TELEGRAPH-DIFFUSION OPERATOR FOR IMAGE ENHANCEMENT

Vadim Ratner and Yehoshua Y. Zeevi

Department of Electrical Engineering
Technion – Israel Institute of Technology, Haifa 32000

ABSTRACT

Image denoising and enhancement problems have many physical analogues that highlight new approaches to novel solutions. One such solution, based on viewing the image as elastic sheet, is presented. A processing scheme for grayscale images is outlined and further considered in the context of color images. Preliminary analysis and simulations on noisy images indicate that multidimensional manifold representation of combined space-color information incorporates the advantages of separate color channel representations. Experimental analysis reveals elastic sheet method to be a powerful and robust denoising tool, which preserves most meaningful details.

***Index Terms*—**Color, image enhancement, image processing, diffusion process, image edge analysis

1. INTRODUCTION

Criteria of minimal surface and maximal similarity (minimum error energy) give image filtering and denoising problems a distinct physical nature. Many physical phenomena behave in a way that favors smooth surfaces over peaks – corrosion (either physical or chemical), particle movement, surface tension, etc. These phenomena were studied thoroughly, and have the advantages of both having strong mathematical basis, and being highly intuitive, since we deal with them in everyday life.

Physical approaches to image processing based on diffusion processes have been proposed, such as anisotropic and nonlinear diffusion [1]-[4], and complex diffusion [5].

In this paper we propose a new approach based on viewing the image as an elastic sheet, contraction of which creates the denoising effect. This method which we call *telegraph-diffusion* method incorporates a new aspect of the geometry of evolving images in the process of enhancement. We will also discuss implementations of the algorithm in color image processing, using novel approach to color image as a manifold embedded in high dimensional space.

This paper is organized as follows: Section 2 presents the image smoothing problem and different approaches to the solution based on diffusion. The *telegraph-diffusion* method for grayscale images is introduced in section 3. Section 4 studies applications of the method for color images. Experimental results and discussion of advantages of the method, as well as future work directions, are presented in sections 5 and 6. Section 7 concludes the discussion.

2. PREVIOUS RELATED STUDIES

Denoising problem addressed in this paper is that of removing white additive noise from input image I. Such noise takes form of a collection of sharp peaks, on a mostly smooth image.

If we regard gray levels as density values at a given point (pixel), then diffusion process will be analogous to smoothing of a grayscale image [1]. Diffusion process is described by:

$$-\nabla\cdot\left(c\left(x,y,t\right)\nabla I\right)+I_t=0 \quad (1)$$

Since the original image is not always smooth, a measure of context dependence is necessary in order not to smoothen out important details. Several such methods were proposed in [1]-[4], and more recently, complex diffusion was proposed in [5]. Denoising process can be described by the following PDE:

$$\begin{aligned} I_t &= \nabla\left(c\left(\left|\mathrm{Im}\left(I\right)\right|\right)\nabla I\right) \\ c\left(s\right) &= \frac{1}{1+s^2} \end{aligned} \quad (2)$$

c being the diffusion coefficient. This choice of c reduces diffusion around edges (Im(I), the imaginary part of I, acting as an edge detector), while increasing it over smooth surfaces. Yet, the diffusion effect is not fully eliminated in the vicinity of edges. The proposed approach of elastic image processing copes successfully with this problem.

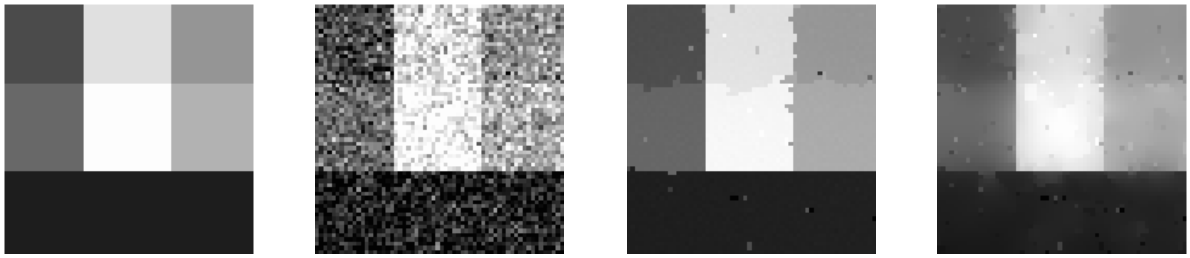

Figure 1: From left to right: original image, heavily noised image ($\sigma = 0.4$), elastic denoising result, diffusion denoising result

3. ELASTIC IMAGE PROCESSING

Let us consider a thin elastic sheet placed in liquid environment. The sheet is held in place by the edges, and is forced to take a certain shape *I(x,y,t=0)* by restraints. After the restraints are removed it contracts and fluctuates. The liquid acts as damping agent, ensuring loss of energy, and, therefore, convergence. As time passes, irregular features of *I* are smoothed, and total surface of the sheet decreases. By defining spatially (and time) variant elasticity and damping coefficients it is possible to locally control the degree of smoothing.

We have limited the movement of pixels to be along color axes only. We assumed elastic forces between the pixels (force proportional to distance). Damping force is proportional to pixel speed. We then derived a PDE describing the shape of the elastic sheet over time, *I(x,y,t)*:

$$a(x,y,t)I_{tt} - \nabla\cdot\left(k(x,y,t)\nabla I\right) + c(x,y,t)I_t = 0 \quad (3)$$

where *k* is the elasticity coefficient and *c* is the damping coefficient. All coefficients are positive. (3) is a parabolic-hyperbolic equation, single dimensional variation of which is known as the *telegrapher's equation* [7]. We call it the *telegraph-diffusion* equation and investigate its inherent properties in the context of image enhancement. In this paper, we mostly refer the case of constant *c*, and *a=1*:

$$I_{tt} - \nabla\cdot\left(k(x,y,t)\nabla I\right) + cI_t = 0 \quad (4)$$

although variation of *a()* may have some advantages, as will be mentioned later.

Initial shape of the sheet is the input image I_0. In order to preserve edges we make the sheet more elastic (higher *k*) in smooth areas and more rigid (lower *k*) around edges:

$$k = \frac{1}{1+\left|\nabla I\right|^2} \quad (5)$$

$\left|\nabla I\right|^2$ acting as an edge detector. It can be proven by minimizing the energy functional,

$$E(t) = \int_0^Y\int_0^X \Big[\left(I_t(x,y)\right)^2 + k(x,y)\left(\left(\nabla I(x,y,t)\right)\right)^2\Big]dxdy \quad (6)$$

that for any *k* dependant on spatial variables only, (4) converges to a solution.

Sampling *u(x,y,t)* at different times leads to multiscale representation of the input image.

It is interesting to compare *telegraph-diffusion* and diffusion equations. (4) is derived from (1) by adding second time derivative of the image. This allows control over the nature of the process. For very small *a()* or very large *k* and *c* the elastic process achieves diffusion behavior. This also happens after very long time, for any choice of coefficients, after transient effects have subsided.

Computational complexity of *telegraph-diffusion* method is the same as that of complex diffusion – O(n).

4. COLOR IMAGES

There are many formats of coding color information. The most common, RGB, represents each pixel as a triplet of red, green and blue color components. Together these convey information about the color and its intensity. Although used extensively in photography and color display technology, this format contains high correlation between color channels, which impedes image processing. Other color spaces exist (IHS, LAB, YCbCr), with lesser degree of inter-channel correlation. Most of these are derived from RGB by axes rotation, and none reduce the correlation to null for all images.

In [6], Sochen and Zeevi discuss representation of images as manifolds embedded in 5D space, e.g. (x, y, c_1, c_2, c_3), c_i representing color information and *(x,y)* – spatial information. Since this format allows simultaneous processing of all color and spatial information, inter color correlation is no longer a problem. Higher dimensions are also possible, incorporating other characteristics aside from intensity and color information.

The simplest way to adapt *telegraph-diffusion* method for use on color images is to treat each color channel as a separate image. This method disregards possible correlation between color channels; therefore the result is affected by color representation scheme one works with.

Figure 2: Elastic sheet denoising, separate color channels. Top row, left to right: original image, RGB result, YCrCb result. Bottom row: noisy image, RGB error image, YCrCb error image.

Another approach is to work with manifolds. Basic idea here is the same: pixels are pulled towards each other by elastic force, proportional to distance between them. The movement is damped by resistance proportional to pixel speed. The only difference is in definition of the distance, which is now a single value incorporating all color information, rather then 3 different ones, for each color channel.

In experiment section we dealt only with L2 norm (Euclidian distance). Other norms are possible, however, as well as mixtures between them [6].

5. EXPERIMENTAL RESULTS

Experiments were conducted on “squares” image (fig. 1, left) and on “street” image (fig. 2, top left), with additive white Gaussian noise (fig. 2, bottom left).

Convergence rates of *telegraph-diffusion* and complex diffusion methods are similar.

Telegraph-diffusion method is highly robust. It functions well even in highly noisy environment (fig. 1).

At a similar (visually) degree of noisiness, *telegraph-diffusion* method leaves much sharper edges then complex diffusion. This effect is clearly visible in figures 1 and 3, where error image of diffusion scheme contains much more meaningful details then that of *telegraph-diffusion*.

Color representation choice affects results of *telegraph-diffusion* method, when used on each color layer separately. With RGB color space, color artifacts begin to appear (fig. 2, top middle), particularly visible in originally gray areas. With YCbCr (luminance (Y) and chrominance (Cb and Cr)) space no new colors appear, but overall color saturation decreases, which can be seen from error image (fig. 2, bottom right) which is considerably more colorful then that of RGB.

Manifold representation of color information has none of the disadvantages and all of the advantages of separate layer processing – the colors are vivid, and no color artifacts appear.

6. DISCUSSION AND FUTURE WORK

As seen from the experiments, and from physical intuition, *telegraph-diffusion* algorithm favors smooth flat surfaces. This property allows the algorithm to be used as edge detector, since, eventually, smooth areas flatten, the noise disappears and we are left with a cartoon-like image, with distinct edges which can be extracted by simple gradient based edge detector.

The *telegraph-diffusion* algorithm is superior to complex diffusion as far as edge preservation is concerned. However it significantly flattens areas with low gradient, which diffusion leaves almost intact. Perhaps some

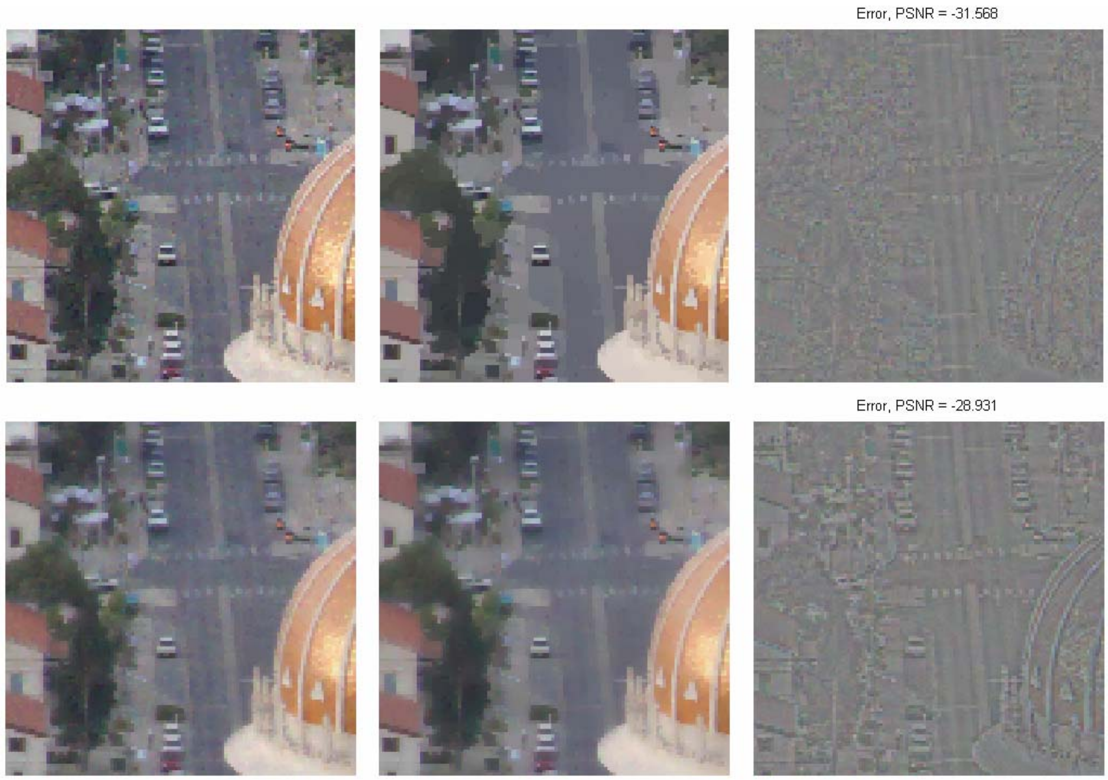

Figure 3: Top row: elastic sheet, 5D color representation. Bottom row: complex diffusion. From left to right: result image after 50 iterations, result image after 100 iterations, error image after 100 iterations.

combination of the two will perform better (can be controlled by the *a()* coefficient).

As any smoothing scheme, *telegraph-diffusion* method defines scale-space representation of the input image. Different scale levels are achieved when sampling the process at different time points. More useful information might be derived by working with several scales rather then single one.

Further exploration of effects of different norms in 5D color space should be performed.

7. CONCLUSION

The advantage of generalizing the diffusion equation to the proposed *telegraph-diffusion* processes is an increased flexibility in the evolution of the image along curves of discontinuities, i.e. edges. This is due to effect of "force" generated by the second derivative in time which represents acceleration. This results in better edge preservation, while yielding better signal-to-noise ratio and low noise sensitivity.

Combining this approach with geometric color representation results in sharper and more coherent color images then those obtained previously by the diffusion approach [4].

8. REFERENCES

[1] P. Perona and J. Malik, "Scale-Space and Edge Detection Using Anisotropic Diffusion," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 629-639, July 1990.

[2] L. Alvarez and L. Mazorra, "Signal and image restoration using shock filters and anisotropic diffusion," *SIAM J. Numer. Anal.*, vol. 31, no. 2, pp. 590–605, 1994.

[3] J. Weickert, "Coherence-Enhancing Diffusion Filtering*", Int. J. Comp.Vision*, vol. 31, pp.111-127, 1999.

[4] J. Weickert, "Coherence-Enhancing Diffusion of Color Images*", Int. J. Comp.Vision*, vol. 17, pp.199-210, 1999.

[5] G. Gilboa, N. Sochen, and Y.Y. Zeevi, "Image Enhancement and Denoising by Complex Diffusion Process," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 26, no. 8, pp. 1020-1036, August 2004.

[6] N. Sochen, and Y.Y. Zeevi, "Images as Manifolds Embedded in a Spatial-Feature Non-Euclidian Space," *EE-Technion report No. 1181*, November 1998.

[7] E. Zauderer, *Partial Differential Equations of Applied Mathematics*, 2ed.,Wiley, New York, 1998.