FAST AND STABLE VECTOR SPLINE METHOD FOR FLUID APPARENT MOTION ESTIMATION

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ABSTRACT

Apparent motion estimated on satellite data is used for example to compute the wind field in meteorology, and surface currents in oceanography. The satellite images display turbulent fluids with strong rotational patterns at different spatial and temporal scales. This specificity necessitates devising adapted methods, allowing to control the divergence and curl of the retrieved motion field. Vector spline methods are very adapted to that purpose. The vector spline problem is defined as finding a motion field that satisfies a temporal conservation equation at selected control points and that minimizes a regularity constraint in all the image domain. An exact solution of this problem can be found for the 2nd order div-curl regularity constraint. The retrieval of the solution does not require an iterative minimization procedure: a dense matrix must be inverted to compute the spline's coefficients. This matrix unfortunately becomes large and ill-conditioned as the number of control points increases, making the vector spline approach unsuitable for processing large satellite images. This paper presents a method called "Partition of Unity and Optical Flow" (PUOF), based on a decomposition of the spatial domain: local vector splines are computed in subdomains of the image, then merged using a partition of unity algorithm. The resulting motion field is a good approximation of the exact vector spline solution, and its retrieval is numerically stable and computationally affordable even when processing large data sets, as demonstrated by results obtained on sequences of synthetic and meteorological images.

***Index Terms*—** Non-rigid motion, Fluid flow, Second order div-curl regularity constraint, Vector spline, Partition of unity, Radial basis function.

1. INTRODUCTION

Estimating the apparent motion field from satellite data has crucial applications in meteorology for estimating wind and clouds motion, and in oceanography for assessing surface current. For these applications, satellite images display fluid turbulent motion. The challenge is to devise a motion estimation method adapted to the strong rotational patterns of the motion field, and computationally affordable for processing sequences of very large satellite images.

The classical approaches of apparent motion estimation attempt to solve a temporal conservation equation in all the image domain. This equation provides information on only one component of the 2D motion field: e.g. the usual luminance conservation equation provides the projection of motion onto the image gradient. This aperture problem is solved by adding a regularity constraint, most often based on the L^2 norm [1]. This general framework have been adapted to the specificities of fluid motion. In [2, 3] the continuity equation is used for a better modelling of compressible flows; in [4] the 1st order div-curl regularity is applied to assess the motion of non deformable objects; in [5] the spatial scales related to turbulence are handled in a multiscale framework and the 2nd order div-curl regularization is used for controlling the divergence and vorticity of the motion field. An alternative tracking methodology for fluid structures (vortices) is proposed in [6]: a model describing the geometry and intensity of vortices is fitted to the data; the tracking is based on analysing the evolution of the model's parameters.

Most of these methods rely on calculus of variations: an energy functional is defined, including a data confidence (the residual of the conservation equation) and a regularity term (the norm of the motion field). The functional is non convex and multi-scale strategies must be implemented for avoiding local minima. Furthermore, if the regularity constraint is based on the 2nd order div-curl norm, minimization involves complex numerical schemes for solving 4th order PDEs, leading to numerical unstability and long computational time.

Vector spline methods constitute efficient alternatives to the classical approaches. They are issued from the data interpolation or approximation domain: a vector field is searched, that minimizes a regularity constraint over the spatial domain, and that interpolates or approximates vector observations at control points. An exact solution of both interpolation and approximation problems can be found if the regularity constraint is based on the 2nd order div-curl norm. This solution is a linear combination of thin-plate radial basis functions, weighted by coefficients obtained by solving a linear system. Vector splines for motion applications have been introduced in [7] to reconstruct the wind field from scattered balloon measurements. In [8] an adaptation of this formalism has been proposed for apparent motion estimation: the control points are defined according to local criteria, such as locations with sufficient contrast. The observations at control points are issued from the luminance conservation equation, and consist of the projection of the motion onto the spatial gradient. This work has been adapted in [9] to the use of the continuity instead of luminance conservation equation, better adapted to compressible fluids.

In practice, the retrieval of a vector spline requires inverting a dense linear system that becomes unstable and computationally prohibitive if the number of control points is large. To overcome this problem, quasi-interpolation methods [10] have been introduced. These methods approximate the exact solution of the interpolation or approximation problems by using bell-shaped instead of thin-plate basis functions. The inversion is thus fast and stable, but it is difficult to apply such techniques for motion estimation: they are hard to adapt to the quasi-interpolation of projected data (observations provided by the conservation equation), and they cannot handle an uneven spatial repartition of control points, thus preventing the definition of control points from local image criteria.

We propose a new method, Partition of Unity for Optical Flow (PUOF), that presents the same advantages as vector spline methods, but numerically stable and computationally affordable for large images. As for quasi-interpolation, the PUOF method provides an approximation of the exact solution of the interpolation/approximation problems. The approach is based on a spatial domain decomposition: local vector spline problems are solved in subdomains, then merged by a partition of unity algorithm.

This paper is structured as follows: in section 2 a short introduction to vector spline methods for fluid motion estimation is given. Readers should refer to [8, 9] for implementation details. The proposed PUOF method is then defined: section 3 details the decomposition of the spatial domain by an adaptative quadtree; section 4 explains how the local vector splines are merged to yield a unique motion field over the whole domain. Results are presented in section 5 for oceanographic and meteorological applications. Finally, conclusions and perspectives are given in section 6.

2. VECTOR SPLINE METHODS FOR FLUID MOTION ESTIMATION

Apparent motion estimation in the vector spline formalism requires (1) selecting control points in the image domain Ω, and (2) defining a conservation equation, to be exactly or approximately satisfied at control points. The conservation equation allows to compute indirect observations of the motion field at the control points, that are interpolated (or approximated) using basis functions minimizing the 2nd order div-curl regularity.

The control points must be selected in regions where a sufficient contrast (in space and time) is available for motion perception, discarding uniform or static areas of the image. Local image criteria are thus used to select control points, as for instance a double thresholding applied to the magnitude of the spatial gradient and to a motion index, defined as the ratio of the temporal derivative and the spatial gradient.

A conservation equation is chosen and assumed valid at the resulting n control points P_i: in the case of incompressible fluids, the usual luminance conservation, and in the case of compressible fluids, the continuity equation. The vector spline formalism enables interpolating observations provided that they can be expressed as a linear operator $\mathcal{L}_i$ applied to the motion field at the i^{th} control point: $\mathcal{L}_i\mathbf{w} = \nabla I \cdot \mathbf{w}$ for the luminance conservation, and $\mathcal{L}_i\mathbf{w} = \nabla I \cdot \mathbf{w} + I\ \mathrm{div}(\mathbf{w})$ for the continuity equation. The conservation equation is then $\mathcal{L}_i\mathbf{w} = -I_t$. The vector spline problem consists of finding $\mathbf{w}$ minimizing:

$$\sum_{i=1}^{n}(\mathcal{L}_i\mathbf{w} + I_t)^2 + \lambda \int_\Omega \alpha\|\nabla\mathrm{div}\mathbf{w}\|^2 + \beta\|\nabla\mathrm{curl}\mathbf{w}\|^2 \quad (1)$$

The λ parameters tunes the confidence on data: the limit case $\lambda \to 0$ corresponds to the interpolation problem. Under specific conditions on the control points (e.g. they must be non aligned) the problem (1) admits a solution, uniquely defined for each combination of control points, conservation equation, α, β and λ parameters. This solution is further referred as the 'exact spline', and is a linear combination of radial basis functions, centered at the control points. The basis functions are built upon derivatives of $r^4 \log r$ (r being the distance to the associated control points) and tend towards infinity with r.

The coefficients of the linear combination are obtained by inverting a dense matrix which size depends on the number of control points. If the latter increases, the required computational cost becomes prohibitive, the system becomes ill-conditioned and its inversion unstable. Practically, it becomes nearly impossible to solve the system with more than 5,000 control points. This limitation motivates the proposed PUOF method, presented below.

3. SPATIAL DOMAIN DECOMPOSITION

The goal is to subdivide the spatial image domain into rectangular cells, using a quadtree strategy driven by the location of control points: cells must be small in regions with important concentration of control points, and reversely large in areas with fewer control points. The algorithm is adapted from the works of [11], devoted to the rendering of very large 3D meshes.

Let's assume that at a given stage of the algorithm, the image is subdivided into rectangular cells. To each cell C^j corresponds a set of N_j control points P_i^j, lying within a ball

B_j centered on the cell and containing it. Neighboring balls overlap each other, avoiding aliasing effects caused by a subdivision into adjacent rectangles.

The subdivision strategy is defined such as each ball contains a number of control points between a low and a high threshold, to enable the computation of the spline, and to ensure the numerical stability. The initial cell is the whole domain. A cell is subdivided into four new cells of equal size if its number of control points is above the high threshold. This subdvision stage is iterated until convergence. For the resulting cells containing not enough control points, the radius of the surrounding ball is increased until this criterion is satisfied.

4. PARTITION OF UNITY

After performing the quadtree subdivision, the image domain is split into M cells, each cell C^j corresponding to a set of control points P_i^j. The motion is computed in each cell as the vector spline minimizing equation (1), providing a set of M vector fields $\mathbf{w}_j$.

Although defined over the whole spatial domain, each vector field $\mathbf{w}_j$ is only valid within its associated cell C_j. These local fields must therefore be combined to define a global motion estimate on the whole image domain.

For that purpose we adopt a partition of unity approach: we define for each cell C^j an influence function ϕ_j as $\phi_j(\mathbf{x}) = \phi(||\mathbf{x} - \mathbf{x}_j||/r_j)$, $\mathbf{x}_j$ being the center of cell C^j, r_j the radius of ball B_j, and ϕ a smooth bell-shaped radial basis function with compact support. In this paper ϕ is defined as in [12] by:

$$\phi(r) = (1-r)_+^6(35r^2 + 18r + 3) \qquad (2)$$

with $(1-r)_+^6$ equals to 0 if $r > 1$. After normalisation of the influence functions, the resulting motion field is computed as:

$$\mathbf{w}(x) = \sum_{j=1}^{M} \Psi_j(\mathbf{x})\mathbf{w}_j(\mathbf{x}) = \sum_{j=1}^{M} \frac{\phi_j(\mathbf{x})}{\sum_{k=1}^{M}\phi_k(\mathbf{x})}\mathbf{w}_j(\mathbf{x})$$

5. RESULTS

We present results obtained on a synthetic sequence, obtained by numerical integration of the OPA ocean circulation model, and simulating satellite acquisitions of Sea Surface Temperature. This sequence provides a reference: the surface current is a state variable of the model, to which the estimated motion fields are compared. A frame of the OPA sequence and the corresponding reference motion field are presented in the upper part of figure 1.

Apparent motion is estimated with PUOF and the exact spline under the same conditions: same set of 955 control points, luminance conservation equation, $\alpha = 0.9$, $\beta = 0.1$, $\lambda = 0$ in eq (1). The resulting motion fields are displayed in the bottom part of figure 1. There is no perceptible difference between the two estimated motion fields: the angular difference between both fields is 0.037 degrees, the relative error on the magnitude of motion less than 2%. PUOF thus provides a good approximation of the exact spline.

The numerical stability and computation time are then compared. PUOF requires a computational time proportional to the number of control points, and its numerical stability is constant. For the exact spline, if the number of control points is increased from 1,000 to 5,000, the computation time is multiplied by 1,000 and the numerical stability is severely affected (conditionning of the system multiplied by 100). These results prove that the PUOF method fulfills its objectives: it has the quality of estimation of the exact spline and is numerically stable and affordable for large sets of control points.

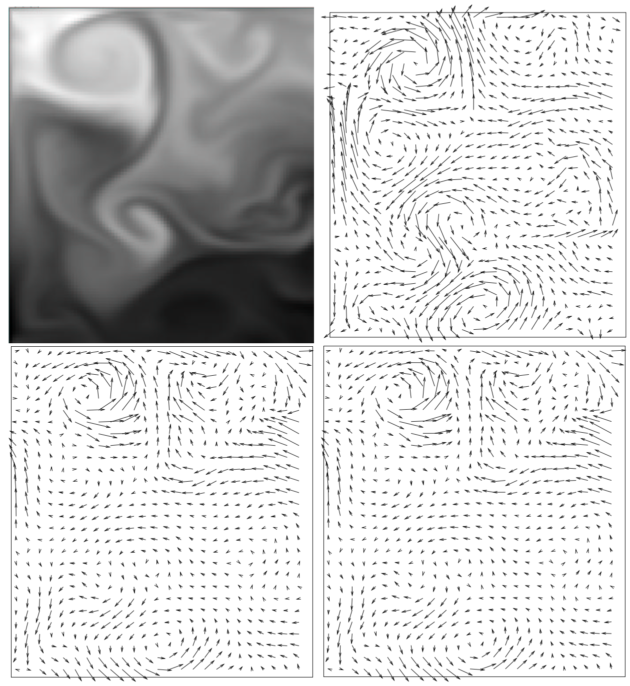

Fig. 1. Top: frame of the OPA sequence and reference motion field. Bottom: exact spline and PUOF.

The following motion estimation methods have been applied to the OPA sequence: (1) the exact spline, (2) PUOF, (3) Corpetti and Mémin's method [5], a variational and multiscale implementation using the 2nd order div-curl constraint, and (4) the classical optical flow [1] with L^2 regularity. The mean errors with the reference are computed and displayed in table 1: angular error E_a and relative error E_n on the magnitude of motion. The smallest angular errors are obtained with PUOF and the exact vector spline. Qualitative assessments furthermore indicate that the spline-based models are more powerful to detect vortices.

Finally, figure 2 displays results of PUOF applied to a real satellite sequence. The motion is computed on an extract of Meteosat-7 waver vapour acquisitions, displaying a large vor-

Method	E_a (degrees)	E_n (%)
Exact spline	42.401	0.871
PUOF	**40.948**	**0.694**
Corpetti & Mémin	47.414	0.587
Horn & Schunck	50.116	0.729

Table 1. Angular and norm errors between models and OPA reference field.

tex. 3 results are shown, corresponding to an increasing number of control points: up to 10,000, which is far beyond the number of control points a standard vector spline can practically manage. With the smallest amount of control points, only the motion of the vortex and a global translation are retrieved. As the number of control points increases, the resulting field presents a greater level of details. This suggests a link between the density of control points and the spatial scale of analysis.

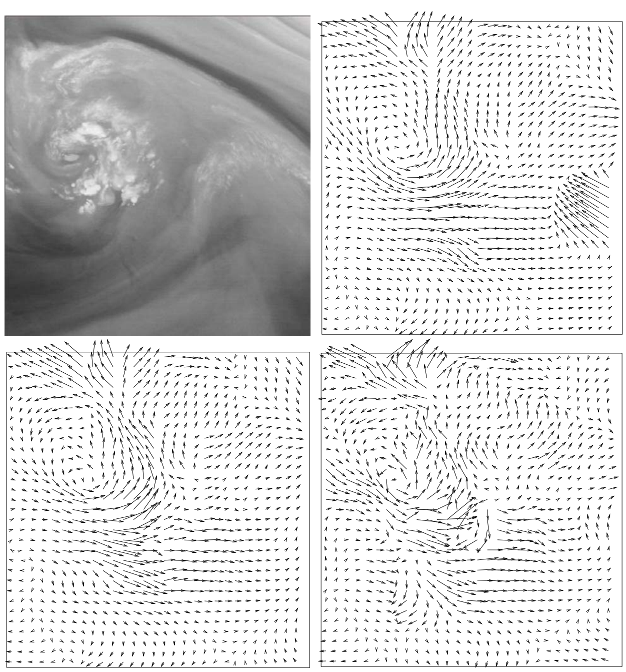

Fig. 2. Top-Left: extract of a METEOSAT-7 WV sequence (c) Eumetsat. Top-right, bottom-left and bottom-right: PUOF with 1871, 3742 and 9354 control points.

6. CONCLUSION

The proposed PUOF method has the same the advantages as vector spline methods for fluid motion estimation: control on the divergence and curl of the retrieved motion field, solution obtained without iterative minimization. PUOF is furthermore computationally affordable and numerically stable even when the number of control points gets large. This makes PUOF a suitable method for motion estimation on sequences of satellite images displaying turbulent fluids. Current work seeks to formulate this approach in a multiscale framework.

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