ON SURFACE TOPOGRAPHY RECONSTRUCTION FROM GRADIENT FIELDS

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ABSTRACT

In this paper, we propose and study surface reconstruction techniques for surfaces with high frequency height variation, which are common for example, in paper and textile manufacturing. Traditionally, photometric stereo methods have been developed and evaluated on objects with additive Gaussian noise. The minimization based methods may perform well on large objects, but they smooth the inherent high frequency variation of machined surfaces in the reconstruction. We extend a Fourier integration method with Wiener filter to reconstruct surfaces from two gradient fields. The experimental results validate that the proposed method performs well on surfaces with high frequency height variation.

Index Terms— Topography, surface reconstruction, gradient fields, Fourier transform, Wiener filter

1. INTRODUCTION

Surface topography is an important quality parameter in many industrial applications, such as paper and textile manufacturing. Undesired surface topography variations can reflect imperfections in manufacturing process, product operational efficiency, and life expectancy. Depth recovery techniques, such as shape from shading (SfS) [1], and photometric stereo (PS) [2], provide surface gradients in a fast and non-contact manner. In order to obtain surface topography, the relative height values of the surface, the surface gradients have to be integrated. However, in practice the surface gradients contain noise, which can be derived from imaging and other measurement errors. Several solutions have been proposed to integrate of the calculated gradient fields. The traditional method for integrating the surface height from gradient information is the Frankot-Chellappa algorithm [3]. The recent developments, such as α surfaces, M-estimators, Regularization and Diffusion, in surface reconstruction from gradient fields have been compared to minimization based Frankot-Chellappa and Poisson [4] methods in [5]. Traditionally, the performance of the surface reconstruction methods have been evaluated on surfaces with objects, such as flower pots, faces, peaks, and ramps with rather strong additive Gaussian noise. However, the monitoring of surface roughness and texture, that is smaller scale variations with limited noise levels, are frequently of interest in manufacturing processes. Recently Hansson [6, 7] has studied two- and three-light photometric stereo in paper surface reconstruction. In his methods, the paper surface topography is calculated from one and three gradient fields in two- and three-light methods, respectively. Unfortunately, he does not provide comparison to alternative surface reconstruction methods.

The contributions of this paper are an extension and comparison of surface reconstruction methods in surface topography reconstruction. A four-light photometric stereo method is introduced, which is developed from Hansson's two-light method. In the experiments, surface reconstruction techniques are evaluated with gradient fields calculated from surfaces containing high frequency variation. The proposed method is shown to preserve the original small scale variation in reconstructed surfaces.

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2. REVIEW OF PHOTOMETRIC STEREO

In photometric stereo, the viewing direction is held constant while the direction of the illumination between successive images is varied. Thus, the correspondence between image points is known *a priori*. The use of the radiance values at a single image location, in successive views, makes the technique photometric. The technique can be used to determine the surface orientation at each image point [2].

For Lambertian surfaces the reflected intensity is independent of the viewing direction. However, the intensity depends on the direction of the light source. Lambert's Law [8] represents the image intensity i at the point (x, y)

$$i = \rho\lambda(\mathbf{l}^T \cdot \mathbf{n}) \,, \qquad (1)$$

where ρ is the surface albedo, λ is the intensity of the light source, $\mathbf{n} = [n_1, n_2, n_3]^T = \frac{[p,q,1]^T}{\sqrt{p^2+q^2+1}}$ is the unit normal to the surface and $\mathbf{l} = [cos(\tau)sin(\sigma), sin(\tau)sin(\sigma), cos(\tau)]^T$ is the unit vector toward the light source. Elements p and q are surface partial derivatives measured along the x and y axes, respectively. τ is the tilt angle of illumination; the angle that the projection of the illuminant vector incident onto the test surface plane makes with an axis in that plane. σ is the slant

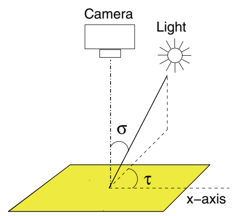


Fig. 1. Geometry of the illumination.

angle that the illuminant vector makes with a normal to the test surface plane. Fig. 1 illustrates the tilt and slant angles. Lambert's Law assumes orthogonal projection and constant illumination over the surface. In orthogonal projection, light rays traveling from the object to the image are parallel, and the focal length is infinite.

3. SURFACE RECONSTRUCTION METHODS

3.1. Extension of Hansson's Two-light Method to Four-light

Hansson and Johansson presented a two-light PS method in [6]. They used the tilt angles $0°$, and $180°$ and derived a directed derivative p of the surface for the respective tilt angles. We propose that Hansson's two-light PS method can be extended to four-light PS by adding tilt angles $90°$, and $270°$, that correspond to calculation of a directed derivative q of the surface.

Hansson used Wiener filter for the computation of the surface height from the directed derivatives. They modeled the imaging as

$$s'_p(x, y) = s_p(x, y) * PSF + n(x, y)\,, \tag{2}$$

where $s_p(x, y)$ is the directional derivative, $n(x, y)$ is the noise, and $s_p(x, y) * PSF$ represents convolution of the signal by a point-spread function. Fourier transform of $s'_p(x, y)$ is

$$S'_p(u, v) = S_p(u, v)OTF(u, v) + N(u, v)\,, \tag{3}$$

where u and v are the spatial frequencies, $S_p(u, v)$ is the Fourier transform of $s_p(x, y)$, $OTF(u, v)$ (optical transfer function) is the Fourier transform of PSF, and $N(u, v)$ is the Fourier transform of $n(x, y)$. Hence, the restoration filter without noise is

$$H_{I,p}(u, v) = i2\pi uOTF(u, v)\,. \tag{4}$$

For the directional derivative q, the restoration filter can be written as

$$H_{I,q}(u, v) = i2\pi vOTF(u, v)\,. \tag{5}$$

Hansson utilized Wiener filter as a restoration filter, and in integration of the filtered signal. The applied Wiener filter is given as follows:

$$H_{R,k} = \frac{H^*_{I,k}}{|H_{I,k}|^2 + SNR(u, v)^{-1}}, \quad k \in \{p, q\}\,, \tag{6}$$

where $SNR(u, v) = |F(u, v)|^2/|N(u, v)|^2$ is the signal-to-noise ratio in the frequency domain. $F(u, v)$ is the Fourier transform of surface height function.

The Fourier transform of the surface height functions is given by

$$F_k(u, v) = S'_k(u, v)H_{R,k}(u, v), \quad k \in \{p, q\}\,. \tag{7}$$

We propose that four-light PS can be obtained by applying weighting functions for surface height functions in Fourier domain. Hansson applied in [7] for three-light PS weight functions, which are proportional to the strength of the surface height functions. However, we found in [9], that symmetric weighting functions are less sensitive to correct estimation of SNR than Hansson's weights. For the four-light case, Symmetrical weight functions are as follows

$$w_p = \left\{ \begin{array}{ll} 1 & -45° \leq \theta < 45° \vee 135° \leq \theta < 225° \\ 0 & \text{otherwise} \end{array} \right\}\,, \tag{8}$$

and

$$w_q = \left\{ \begin{array}{ll} 1 & 45° \leq \theta < 135° \vee 225° \leq \theta < 315° \\ 0 & \text{otherwise} \end{array} \right\}\,, \tag{9}$$

where θ is the angle with respect to the x-axis in the test surface plane. Symmetrical weighting functions are not dependent on the surface height functions, but the signals from illumination direction are assumed to provide the most correct information from respective direction.

Using the weight functions, the Fourier transformed surface height functions can be integrated to common surface height function in the frequency plane

$$F = F_p w_p + F_q w_q\,. \tag{10}$$

By using inverse Fourier transform on F, the reconstructed topography of the surface is obtained.

3.2. Frankot-Chellappa and Poisson algorithms

The common approaches for reconstructing surface from gradient fields are Frankot-Chellappa [3] and Poisson solver [4]. Both methods minimize the reconstruction error in least square sense given by

$$J(Z) = \int\int ((Z_x - p)^2 + (Z_y - q)^2)dxdy\,,$$

where Z is the surface to be obtained, $\{Z_x, Z_y\}$ gradient field of Z, and $\{p, q\}$ the given non-integrable gradient field. The gradient field of Z can be written as $\{Z_x, Z_y\} = \{p, q\} + \{\epsilon_x, \epsilon_y\}$, where $\{\epsilon_x, \epsilon_y\}$ denote the correction gradient field, which makes the non-integrable field to integrable. The Poisson solver minimizes the norm of the correction gradient field. In Frankot-Chellappa method, the non-integrable gradient field is projected on to set of integrable slopes using the Fourier basis functions [5].

4. EXPERIMENTS

The purpose of the experiments was to study the reconstruction of inhomogeneous planar surfaces from gradient fields using several surface reconstruction techniques. Six paper and cardboard samples were measured using a laser profilometer and four-light photometric stereo method. Correlations, surface reconstruction errors, and power spectra were calculated for reconstructed topographies.

Topographies were reconstructed using the four different approaches: Fourier domain integration 1) using Hansson's two-light method, and 2) using symmetrical weighting functions in order to extend Hansson two-light method to four-light one, 3) Frankot-Chellappa, and 4) Poisson Solver, denoted as Two-light, Symmetric, Frankot-Chellappa, and Poisson. The results are calculated from topographies, which were scaled to the same mean and variance as profilometer topography, since gradient fields do not provide information on the scale of the measurements. The computational complexities of reconstruction methods are log linear, since all the methods are based on Fast Fourier Transform (FFT).

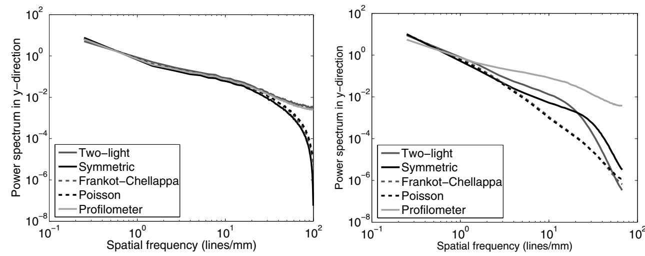
Experiments were performed using three sample sets: 1) two light weight coated (LWC) paper samples, 2) two supercalendered (SC) paper samples, and 3) two base cardboard samples. The first two sample sets, LWC and SC are similar in roughness, while the cardboard is significantly rougher.

An optical profilometer was used as a reference topography measurement device. The applied profilometer was Rodenstock RM-600 3-D/C laser profilometer with resolution of 5 μm in the profile direction, and 5 μm between profiles. Each sample was measured with a laser profilometer, and the imaged area was 15 $mm \times 15\ mm$, which corresponds to image size of 3000 $\times$ 3000 pixels.

4.1. Profilometer data

The purpose was to evaluate methods on high frequency containing data without PSF and controlled noise level. First the surfaces were reconstructed from gradients fields, which were calculated from profilometer measured topographies. Therefore, the OTF and SNR -functions were omitted, and the Wiener filter was replaced by a direct integration in Fourier domain integration methods.

The results of surface reconstruction techniques are presented in Table 1, and Fig. 2 (a). Table 1 shows average values for each sample set on reconstruction error, MSE, the absolute error of arithmetic mean deviation of surface, S_a, on 80 μm wavelength [10], $|\epsilon_{Sa}|$, and correlation, r, to profilometer measured surface. S_a is the industry standard measure for surface roughness. The reconstruction errors are largest for Two-light and Symmetric for rougher Cardboard samples, but for smoother samples errors are in the same level with Frankot-Chellappa. Absolute S_a error measures reconstruction errors in high frequencies. Errors are slightly larger for Symmetric than for other methods. The linear correlations are strong for all the methods, especially for Poisson. Figure 2 (a) shows normalized power spectra of reconstructed surfaces and profilometer measured topography from a Cardboard sample. We computed 2D spectra using FFT, and 1D spectra in x-direction was obtained by summing columns of 2D spectrum. The power spectrum of the original profilometer measured topography and two-light reconstructed one were almost identical. For the other methods, the power spectra were weaker in higher frequencies.



(a) Profilometer (b) Photometric Stereo

Fig. 2. Power spectra from a Cardboard sample.

4.2. Photometric Stereo

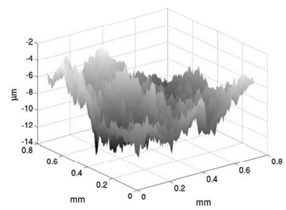
In the second phase, surfaces were reconstructed from gradient fields calculated using photometric stereo approach. The results were contrasted to profilometer measurements. The OTF and SNR functions for Two-light, and Symmetric methods were as Hansson proposed in [6].

The images for photometric stereo were acquired using a CCD camera with resolution of 2048 x 2048 pixels with 12 bits per pixel. In the experiments, the image area was 15 mm x 15 mm. The images were acquired using slant angle of 60° for the illumination. The reconstructed surfaces were registered with the profilometer measurements using cross-correlation based method [11]. Profilometer measurements were pointwise aligned to the photometric stereo measurements using geometric affine transformation and interpolation.

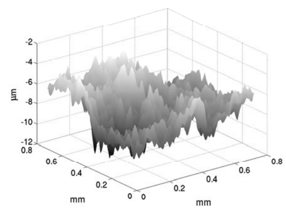
The reconstruction errors, absolute S_a errors, and correlations are in Table 1. The reconstruction errors are in the same level for all the methods, whereas absolute S_a errors are significantly larger for Frankot-Chellappa and Poisson compared to Two-light and Symmetric. This can be seen also from Fig. 3, which shows a fragment of the reconstructed surfaces from a LWC sample. Minimization based methods, such as Frankot-Chellappa and Poisson, produce clearly smoother surfaces than Fourier integrating methods, which restore the original signal using Wiener filter with estimated SNR and OTF functions. Also power spectra of Frankot-Chellappa and Poisson are in lower level, see Fig. 2 (b). In general, Symmetric exhibits larger absolute S_a errors but smaller reconstruction errors than Two-light. The correlations in Table 1 are slightly better for minimization based methods, than for Fourier integrating methods.

	Hansson two-light			Symmetric			Frankot-Chellappa			Poisson		
Profilometer	MSE	$\lvert\epsilon_{Sa}\rvert$	r	MSE	$\lvert\epsilon_{Sa}\rvert$	r	MSE	$\lvert\epsilon_{Sa}\rvert$	r	MSE	$\lvert\epsilon_{Sa}\rvert$	r
Cardboard	3.50	0.13	0.90	2.30	0.44	0.96	1.69	0.24	0.98	1.52	0.30	0.98
SC	1.73	0.04	0.82	1.53	0.21	0.86	1.28	0.13	0.90	0.49	0.15	0.99
LWC	1.17	0.01	0.90	1.33	0.17	0.87	0.98	0.12	0.93	0.36	0.11	0.99
Photometric stereo	MSE	$\lvert\epsilon_{Sa}\rvert$	r	MSE	$\lvert\epsilon_{Sa}\rvert$	r	MSE	$\lvert\epsilon_{Sa}\rvert$	r	MSE	$\lvert\epsilon_{Sa}\rvert$	r
Cardboard	7.11	1.60	0.60	6.48	1.68	0.66	6.05	2.08	0.71	5.96	2.10	0.71
SC	2.69	0.05	0.57	2.67	0.07	0.58	2.49	0.39	0.63	2.51	0.40	0.63
LWC	2.46	0.04	0.52	2.40	0.07	0.54	2.31	0.27	0.58	2.30	0.29	0.58

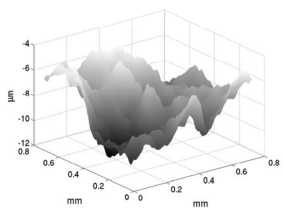
Table 1. Results from profilometer and PS calculated gradient fields. MSE is the reconstruction error, $|\epsilon_{Sa}|$ the absolute error of arithmetic mean deviation of surface, S_a, on 80 μm wavelength, and r correlation to profilometer measured surface. Results are average values for each sample set.



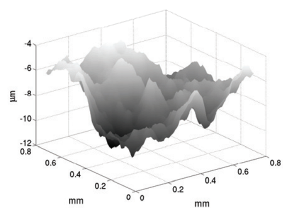
(a) Profilometer



(b) Symmetric



(c) Frankot-Chellappa



(d) Poisson

Fig. 3. A fragment of reconstructed paper surface topographies from a LWC sample. Note the different axis scales.

5. CONCLUSIONS

In this work, surface reconstruction methods from gradient fields have been studied and further developed. The gradient fields of paper surfaces were calculated from profilometer measured topography and using photometric stereo.

Hansson's two-light photometric stereo method was extended to four-light one, and a comparison to traditional Frankot-Chellappa and Poisson methods was performed. On gradient fields calculated on profilometer measured topography, Poisson achieved very strong correlations. However, minimization based methods, such as Poisson and Frankot-Chellappa, smooth the higher frequency variation, as it was observed on surfaces reconstructed from gradient fields of photometric stereo. However, the small scale variation, that is the roughness, is of interest in many manufacturing processes. Fourier domain integration based Hansson's two-light method and our four-light extension could preserve the original surface topography. The proposed four-light method exhibited larger reconstruction errors in high frequencies than two-light method. This may derive from boundary value problems when using discrete step functions as weighting functions in surface integration.

In future work, the integration parameters, such as OTF, SNR, and weighting functions for Fourier domain integration methods, will be further developed, since the high frequency height variation is still attenuated in reconstructed topographies.

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