ENHANCEMENT OF MEDICAL IMAGES BY THE PAIRED TRANSFORM

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ABSTRACT

In this paper, we discuss the application of the two-dimensional paired representation for processing medical images. This representation leads to the effective solution of the discrete as well as continuous model of image reconstruction from their projections, and to the image enhancement. These two applications can be combined in order to receive high quality images. The method of paired representation of two-dimensional (2-D) images is considered with respect to the 2-D discrete Fourier transform (DFT). Basis functions of the paired transformation are defined completely by parallel projections. At the same time, the paired representation describes the image as a set of short 1-D real signals (splitting-signals) which completely determine the 2-D DFT of the image at disjoint subsets of frequency-points. The image enhancement procedure is thus can be reduced to processing splitting-signals and such process requires only a few spectral components of the image. For instance, the traditional α-rooting method of image enhancement can be fulfilled through processing the splitting-signals defined for only $3N-2$ frequency-points, when the image has the size $N \times N$, and N is a power of two. It is shown, that processing one or a few splitting signals leads to a high image enhancement.

Index Terms – Image enhancement, paired representation, fast paired transform, Fourier transform.

1. INTRODUCTION

In medical imaging, such as the computer tomography and magnetic resonance, two- and three-dimensional images (or stack of two-dimensional images) of different organs and tissues are produced. There are many sources of interference in the production of medical images, such as the movement of a patient, insufficient performance and noise of imaging devices. The quality of many images is poor in their contrast, and to improve the quality of images, enhance edges, to see clearly enough critical details, and reduce the noise for diagnostic purposes, methods of enhancement can be used. The purpose of image enhancement is thus to improve a digital image quality and to support the human perception [13-19].

We consider the Fourier transfer-based image enhancement, although other transforms such as the Hadamard and cosine transforms can be used for image enhancement as well [11, 12]. Our focus is on the new method of the α-rooting [8] which is used in the 1-D form for enhancing splitting-signals of the paired representation of the image. Such application of the paired transform together with the α-rooting results is very effective in image enhancement. The advantage of such application of the paired transform is in the fact, that this transform splits, or reveals the structure of the 2-D DFT in an optimal way, which allows us to not only to calculate faster the 2-D DFT, but to fulfil many operations over the spectrum through the splitting-signals. Indeed, the calculation and analysis of the two-dimensional discrete Fourier transform (2-D DFT) are the main steps of the image enhancement. In the paired representation, an image is considered as a certain totality of 1-D signals (which we call splitting-signals, or image-signals) that carry the spectral information of the 2-D DFT of the image at frequency-points of different and disjoint subsets in the frequency domain. The problem of 2-D image enhancement can thus be reduced to the split α-rooting method, when splitting-signals are processed separately to achieve high quality enhanced images, even by processing only one of a few such signals.

In this paper, a new effective paired-transform-based split α-rooting method of image enhancement is presented, when only one splitting signals is used enhancement, and which is performed without 2-D DFT. Each signal also can be modified by using only one enhancement coefficient instead of maximum $N/2$. It will be shown, that the image enhancement can be performed by using maximum $3N-2$ coefficients of the 2-D DFT of image ($N \times N$), when N is a power of two. The image splitting which is performing by the paired transform that does not require operations of multiplication.

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1.1. Frequency Domain Methods

The Fourier transform-based method of image enhancement consists in computing the 2-D DFT of the image, manipulating the transform coefficients by a specific operator $\mathbf{M}$, and performing then the inverse 2-D transform. We stand here on the case when $\mathbf{M}$ is an operator of magnitude and the enhancement is described by

$$\{f_{n,m}\} \to \{F_{p,s}\} \to \{G_{p,s} = M\left[|F_{p,s}|\right] e^{-j\vartheta_{p,s}}\} \to \{g_{n,m}\} \quad (1)$$

where $\vartheta_{p,s}$ is the phase spectrum of the image.

In the α-rooting method of image enhancement [8], the magnitude of the Fourier transform of the image is transformed as $M\left[|F_{p,s}|\right] = |F_{p,s}|^\alpha$ at frequency-point (p,s), and the parameter α is taken from the interval $(0,1)$. In other words, the components of the Fourier transform, $F_{p,s}$, are multiplied by coefficients $C(p,s) = |F_{p,s}|)|^{\alpha-1}$ which we call *enhancement coefficients*.

1.2. Paired Representation

In the paired representation transform [10], the 2-D $N \times N$-point DFT can be split into a maximum number of short 1-D DFTs. In the $N = 2^r$ case, where $r > 1$, the transform is defined by the following paired functions

$$\chi'_{p,s,t}(n,m) = \begin{cases} 1, & \text{if } np + ms = t \bmod N \\ -1, & \text{if } np + ms = (t + N/2) \bmod N \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where $n, m = 0 : (N-1)$. There exists such a set U of triples (p, s, t), that the totality of paired functions $\chi' = \{\chi'_{p,s,t}; (p,s,t) \in U\}$ compose a basis in the linear space of discrete images of size $N \times N$ (see [ ] for detail). In numbering of the paired functions, two parameters relate to the frequency, and one to the time. The paired splitting-signal with number (p, s),

$$f_{T'_{p,s}} = \{f'_{p,s,0}, f'_{p,s,1}, f'_{p,s,2}, \ldots, f'_{p,s,N/2-1}\}.$$

is defined by

$$f'_{p,s,t} = \chi'_{p,s,t} \circ f = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \chi'_{p,s,t}(n,m) f_{n,m} \tag{3}$$

where $t = 0 : (N/2 - 1)$ with step 1, or $2^n = g.c.d(p, s)$. The following holds:

$$F_{\overline{(2m+1)p}, \overline{(2m+1)s}} = \sum_{t=0}^{N/2-1} \left(f'_{p,s,t} W^t\right) W^{mt}_{N/2} \tag{4}$$

for $m = 0 : (N/2 - 1)$. Thus the 2-D DFT of f at frequency-points of the following subset

$$T'_{p,s} = \left\{(p,s), (\overline{3p}, \overline{3s}),, (\overline{5p}, \overline{5s}) \ldots, (\overline{(N-1)p}, \overline{(N-1)s})\right\} \tag{5}$$

If $g.c.d(p, s) = 2^n$, where $n \geq 0$, then the components $f'_{p,s,t} = 0$ if t is not divisible by 2^n. The splitting-signal $f_{T'_{p,s}}$ is considered as the signal of length $N/2^{n+1}$,

$$f_{T'_{p,s}} = \{f'_{p,s,0}, f'_{p,s,2^n}, f'_{p,s,2\cdot 2^n}, \ldots, f'_{p,s,N/2-2^n}\}$$

The $N/2$-point DFT in the right side of (4) represents the $N/2^{n+1}$-point DFT of the splitting-signal $f_{T'_{p,s}}$ modified by the vector of twiddle factors $\{W^t_{N/2^n}; t = 0 : (N/2^{n+1} - 1)\}$.

The $2^r \times 2^r$-point DFT is split into $2^r 3 - 2$ short DFTs, namely, $3 \cdot 2^{r-1}$ 2^{r-1}-point DFTs, $3 \cdot 2^{r-2}$ 2^{r-2}-point DFTs, ..., and six 2-point DFTs. The totality of $3N - 2$ splitting-signals $\{f_{T'}, T' \in \sigma'\}$ consists of $2^{r-1}3$ signals of length 2^{r-1}, $2^{r-2}3$ signals of length 2^{r-2}, and so on. The summary length of all splitting-signals equals N^2.

As an example, Figure 1 illustrates the image of size 128×128 in part a, along with the splitting-signal $f'_{T_{6,1}}$ of length 64 in b, the 1-D DFT over this splitting-signal in c, and frequency-points of subset $T'_{6,1}$ at which the 2-D DFT of the image is filled by the 1-D DFT in d.

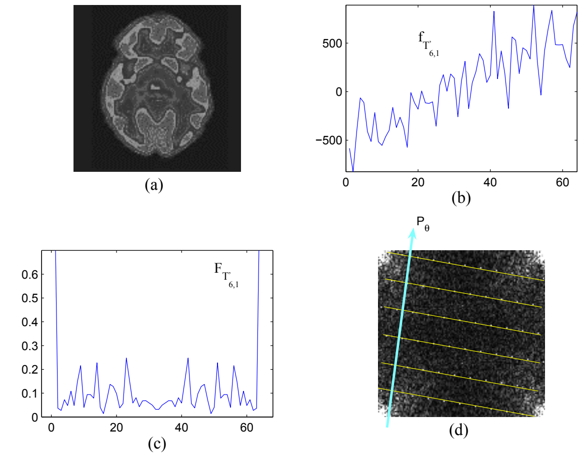

Fig. 1. (a) Original image. (b) Splitting-signal $f_{T_{6,1}}$. (c) The 1-D DFT of the splitting-signal [no shifting in the center]. (d) Arrangement of values of the 1-D DFT in the 2-D DFT of the image at frequency-points of the subset $T'_{6,1}$.

2. EXPERIMENTAL RESULTS

2.1. Concept of 1-D $\alpha-$rooting

$\alpha-$rooting also named as root filtering is first introduced by Ersoy [8]. The purpose of root filtering is enhancement of details in the image that to be processed. Root filtering is adaptive to pixels or elements. The basic idea of root filtering can simply be explained as emphasizing the high frequency components more than low frequency components. The mathematical background of root filtering is given by the below equation. Let f_n be the 1-D signal and F_k be the 1-D DFT of f_n. Than root filtering can be defined as:

$$F^*_k = |F_k|^{(\alpha-1)}.F_k$$

where F^*_k is the processed signal. As it can be seen from the root filtering equation each element of signal is multiplied by coefficients depend on element itself. Figure 2 shows a sample application of root filtering. The coefficients defined as C_k refers to $|F_k|^{(\alpha-1)}$.

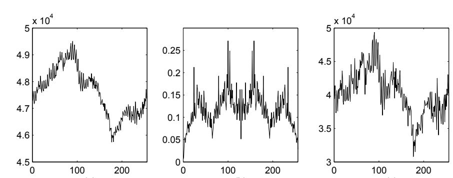

Fig. 2. (a) Original signal f_n of length 256. (b)Coefficients C_k of length 256 (c) Enhanced signal f'_n of length 256.

The high frequency elements in signal is enhanced by root filtering as can be seen in figure 2.

2.2. Method of Paired enhancement

The algorithm used in method is paired enhancement is as follows:

Step 1: Calculate paired splitting signals $f'_{T'_{p,s}}$.

Step 2: Multiply the paired signal by exponential weights

$$\widetilde{f}_{T'_{p,s}} = f'_{T'_{p,s}} W^{-t/N1}$$

where $N1 = N/2$ and $t = 0 : (N-1)$.

Step 3: Perform the 1-D DFTs of the weighted paired splitting-signals

$$\widetilde{f}_{T'_{p,s}} \rightarrow F_k = \sum_{t=1}^{N-1} \widetilde{f}_{p,s,t} W^{kt}, \quad k = 0 : (N-1).$$

Step 4: Multiply coefficients of the transforms of splitting-signals by coefficients $C_k = A|F_k|^{\alpha-1}$.

Step 5: Fill the 2-D DFT by new 1-D DFTs at points of sets $T'_{p,s}$.

Step 6: Perform the inverse 2-D DFT.

In paired method enhancement with using different α's, the paired signal is decomposed into parts and each part is processed with a different optimum α. The optimality is with respect to the enhancement measure QME [12].

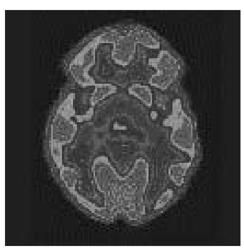

Fig. 3. Original PET image.

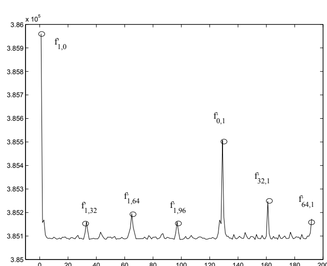

Fig. 4. Energy graph of splitting signals of PET image.

In figure 3 the original Pet image is shown. In figure 4 the energy values of $3N/2$ paired signals are plotted. It can be seen that the energy is usually condensed on the signals which has p or s as 2's power.

Figure 5 shows the results of enhancement respectively by $\alpha-$rooting [a] 2-D image ,[b] tensor splitting-signal $f_{T_{6,1}}$ [c] paired splitting-signal $f'_{T'_{6,1}}$ $\alpha = 0.92$ (optimum for traditional $\alpha-$rooting). In [d] paired splitting-signal $f'_{T'_{6,1}}$ is used however different optimum α's are used for each decomposition of it. It can be seen here the paired $\alpha-$rooting with different optimum α's outperforms 2-D $\alpha-$rooting with respect to enhancement of contrast measure QME. Figure 6 shows the same methods with different splitting-signal $f_{T_{0,1}}$ and different $\alpha = 0.95$.

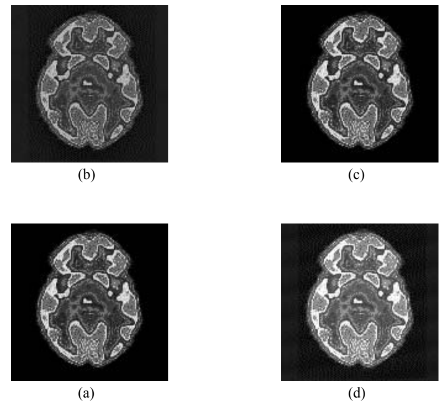

Fig. 5. (a) Enhanced by traditional alpha rooting QME=27.22 (b) Enhanced by tensor splitting signal (1,6) QME=12.94 $\alpha = 0.92$ (c) Enhanced by paired by paired splitting signal (1,6) with same $\alpha = 0.92$ QME=12.92 (d) Enhanced by paired splitting signal (1,6) with different optimum α's for each decomposition of paired signal QME=27.68

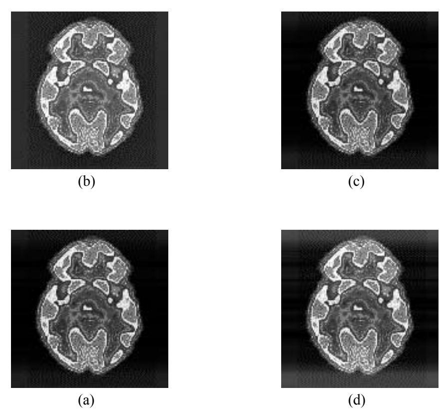

Fig. 6. (a) Enhanced by traditional alpha rooting QME=19.77 (b) Enhanced by tensor splitting signal (0,1) QME=16.27 $\alpha = 0.95$ (c) Enhanced by paired tensor splitting signal (0,1) with same $\alpha = 0.95$ QME=19.98 (d) Enhanced by paired tensor splitting signal (0,1) with different optimum α's for each decomposition of paired signal QME=15.29

In figure 7 the same methods are applied to luminance component of color Pet image and visually both 1-D paired methods outperforms 2-D traditional α rooting.

Figures 8 and 9 show the application of 2-D $\alpha-$rooting and proposed methods to a mammogram image and Fish image. The improvement of the contrast is seen clearly. Enhancement measure QME also shows this improvement. Proposed methods work slightly better than traditional method.

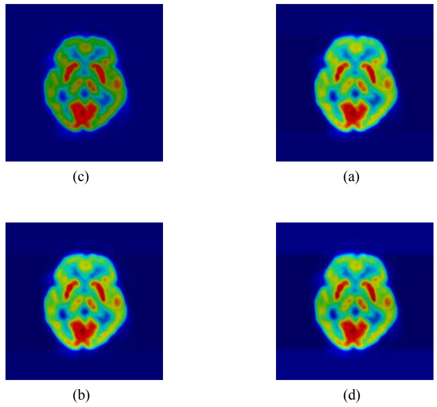

Fig. 7. (a) Enhanced by traditional alpha rooting (b) Enhanced by tensor splitting signal (1,1) $\alpha = 0.9$ (c) Enhanced by paired tensor splitting signal (1,1) with same $\alpha = 0.9$ (d) Enhanced by paired tensor splitting signal (1,1) with different optimum α's for each decomposition of paired signal

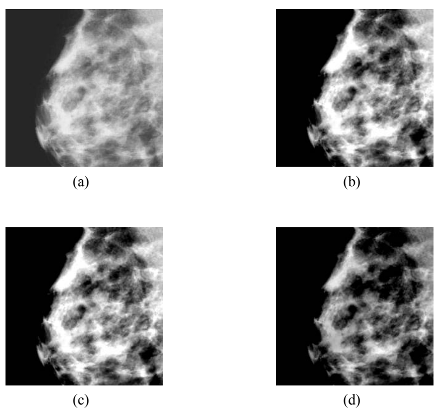

Fig. 8. ((a) Original image QME=3.43 (b) Enhanced traditional $\alpha-$ rooting QME=7.03 $\alpha = 0.92$ (c) Enhanced by paired splitting signal (1,1) with same $\alpha = 0.92$ QME=7.56 (d) Enhanced by paired splitting signal (1,1) with different optimum α's for each decomposition of paired signal QME=8.00

3. CONCLUSIONS

Two methods of enhancement are proposed here. Enhancement by paired splitting signal with same α for all decompositions of splitting-signal and with different α's for each decomposition of splitting-signal. Each proposed method decreases the burden in processing image two dimensionally. These methods will be improved by developing of processing of splitting-signal with one α weighted coefficient.

4. REFERENCES

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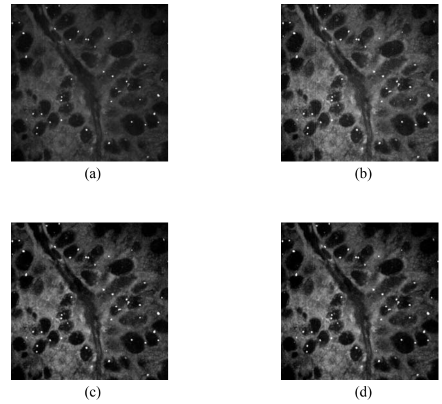

Fig. 9. ((a) Original image QME=15.84 (b) Enhanced traditional $\alpha-$ rooting QME=21.68 $\alpha = 0.92$ (c) Enhanced by paired splitting signal (1,0) with same $\alpha = 0.92$ QME=18.52 (d) Enhanced by paired splitting signal (6,1) with different optimum α's for each decomposition of paired signal QME=22.09

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