under a normality assumption. If either the absolute value of DFFITS_i or the RSTUDENT value is larger than the given threshold, then we regard the corresponding sample as a 'possible outlier'. If the number of outliers is significant, than we repeat the regression excluding the possible outliers for a better fitting. Next step is to refine the possible outliers into the final outliers. Supposing that the outliers form an occlusion region, we remove isolated outliers by using a morphological filter. The proposed intensity compensation is summarized as follows:

Intensity Compensation Excluding Outliers
1) Perform regression.
2) Possible outliers: $|\text{DFFITS}_i| > 2\left[(t+1)/(m-t-1)\right]^{1/2}$ or RSTUDENT > 2.
3) Excluding the possible outliers, repeat Steps 1) and 2).
4) Morphological operations, erosion and then dilation, to decide the final outliers.
5) Rerun the regression excluding the final outliers.

In the proposed intensity compensation algorithm, Steps 1) and 2) are applied two times when the number of outliers is significant. From numerical experiments, we found that more repetition is required for a further accurate detection of outliers depending on images.

3. JOINT REGISTRATION AND COMPENSATION

In this section, a joint optimization problem of the spatial registration and the intensity compensation is considered based on the Gauss-Newton method.

For the description of the spatial registration, we represent the pixel values at a location $\boldsymbol{x} \in \mathbb{R}^2$ as $U(\boldsymbol{x})$ and $V(\boldsymbol{x})$, for the reference and input images, respectively. Note that this description on images is more convenient than the U_i and V_i case of the previous section. Let a map $\boldsymbol{\phi}(\boldsymbol{x};\boldsymbol{p}) = (\phi_x, \phi_y) \in \mathbb{R}^2$ denote the warp for given $\boldsymbol{x} \in \mathbb{R}^2$, where $\boldsymbol{p} := (p_1, \ldots, p_s) \in \mathbb{R}^s$ is a vector of s parameters. The map $\boldsymbol{\phi}$ takes a location $\boldsymbol{x}$ in the coordinate frame of the template image T and maps it to the sub-pixel location $\boldsymbol{\phi}(\boldsymbol{x};\boldsymbol{p})$ in the coordinate frame of the input image V. Here, the template image T is a part of the reference image U and has m_0 $(\leq m)$ pixels. We suppose that, within the relationship of a warp, T and a part of V have the same scene. The homomorphic warp with $s = 8$ is usually used.

In order to jointly optimize the spatial registration and the intensity compensation, we consider the following empirical error:

$$\delta(\boldsymbol{\beta}) := \frac{1}{m_0}\sum_{\boldsymbol{x}}\left[\eta\big(V(\boldsymbol{\phi}(\boldsymbol{x};\boldsymbol{p}));\boldsymbol{q}\big) - T(\boldsymbol{x})\right]^2, \qquad (2)$$

where $\boldsymbol{\beta} \in \mathbb{R}^{s+t+1}$ is a parameter vector composed of $\boldsymbol{p}$ and $\boldsymbol{q}$ as $\boldsymbol{\beta} := (p_1, \cdots, p_s, q_0, \cdots, q_t)$, and the summation is over all pixels in T. To reach a solution set $\Gamma := \{\boldsymbol{\beta} : \nabla\delta(\boldsymbol{\beta}) = \boldsymbol{0}\}$, we may use the steepest descent method [3].

Since δ is a sum of squares of arbitrary functions, the *Gauss-Newton method* [5, p. 299] can efficiently minimize δ without calculating the Hessian matrix. Based on this method, Mann [4] considered an affine function with the homomorphic warp. In this section, the current joint optimization methods, which are based on the Gauss-Newton method, are extended to further generalized maps $\boldsymbol{\phi}$ and η. Letting $g(\boldsymbol{x};\boldsymbol{\beta}) := \eta\big(V(\boldsymbol{\phi}(\boldsymbol{x};\boldsymbol{p}));\boldsymbol{q}\big) - T(\boldsymbol{x})$, δ can be rewritten by $\delta(\boldsymbol{\beta}) = m_0^{-1}\sum_{\boldsymbol{x}}[g(\boldsymbol{x};\boldsymbol{\beta})]^2$. Let $\nabla g(\boldsymbol{x};\boldsymbol{\beta})$ denote the gradient of g with respect to $\boldsymbol{\beta}$ as a row vector given by

$$\nabla g(\boldsymbol{x};\boldsymbol{\beta}) := \left(\frac{\partial g}{\partial p_1} \cdots \frac{\partial g}{\partial p_s}\ \frac{\partial g}{\partial q_0} \cdots \frac{\partial g}{\partial q_t}\right). \qquad (3)$$

For small $\|\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)}\|$, we have an approximation from the Taylor series as

$$g(\boldsymbol{x};\boldsymbol{\beta}) \approx g(\boldsymbol{x};\boldsymbol{\beta}^{(k)}) + \nabla g(\boldsymbol{x};\boldsymbol{\beta}^{(k)})(\boldsymbol{\beta} - \boldsymbol{\beta}^{(k)}).$$

Hence, a subsequent guess $\boldsymbol{\beta}^{(k+1)}$ for the parameter vector is then obtained by the recurrence relation:

$$\boldsymbol{\beta}^{(k+1)} = \boldsymbol{\beta}^{(k)} - J(\boldsymbol{\beta}^{(k)})^{-1}\sum_{\boldsymbol{x}}\nabla g(\boldsymbol{x};\boldsymbol{\beta}^{(k)})^{\top} g(\boldsymbol{x};\boldsymbol{\beta}^{(k)}), \qquad (4)$$

where $J(\boldsymbol{\beta}) := \sum_{\boldsymbol{x}}\nabla g(\boldsymbol{x};\boldsymbol{\beta})^{\top}\nabla g(\boldsymbol{x};\boldsymbol{\beta})$ is an $(s+t+1) \times (s+t+1)$ matrix, and $\top$ denotes the transpose of matrices. Before entering the main iteration, a template T is extracted from U with an initial warp. To set a template, a center part of V is first selected and then find the corresponding part of U using a simple translation-based alignment. We use these corresponding part and the translation parameters for setting the template T and an initial warp parameter $\boldsymbol{p}^{(0)}$, respectively. Here, the initial parameter for the intensity compensation is given by $\boldsymbol{q}^{(0)} = (0, 1, 0, \ldots, 0)$, which implies an affine function. We call the joint method, which is based on

[1] A special case of polynomials for $t = 1$ is the affine function, which is usually used to correct brightness differences [2].

(4), the Gauss-Newton method in this paper.

Gauss-Newton (GN)

0) Set a constant $\epsilon > 0$ and a template T. Letting $k = 0$, choose an initial $\boldsymbol{\phi}$ and η with $\boldsymbol{\beta}^{(0)}$.
1) Compute $\boldsymbol{\beta}^{(k+1)}$ from (4).
2) If $\|\delta(\boldsymbol{\beta}^{(k+1)}) - \delta(\boldsymbol{\beta}^{(k)})\| < \epsilon$, then stop. Otherwise, $k \leftarrow k + 1$ and goto Step 1).

In (4), the parameter vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are simultaneously updated. Instead of such an update, we can derive a closed form of update on $\boldsymbol{p}$ as a function of an optimal $\boldsymbol{q}$. Based on this notion, Candocia [1] derived an update on $\boldsymbol{p}$ as a function of $\boldsymbol{q}$ that constructs a continuous piecewise-linear fitting for a better exposure compensation. It is clear that such a derivation is more difficult than the joint update case of (4). If we can separate the registration and intensity compensation parts in the joint optimization, then we can significantly simplify the optimization problem. In the following section, we will do such a separation to develop an efficient joint optimization scheme.

4. BLOCK-COORDINATE METHOD FOR JOINT OPTIMIZATION

The *block-coordinate method* [3] is based on decomposing the parameters into several blocks and producing optimization steps in the respective block subspaces in a sequential manner. Even though there are possibilities of slow convergency and local minimum problems, complicate joint optimizations can be simplified, and appropriate operations, such as scaling and preconditioning, can be applied depending on the parameters so as to obtain stable solutions. Based on the block-coordinate method, we propose dividing the parameters of $\boldsymbol{\beta}$ into two blocks for the spatial registration $\boldsymbol{p}$ and the intensity compensation $\boldsymbol{q}$, respectively.

4.1. Block Coordinate Methods

We now describe the block-coordinate method for the Gauss-Newton case as follows. Let the gradients $\nabla_1 g(\boldsymbol{x};\boldsymbol{\beta})$ and $\nabla_2 g(\boldsymbol{x};\boldsymbol{\beta})$ denote row vectors as $(\partial g/\partial p_1 \cdots \partial g/\partial p_s)$ and $(\partial g/\partial q_0 \cdots \partial g/\partial q_t)$, respectively. The first phase, which is for updating the registration parameters, is then given by

$$\boldsymbol{p}^{(k+1)} = \boldsymbol{p}^{(k)} - J_1(\boldsymbol{\beta}^{(k)})^{-1} \sum_{\boldsymbol{x}} \nabla_1 g(\boldsymbol{x};\boldsymbol{\beta}^{(k)})^\top g(\boldsymbol{x};\boldsymbol{\beta}^{(k)}), \tag{5}$$

and the second phase, which is for updating the compensation parameters, is given by

$$\boldsymbol{q}^{(k+1)} = \boldsymbol{q}^{(k)} - J_2(\boldsymbol{\gamma}^{(k)})^{-1} \sum_{\boldsymbol{x}} \nabla_2 g(\boldsymbol{x};\boldsymbol{\gamma}^{(k)})^\top g(\boldsymbol{x};\boldsymbol{\gamma}^{(k)}), \tag{6}$$

where $J_1(\boldsymbol{\beta}) := \sum_{\boldsymbol{x}} \nabla_1 g(\boldsymbol{x};\boldsymbol{\beta})^\top \nabla_1 g(\boldsymbol{x};\boldsymbol{\beta})$ and $J_2(\boldsymbol{\beta}) := \sum_{\boldsymbol{x}} \nabla_2 g(\boldsymbol{x};\boldsymbol{\beta})^\top \nabla_2 g(\boldsymbol{x};\boldsymbol{\beta})$. In (6), an intermediate parameter vector $\boldsymbol{\gamma}^{(k)} \in \mathbb{R}^{s+t+1}$ is defined as

$$\boldsymbol{\gamma}^{(k)} := (p_1^{(k+1)}, \cdots, p_s^{(k+1)}, q_0^{(k)}, \cdots, q_t^{(k)}).$$

The block-coordinate method is now summarized as follows:

Block-Coordinate Gauss-Newton (BCGN)

0) Set a constant $\epsilon > 0$ and a template T. Letting $k = 0$, choose an initial $\boldsymbol{\phi}$ and η with $\boldsymbol{\beta}^{(0)}$.
1) Registration: Compute $\boldsymbol{p}^{(k+1)}$ from (5).
2) Compensation: Compute $\boldsymbol{q}^{(k+1)}$ from (6).
3) If $\|\delta(\boldsymbol{\beta}^{(k+1)}) - \delta(\boldsymbol{\beta}^{(k)})\| < \epsilon$, then stop. Otherwise, $k \leftarrow k + 1$ and goto Step 1).

The Gauss-Newton update of (4) is divided into two steps, Steps 1) and 2) in BCGN. Note that we can perform separate matrix inverses for $\boldsymbol{p}$ and $\boldsymbol{q}$, respectively. Hence, we can easily apply an appropriate scaling for good inversions depending on the parameter properties. On the other hand in the inverse for (4), we should deliberate over combining two different parameter vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ for successful matrix inversions. Note that we can employ different updates especially for the compensation step instead of the Gauss-Newton update (6). In the following section, such a notion will be discussed in a frame work of regression analysis.

4.2. Block-Coordinate Gauss-Newton/Regression Method

In the following algorithm, we use the regression model of Section 2.1 instead of the Gauss-Newton method of (6) to update the compensation parameter vector $\boldsymbol{q}$. The proposed algorithm is now summarized as follows:

Block-Coordinate Gauss-Newton/Regression (BCGNR)

0) Set a constant $\epsilon > 0$ and a template T. Letting $k = 0$, choose an initial $\boldsymbol{\phi}$ with $\boldsymbol{p}^{(0)}$.
1) Registration (Gauss-Newton): Compute $\boldsymbol{p}^{(k+1)}$ from (5).
2) Compensation (regression): $\boldsymbol{q}^{(k+1)} = \min_{\boldsymbol{q}}^{-1} \delta\big((p_1^{(k+1)}, \ldots, p_s^{(k+1)}, q_0, \ldots, q_t) \big).$
3) If $\|\delta(\boldsymbol{\beta}^{(k+1)}) - \delta(\boldsymbol{\beta}^{(k)})\| < \epsilon$, then stop. Otherwise, $k \leftarrow k + 1$ and goto Step 1).

The first phase is searching for an update $\boldsymbol{p}^{(k+1)}$ that minimizes $\delta(\boldsymbol{\beta})$ for fixed $\boldsymbol{q}^{(k)}$. The second phase is then searching for a map η with $\boldsymbol{q}^{(k+1)}$ that minimizes δ for a fixed $\boldsymbol{p}^{(k+1)}$. Consequently the compensation-registration error δ of (2) decreases to a limit. The block-coordinate method, which is based on the steepest descent method, can reach a limit that belongs to the solution set Γ [3, p. 228].

5. NUMERICAL RESULTS

We now numerically compare GN with the block-coordinate methods, BCGN and BCGNR. A line search step is considered in the Gauss-Newton updates to alleviate the effect from

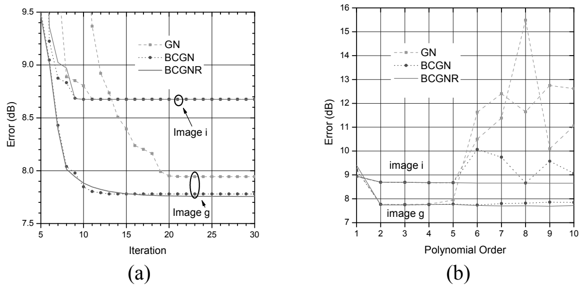

Fig. 1. Comparison of proposed BCGN and BCGNR with GN. (a) Error in decibel with respect to iteration ($t = 5$). (b) Error in decibel with respect to the polynomial order.

Fig. 2. Mosaicked image based on BCGNR.

Fig. 3. Mosaicked images. (a) Conventional registration with BCGNR. (b) BCGNR excluding outliers.

the quadratic approximation error [3, p. 226]. Furthermore, for the matrix inversions, such as $J(\boldsymbol{\beta})^{-1}$, an appropriate scaling scheme is applied to the data to be fitted to polynomials. Several image pairs are tested for the registration, where the size of the image is given by 320×240 pixels with 8 b/pixel. In each image pair, the images have different exposure settings and are related by the homomorphic warp. The empirical error $\delta(\boldsymbol{\beta})$ with respect to the iteration is shown in Fig. 1(a). Here, we can notice that the block-coordinate methods, BCGN and BCGNR, show comparable performances to the GN case and a faster convergence property. Furthermore, from Fig. 1(b), especially BCGNR shows a stable performance with respect to the polynomial order for some bad conditions compared to the GN and BCGN cases. Therefore, besides reducing the computational complexity, we can obtain a stable solution if we using BCGNR. A panorama image is constructed in Fig. 2 based on BCGNR using 5 images. In Fig. 3(a) is a mosaicked image based on BCGNR excluding outliers as shown in Section 2.1. This result shows a good aligned result compared to the conventional method that does not consider the outlier problem.

6. CONCLUSION

In this paper, we conducted a joint optimization of the spatial registration and the intensity compensation based on the block-coordinate Gauss-Newton/regression method. The proposed algorithm shows a comparable result with the conventional joint optimization approaches, and further provides a stable and fast result. By excluding outliers, which are detected based on RSTUDENT and DFFITS_i, we can successfully align the images having occuluded parts.

7. REFERENCES

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