SKELETONIZATION BY GRADIENT DIFFUSION AND REGULARIZATION

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ABSTRACT

This paper describes a skeletonization process for grayscale or color images which uses the diffusion of the image gradient vectors. We propose to diffuse the native gradients of the color images to obtain the skeleton of the main contrasted objects. Contrary to a distance transform or thinning based skeleton, the gradient vectors diffusion is a straightforward, simple, and efficient approach to compute the skeleton, which requires no complex parameters or predefined criteria and no prior image binarization. In comparison to other approaches based on potential field functions, our method does not require the segmentation of the objects or the precise localization of their contours. Our method is very simple to implement and can be applied to natural noisy color images. Some results on real color and grayscale images are reported.

Index Terms— skeletons, potential field, diffusion, gradient

1. INTRODUCTION

Skeletons are widely used in many computer vision applications. They provide a simple and compact representation of 2D and 3D shapes that preserves original object's topologies. The skeleton can be defined in several ways either by the centers of the maximal disks contained in the original object [1] or by the intuitive grassfire paradigm introduced by Blum [2]. Morphology introduced by Serra [1] provides a well-founded theory to define and compute skeletons of binary images and it was later extended to grayscale images [3].

1.1. Skeleton of binary images

There are mainly four different approaches to compute binary images' skeletons of delimited objects by their area or contour :

1) methods based on thinning or grassfire approach
They are generally computed by iterative conditional thinning which iteratively deletes the non-skeleton points [4]. Heuristics and complex criteria are often used to stop the process and preserve the skeleton continuity and width.
2) The distance transform The distance transform is defined for each point of an object as the smallest distance from that point to the boundary of the object [5]. Skeletons and medial lines of objects can be computed by finding the local maxima of the distance map. The object can be entirely reconstructed by replacing each point of the skeleton by a discrete disc with a radius given by the distance transform.

3) Geometric methods based on Voronoi diagram.
The skeleton is computed from the Voronoi graph of a set of points located on the boundary of the object [6]. This approach is theoretically well defined in a continuous space and provides fully connected skeletons. The main drawbacks of this approach are the sampling of the boundaries which defines the quality of the Voronoi diagram and the pruning of branches by using complex post-processing stages.

4) Methods based on Potential Field functions
Skeletonization approaches based on potential fields for 2D and 3D objects identify skeleton by using a potential model instead of the distance transform. The pixels of the boundary are considered point charges generating the potential field inside the object using an electrostatic field function [7] or the Newton law [8]. But these approaches have some drawbacks such as the necessity to consider the distance to the border, or to verify the visibility of each point to the border or the localization of the contour curvatures.

Generalized Potential field function has been used for 2D objects [9] and 3D objects [10]. For this approach, the potential at a point interior to the object is determined as a sum of potentials generated by the boundary of the object. This approach gives much smoother skeletons which are less sensitive to the noise.

In conclusion, most of the previous methods lead to difficult problems due to the nature of the discrete space. Most of previous methods are sensitive to noise and complex pruning operation must be applied to clean the final skeleton. Nevertheless, the approaches based on generalized potential fields are interesting for noisy images. In this paper, we show that these methods can be extended to the gradient vectors of the grayscale or color images.

1.2. Skeleton of grayscale images

There is limited number of papers about the skeletonization using directly grayscale information without image segmentation. The recent trend is using potential field approaches by analogy either with electromagnetism or Newton law. These approaches seek to diffuse potential fields using various diffusion equations in order to define an edge strength function (also called skeleton strength function or pseudo distance map) [11][13][14] [15]. The skeleton is extracted by tracking the ridges of this edge strength map. An homothetic erosion of the grayscale image is also used to extract skeletons [12]. However, this approach is applicable on images characterized by objects brighter or darker than the background.

The skeletons from grayscale images are interested for many applications and are robust to image noise. But those authors are focused on the physical theory (Newton's law of universal gravitation or Electromagnetism laws) that defines the underlying mathematical model of vector diffusion. We propose to simply abandon the physical framework and develop a different skeletonization algorithm, which uses directly the diffusion of the image gradients from grayscale or color images.

2. PROPOSITION

Our objective consists to find a sketonization approach suited for noisy images which use color or grayscale information. Our contribution can be classified into potential field approaches. We have fixed four objectives to reach:

- To use of a simple diffusion framework (avoid heuristics, parameters and the definition of masks)
- To initialize the potential field with all gradients of the images without the selection of particular points of the object or its contour.
- To get a stationary diffusion which do not require any explicit stop function.
- To be less sensitive to image noises.

In order to achieve these goals, we need to make some assumptions. First of all, we assume that the contour of the objects to skeletonize has a higher gradient magnitude than in the background or the interior of the object. Secondly we assume that the gradients keep almost the same magnitude along the contour of the same object. These assumptions are not too restrictive for many applications.

2.1. Diffusion of gradient vectors

We propose to use the color gradients of the entire image to initialize the potential field. In a second step, we diffuse the gradient vector field by using a regularization process which preserves the divergence of the gradient vector field around the skeleton of the objects. We notice that in most analytic formula of vector diffusion process, a regularization of the vector field is achieved. We find this regularization step in the isotropic or anisotropic vector diffusion or during the calculation of motion vectors. We propose to apply directly the regularization of the gradient vector field as a diffusion process without using any analogy to physical laws. If we consider a 1D signal (fig. 1), its first order derivative cross zero precisely for local extrema which correspond exactly to both the skeleton SQ and it's complementary $\overline{SQ}$. Figure 1 also shows the diffusion of gradient information into flat zones where the gradient is null or low by using a simple regularization process (1). The gradient vector field will flow into flat zones and the opposite vectors indicate the location of the skeleton and its complementary. The vector regularization (1) can be used to diffuse the gradient vectors into flat zones without computing distance map or singular points of the contours around high curvatures like for the Hamilton Jacobi skeletons [8] or potential fields approaches.

$$(1)\quad \nabla I^{n+1}(p) = \frac{1}{|N(p)|}\sum_{h\in N(P)} \nabla I^{n}(h) \quad \text{with } \nabla I = \begin{pmatrix} Ix \\ Iy \end{pmatrix}$$

The regularization can be considered as an iterative smoothing of the gradient vectors using the 8-connected neighbors N of each point p, with a uniform kernel.

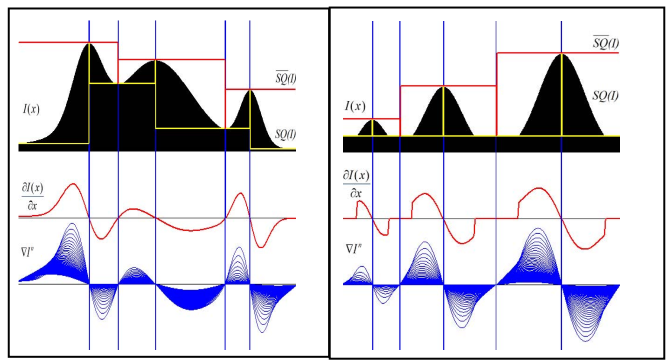

Fig. 1. Gradient vectors diffusion

But the intensive regularization of the gradient vector field will progressively diverge. Points which are not the center of exact opposite gradients of the same magnitude will progressively disappear. To keep the stability of the gradient field, we need either to use a grassfire algorithm or a stop function to block the diffusion before reaching the loss of stability of the field. We choose to control precisely the regularization by stopping the flow (1) when the number of iteration n for non null gradient $\nabla I^n \neq 0$ reach a user defined limit σ. The number of iteration for null gradient around flat zones must be unlimited. If $\sigma = 1$ we have a classical grassfire algorithm which provides a sharp skeleton with spurious branches. For a higher value of σ, we obtain a smooth skeleton, robust to noise and suited for real images. Hence, we can increase the smoothness parameter σ to process particular noisy images without reaching the loss of stability of the gradient field.

2.2 Skeleton from the diffused gradient vector field

There are several approaches to extract the skeleton from a diffused gradient vector field. Originally, the skeleton is located along lines where the divergence of the gradient vector field is null. We choose to compute the maximal difference of the orientation of two adjacent symmetric pair of gradient vectors in a 3X3 neighborhood and we keep this value in a *Skeleton Strength* map *SS* (2).

$$(2) \quad SS(I) = \underset{\substack{h,k \in N(p) \\ h,k \ \text{symmetric} \quad \text{pair}}}{Max} \left\{ \left| \theta_h - \theta_k \right| \right\}$$

where θ_h and θ_k are the gradient orientations of the symetric pair of adjacent gradient. If the Skeleton Strength reaches a maximal value of 180°, then the point belong to the skeleton of a line shape. If the skeleton strength decreases around 45° then the point belong either to the skeleton of a triangle shape or to a redundant skeleton branches. Fig. 2b gives the initial gradient orientation along the contour obtained by a Sobel convolution. Fig. 2c shows the diffusion by using (1) for σ=1. Fig. 2e and 2f show respectively the diffusion and the Skeleton for σ=20. Figure 3 shows the robustness against heavy variations of the objects contours with σ=20.

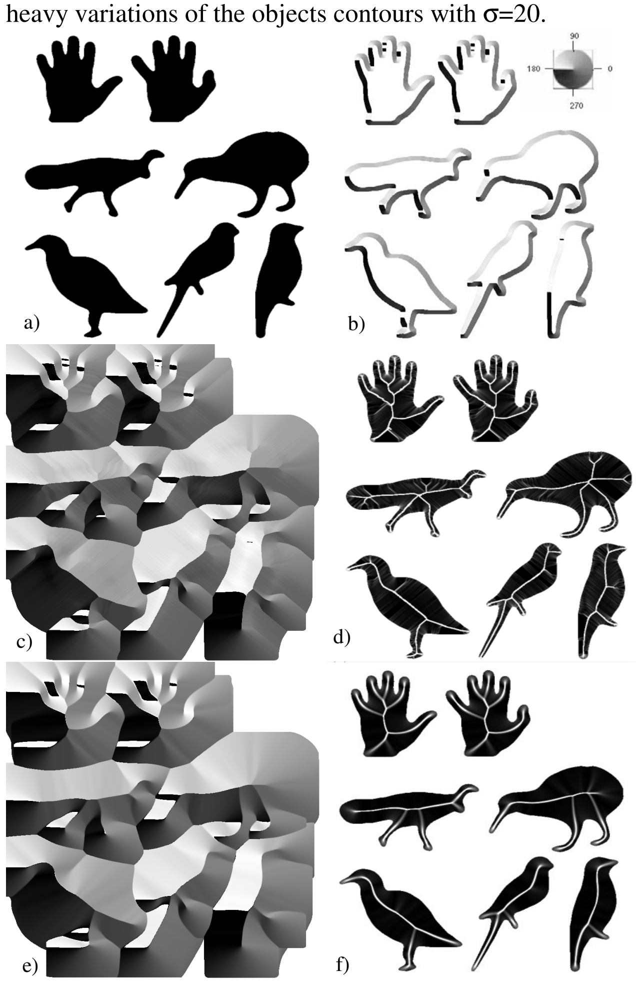

Fig. 2. a) original image b) initial gradient orientation c) gradient diffusion σ=1 d) Strength Skeleton of c, e) gradient diffusion σ=20 f) Strength Skeleton of e.

As we do not use the segmentation of the objects from the background, the skeletonisation process is applied for the whole image. The differentiation between the *SQ* and the complementary $\overline{SQ}$ (skeleton of the background) must use the convergence or the divergence of the gradient field. As the gradient flows from dark areas toward brighter part of the image, *SQ* corresponds to a divergent gradient field and $\overline{SQ}$ to a convergent gradient field (Fig 4).

Fig. 3. Robustness to the contour deformation.

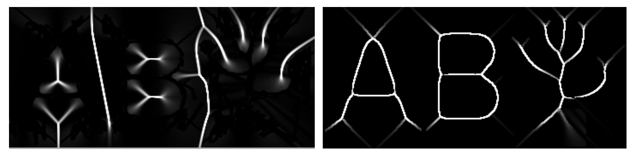

Fig. 4. $\overline{SQ}$: convergent field   *SQ* : divergent field

2.3. Extension to grayscale and color images

We use all the gradients of the image even for pixels from flat areas where the gradient orientation is not well defined. We have noticed that gradient vectors which are individually not correctly oriented around flat zones are statistically "on average" almost well oriented. Our key idea is that the apparent disorder of the gradient orientation around flat zones can be reoriented by the regularization of the gradient vector field by using (1). Fig. 5 shows the gradient field from a noisy color image and the regularized field obtained after 100 iterations with σ=10. By using the regularization as a diffusion process, the gradient vectors having higher magnitude will impose progressively their orientation to the other gradient vectors. This property is important to process noisy images and to reorient correctly the gradients from nearly flat zones.

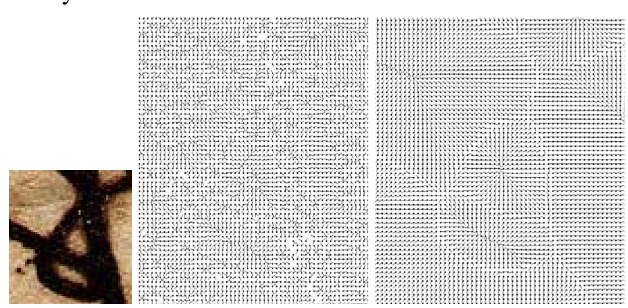

Fig. 5. a) Initial color gradients b) their regularization

3. RESULTS

We apply our skeletonization on several images from various domains. Our approach provides smoothed skeletons similar to those obtained by potential field functions. The method is robust to noise and do not require the object segmentation (fig. 6). But the skeleton will be distorted for objects which are not uniformly lightened.

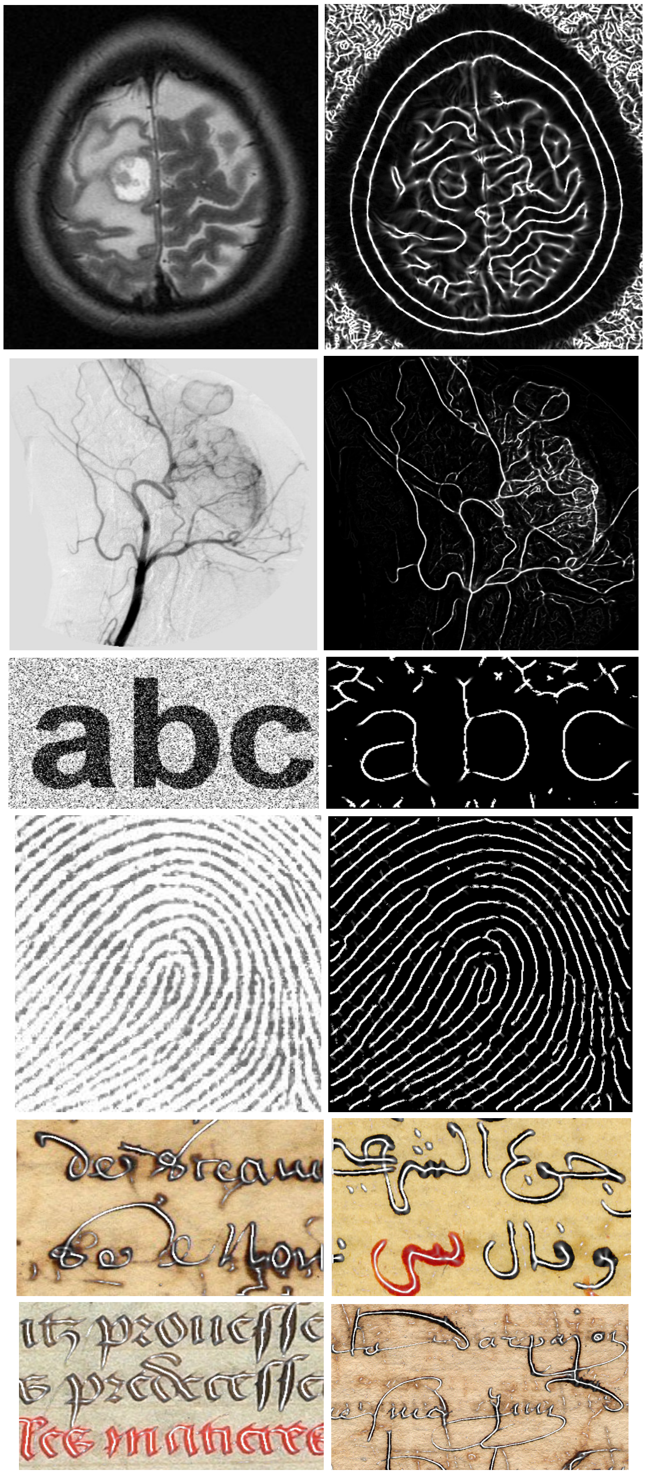

Fig. 6. Some results on real images

4. CONCLUSION

We have presented a simple skeletonization approach, which uses the gradient vector regularization to simultaneously diffuse gradient information into flat areas and smooth the gradient vector disorder due to the image noise. The diffusion process is simple and requires no computation of a distance map. After several iterations, the gradient vectors flow toward the image skeletons. This simple method can be applied directly to grayscale or color natural images from various domains. It requires a parameter σ which controls the smoothness of the skeleton and limit the number of iterations for each non-null gradient vectors.

5. REFERENCES

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