POCS-BASED ITERATIVE RECONSTRUCTION ALGORITHM OF MISSING TEXTURES

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ABSTRACT

In this paper, a new framework for texture reconstruction of missing areas, which exist all over the target image, is presented. The framework is based on a projection onto convex sets (POCS) algorithm including a novel constraint. In the proposed method, a nonlinear eigenspace of each cluster obtained by texture classification is applied to the constraint. Furthermore, by monitoring the errors converged by the POCS algorithm, selection of the optimal cluster for the target texture including missing intensities is realized in order to reconstruct it adaptively. Then, iterating the POCS-based procedures, our method renews the nonlinear eigenspaces and the reconstruction image, and outputs the reliable result. This approach provides a solution to the problem in traditional methods of not being able to perform adaptive reconstruction of the target textures due to the missing intensities. Experimental results show subjective and quantitative improvement of the proposed reconstruction technique over previously reported reconstruction techniques.

Index Terms— Image restoration, image texture analysis, interpolation, nonlinear estimation.

1. INTRODUCTION

In the field of image restoration, reconstruction of missing areas in digital images is a very important issue because it has a number of fundamental applications. For example, it is applied to removal of unnecessary objects such as superimposed text, restoration of corrupted old films, and interpolation of missing blocks transmitted in an error-prone environment for video communications.

In recent work, the study of missing texture reconstruction has developed rapidly and its achievements have become a center of attraction [1, 2]. Most algorithms reported in the literature reconstruct the missing areas by utilizing known textures within the target image as training patterns. However, when the missing areas exist all over the target image, they cannot obtain enough training patterns and their performance becomes worse. Further, they must be based on the assumption that the squared distances between arbitrary local areas within the target image are small enough. This means that the target image must consist of only one type of texture. When various kinds of textures exist in the target image, it is desirable that the missing textures be reconstructed from only the same kinds of textures. Unfortunately, these textures cannot be selected by traditional methods since the distance between the target area including the missing intensities and the other areas cannot be calculated.

In this paper, a novel texture reconstruction method based on the theory of projections onto convex sets (POCS) is proposed to solve the traditional problems. The POCS algorithm has been applied to blocking artifact reduction in coded images as a nonlinear image restoration method [3]. In the proposed method, the following two novel approaches have been introduced into the POCS algorithm. One approach is the use of a new constraint in the POCS algorithm. Texture classification is performed for local images, and the nonlinear eigenspace [4] of each cluster is applied to the constraint. The main advantage of this approach is that each nonlinear eigenspace correctly approximates the local images classified into the same cluster. Furthermore, by introducing the other approach into the POCS algorithm, optimal cluster selection for the target local image including missing intensities can be performed. Special attention is given to the errors converged by the POCS algorithm. These values correspond to the distances from the eigenspace of each cluster, and the optimal cluster can therefore be selected even if the target local image includes missing intensities. Therefore, the POCS algorithm, whose constraint utilizes the selected cluster's eigenspace, can adaptively reconstruct the target local image. Then, iterating the POCS-based procedures, we renew the nonlinear eigenspaces and the texture reconstruction result. Consequently, all local images in the whole image can be utilized as the training patterns, so that our method restores the missing textures more successfully than the traditional ones for the images including missing areas all over.

This paper is organized as follows. The POCS algorithm is explained in Section 2. In Section 3, a novel texture reconstruction method using the POCS algorithm is presented. Experimental results that verify the high performance of the proposed method are presented in Section 4.

2. POCS ALGORITHM

This section explains the POCS algorithm. The theory of POCS was first introduced in the field of image restoration by Youla and Webb. This algorithm estimates the original image f in the Hilbert space H from its known properties. Given n properties of the original image f, these properties generate n well-defined closed convex sets C_i ($i = 1, 2, \cdots, n$). Further, the original image f should be included in all the C_i's and also the intersection of all the C_i's C^*, i.e.,

$$f \in C^* = \bigcap_{i=1}^{n} C_i. \tag{1}$$

It is clear that the intersection C^* is a closed and convex set. Consequently, the estimation of f from the n properties is equivalent to finding at least one point f^* belonging to C^*. Unfortunately, C^* may be nonlinear and complex in structure so that a direct estimation of f^* is almost infeasible. However, given the projection operator P_i onto C_i, the iteration

$$f_t = P_n P_{n-1} \cdots P_2 P_1 f_{t-1} \quad t = 1, 2, \cdots \tag{2}$$

converges to a limiting point f^* of the intersection C^* for an arbitrary initial element f_0 in H. Note that the operator P_i satisfies

$$||f - P_i f|| = \min_{g \in C_i} ||f - g||, \tag{3}$$

where $||\cdot||$ denotes the norm in H. Then, we can calculate f^* from the n properties of the original image by using Eq. (2).

3. POCS-BASED TEXTURE RECONSTRUCTION

A POCS-based texture reconstruction method is presented in this section. First, in order to know which parts have the same kinds of textures, the proposed method performs texture classification of local images within the target image (See 3.1). From the obtained results, a target local image including missing textures is reconstructed by using the POCS algorithm. In this procedure, the following two novel approaches are introduced into the POCS algorithm:

(i) Introduction of the nonlinear eigenspace of each cluster obtained by the texture classification into the constraint of the POCS algorithm.

(ii) Adaptive selection of the optimal cluster for the target local image based on errors caused by the POCS algorithm.

The first approach aims at accurate texture approximation of the local images including the same kinds of textures. Further, the second approach is necessary to restore the target local image from the nonlinear eigenspace which correctly reconstructs the same kinds of textures as the target one. By utilizing the above approaches, adaptive reconstruction of the missing textures can be achieved, so that successful reconstruction of the target image including various kinds of textures can be expected. Then, the proposed method iterates the above procedures, and renews the nonlinear eigenspaces and the reconstruction result (See 3.2). Consequently, we can utilize all local images in the whole image as the training patterns for the nonlinear eigenspaces. Therefore, our method successfully reconstructs the target image even if it includes missing areas all over.

3.1. Texture Classification Algorithm

In this subsection, classification of textures within the target image into K clusters as preprocessing for reconstruction of the missing textures is described. First, we clip N local images f_i ($w \times h$ pixels, $i = 1, 2, \cdots, N$) at even interval from the target image and generate vectors $\mathbf{x}_i \in \mathbf{R}^{wh}$, whose elements are the raster scanned intensities of f_i. Then, we map $\mathbf{x}_i$ into the feature space $\mathbf{F}$ via the nonlinear map [4] and obtain $\phi(\mathbf{x}_i)$. The proposed method regards the mapped results $\phi(\mathbf{x}_i)$ ($i = 1, 2, \cdots, N$) as texture feature vectors and performs their classification that minimizes the following new criterion:

$$D = \sum_{k=1}^{K} \sum_{j=1}^{M} ||\phi_j^k - \mathbf{U}^k \mathbf{U}^{k\prime} \phi_j^k||^2, \tag{4}$$

where ϕ_j^k ($j = 1, 2, \cdots, M$) is $\phi(\mathbf{x}_i)$ ($i = 1, 2, \cdots, N$) included in cluster k. Furthermore, $\mathbf{U}^k = [\mathbf{u}_1^k, \mathbf{u}_2^k, \cdots, \mathbf{u}_D^k]$ is a projection matrix onto the eigenspace spanned by the eigenvectors $\mathbf{u}_d^k$ ($d = 1, 2, \cdots, D$) and satisfies the following equation of the singular value decomposition:

$$\mathbf{\Xi}^k \mathbf{H} \cong \mathbf{U}^k \mathbf{\Lambda}^k \mathbf{V}^{k\prime}. \tag{5}$$

In the above equation, $\mathbf{\Xi}^k = [\phi_1^k, \phi_2^k, \cdots, \phi_M^k]$, and $\mathbf{H} = \mathbf{I} - \frac{1}{M}\mathbf{1}\mathbf{1}'$, where $\mathbf{I}$ is the $M \times M$ identity matrix and $\mathbf{1} = [1, 1, \cdots, 1]'$ is an $M \times 1$ vector. From Eq. (5), the following equation can be obtained.

$$\mathbf{U}^k \cong \mathbf{\Xi}^k \mathbf{H} \mathbf{V}^k \mathbf{\Lambda}^{k^{-1}}. \tag{6}$$

Then, Eq. (4) can be rewritten as follows:

$$D \cong \sum_{k=1}^{K} \sum_{j=1}^{M} ||\phi_j^k - \mathbf{\Xi}^k \mathbf{H} \mathbf{V}^k \mathbf{\Lambda}^{k^{-2}} \mathbf{V}^{k\prime} \mathbf{H} \mathbf{\Xi}^{k\prime} \phi_j^k||^2. \tag{7}$$

Since the eigenvectors $\mathbf{u}_d^k$ ($d = 1, 2, \cdots, D$) of $\mathbf{U}^k$ in Eq. (4) are high-dimensional, we cannot calculate $\mathbf{U}^k$ directly. Therefore, we use Eq. (7) for the calculation of D.

In Eq. (4), $\mathbf{U}^k$ is the projection matrix onto the eigenspace spanned by its eigenvectors. Therefore, criterion D represents the sum of the approximation errors of ϕ_j^k ($j = 1, 2, \cdots, M$) in their eigenspaces. This means that the squared error in Eq. (4) corresponds to the distance from the nonlinear eigenspace of each cluster. Furthermore, since we regard $\phi(\mathbf{x}_i)$ ($i = 1, 2, \cdots, N$) as the texture feature vectors of $\mathbf{x}_i$, the new criterion D is useful for classification of the textures.

3.2. Texture Reconstruction Algorithm

In this subsection, we show reconstruction of the missing textures in the target image by the POCS algorithm from the classification results presented in the previous subsection. From the target image, we clip a local image f_0 ($w \times h$ pixels) including missing textures and generate a vector $\mathbf{x}$ whose elements are its raster scanned intensities. For $\mathbf{x}$, the proposed method calculates its reconstruction result $\hat{\mathbf{x}}$, which satisfies the following novel constraints.

[Constraint 1]
In vector $\hat{\mathbf{x}}$, the known original intensities are fixed.

[Constraint 2]
In the feature space, $\phi(\hat{\mathbf{x}})$ is in the eigenspace spanned by its eigenvectors $\mathbf{u}_1^k, \mathbf{u}_2^k, \cdots, \mathbf{u}_D^k$ of cluster k. Then $\hat{\mathbf{x}}$ satisfies

$$\hat{\mathbf{x}} = \phi^{-1}\left(\mathbf{U}^k \mathbf{U}^{k\prime} \phi(\hat{\mathbf{x}})\right). \tag{8}$$

In the above equation, ϕ^{-1} represents the inverse mapping, which is called pre-image, from the feature space back to the input space. However, an exact pre-image typically does not exist [5]. Therefore, the proposed method newly defines a linear map $\mathbf{A}^k$ and calculates the approximate solution ϕ^{-1} as follows:

$$\hat{\mathbf{x}} \cong \mathbf{A}^k \mathbf{U}^k \mathbf{U}^{k\prime} \phi(\hat{\mathbf{x}}), \tag{9}$$

where $\mathbf{A}^k$ satisfies the following equation:

$$\mathbf{X}^k = \mathbf{A}^k \mathbf{\Xi}^k. \tag{10}$$

Then from Eqs. (5) and (10), $\mathbf{A}^k$ can be obtained as follows:

$$\mathbf{A}^k \cong \mathbf{X}^k \mathbf{H} \mathbf{V}^k \mathbf{\Lambda}^{k^{-1}} \mathbf{U}^{k\prime}. \tag{11}$$

Thus, by using Eqs. (6) and (11), Eq. (8) can be rewritten below.

$$\hat{\mathbf{x}} \cong \mathbf{X}^k \mathbf{H} \mathbf{V}^k \mathbf{\Lambda}^{k^{-2}} \mathbf{V}^{k\prime} \mathbf{H} \mathbf{\Xi}^{k\prime} \phi(\hat{\mathbf{x}}). \tag{12}$$

Utilizing the POCS algorithm, the proposed method calculates the vector $\hat{\mathbf{x}}$ that satisfies the above two constraints from the initial vector $\mathbf{x}$. Specifically, we respectively utilize these constraints as the closed convex sets C_1, C_2 in Eq. (1) and calculate $\hat{\mathbf{x}}$ by their projection operators P_1, P_2 in Eq. (2). Note that in the least-squares sense, the nonlinear eigenspace used in [Constraint 2] correctly approximates ϕ_j^k ($j = 1, 2, \cdots, M$) included in the same cluster k. Therefore, if we can classify $\phi(\mathbf{x})$ of the target local image f_0, the proposed method accurately reconstructs it by using the eigenspace of the cluster including $\phi(\mathbf{x})$. However, since $\mathbf{x}$ contains missing intensities, $\phi(\mathbf{x})$ cannot be classified by the algorithm shown in 3.1. Thus,

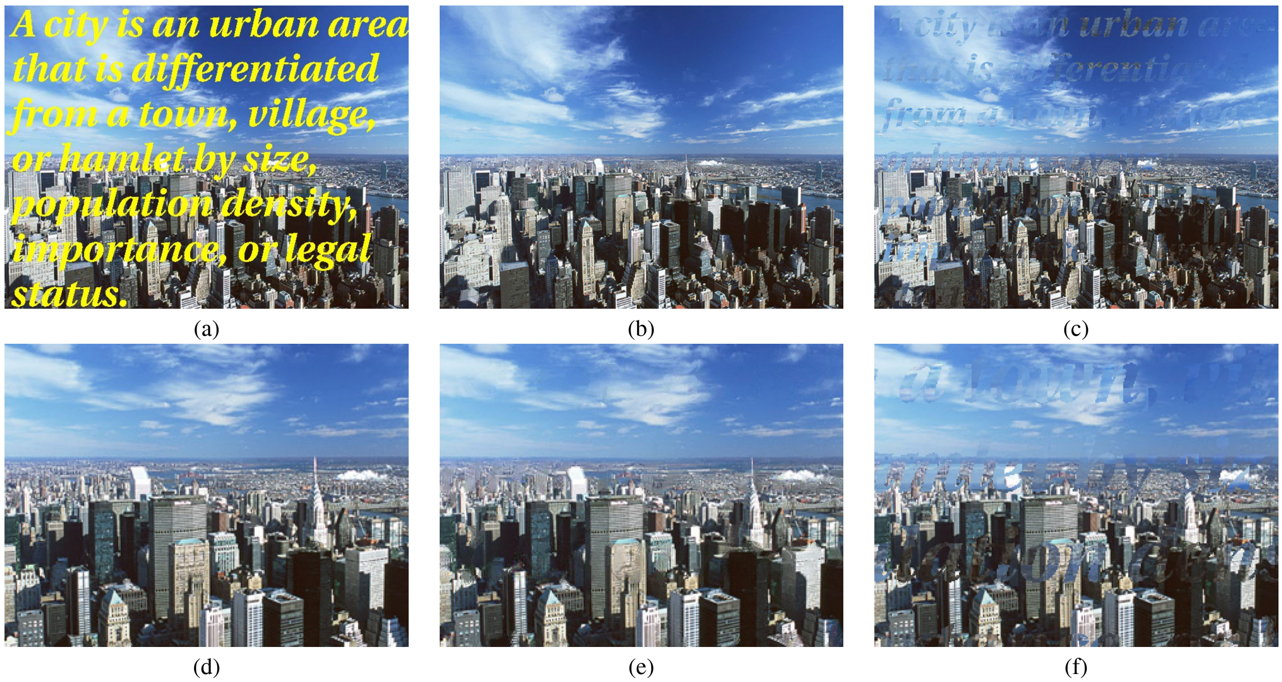


Fig. 1. (a) Corrupted image including text regions (18.5 % loss), (b) Reconstructed image by the proposed method (27.10 dB), (c) Reconstructed image by the traditional method (23.22 dB), (d) Zoomed portion of the original image, (e) Zoomed portion of (b), (f) Zoomed portion of (c).

in order to achieve the classification of $\mathbf{x}$, the proposed method utilizes the following novel criterion as a substitute for Eq. (4).

$$E^k = \frac{1}{\mathrm{diag}\left(\bar{\mathbf{\Sigma}}\right)} ||\phi(\bar{\mathbf{\Sigma}}\hat{\mathbf{x}}) - \phi(\bar{\mathbf{\Sigma}}\mathbf{X}^k\mathbf{H}\mathbf{V}^k\mathbf{\Lambda}^{k^{-2}}\mathbf{V}^{k\prime}\mathbf{H}\mathbf{\Xi}^{k\prime}\phi(\hat{\mathbf{x}}))||^2 \tag{13}$$

where $\bar{\mathbf{\Sigma}}$ is a diagonal matrix whose diagonal elements are zero or one and satisfies $\bar{\mathbf{\Sigma}}\mathbf{x} = \bar{\mathbf{\Sigma}}\hat{\mathbf{x}}$. The criterion E^k exactly corresponds to the squared error converged by the POCS algorithm in the feature space and also the distance from the eigenspace of cluster k. Therefore, this criterion E^k as well as $\tilde{D}$ in Eq. (4) is applicable for classification of the textures. Selection of the optimal cluster for the target local image including missing intensities then becomes possible. Furthermore, the proposed method regards the result $\hat{\mathbf{x}}$ obtained by the eigenspace of the selected cluster as the output. Consequently, by performing the non-conventional approach, which adaptively selects the optimal eigenspace for the missing texture, we can restore all of the missing textures in the target image accurately.

As described above, we can reconstruct the missing texture in the target local image. The proposed method clips local images ($w \times h$ pixels) including missing textures in a raster scanning order and reconstructs them by using the POCS algorithm. Note that each restored pixel has multiple estimation results if the clipping interval is smaller than the size of the local images. In this case, the proposed method regards the result minimizing Eq. (13) as the final one. Further, the proposed method uses the obtained reconstruction result as a new target image and iterates the classification and reconstruction procedures respectively shown in 3.1 and this subsection. Then, we renew the nonlinear eigenspaces utilized for the constraint and the reconstruction image, and obtain its converged result. The nonlinear eigenspaces, which are first calculated from the image including missing areas all over, are not necessarily correct. Thus, we need to perform the iteration procedure in order to obtain the reliable nonlinear eigenspaces.

4. EXPERIMENTAL RESULTS

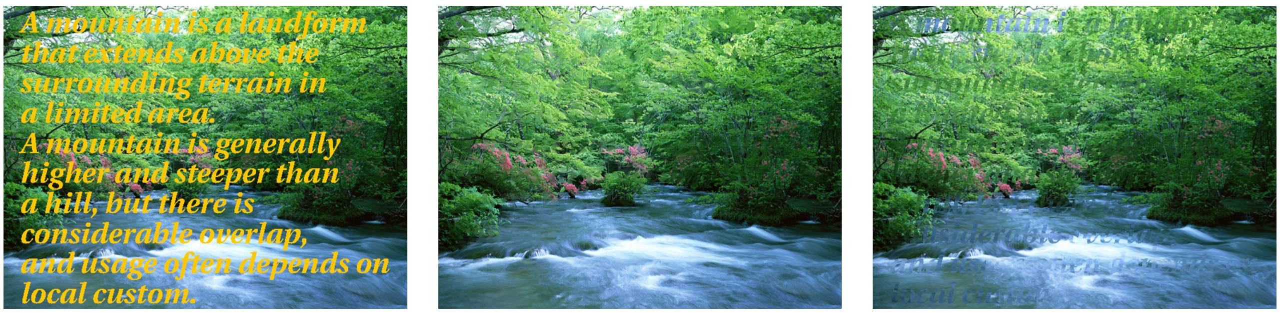
The performance of the proposed method is shown in this section. Figure 1(a) is a test texture image (480×359 pixels, 24-bit color levels) that includes the text regions all over. Figure 1(b) shows the results of reconstruction by the proposed method. In order to accurately restore edge areas, we use the result restored by our previously reported method [6] as a initial image for the proposed method. For comparison, Fig. 1(c) shows the results obtained by the traditional eigenspace method using projection of the nonlinear subspace obtained by the kernel principal component analysis in [4]. For better subjective evaluation, the enlarged portions around the middle of the images are shown in Figs. 1 (d)–(f). It can be seen that the use of the proposed method has achieved noticeable improvements.

In the conventional method, different kinds of textures affect the reconstruction of the target missing textures since the total number of training data is not enough. On the other hand, we introduce the iterative renewal approach into the reconstruction procedures, and solve the traditional problem. Further, by selecting the optimal cluster including the same kinds of textures, the proposed method can adaptively reconstruct the missing textures from only the reliable ones. Therefore, the proposed method has higher performance than that of the conventional method.

Different experimental results are shown in Figs. 2 and 3. Compared to the results obtained by using the conventional method, it can be seen that various kinds of textures can be accurately restored by using the proposed method. Further, in order to quantitatively eval-

(a) (b) (c)

Fig. 2. (a) Target image including text (19.8 % loss), (b) Reconstructed image by the proposed method (27.28 dB), (c) Reconstructed image by the traditional method (19.16 dB).



(a) (b) (c)

Fig. 3. (a) Target image including text (17.6 % loss), (b) Reconstructed image by the proposed method (27.12 dB), (c) Reconstructed image by the traditional method (23.47 dB).

uate the performance of the proposed method, we show the PSNR[1] of the reconstruction results in the captions of Figs. 1–3. It can be seen that our method has achieved an improvement of 3.65–8.12 dB over the conventional method. Therefore, high performance of the proposed method was verified by the experiments.

5. CONCLUSIONS

In this paper, we have proposed a new POCS-based framework for reconstruction of missing textures in still images. The proposed method introduces the nonlinear eigenspace of each cluster obtained by the texture classification into the constraint of the POCS-based reconstruction algorithm. Furthermore, by monitoring the errors converged by the POCS algorithm, selection of the same kinds of textures for the target local image including missing intensities can be achieved. The above two novel approaches enable adaptive restoration of the missing textures, and the proposed method therefore has high performance. Then, iterating the POCS-based reconstruction procedures, the proposed method renews the nonlinear eigenspaces and the restoration result. Consequently, since the reliable eigenspaces are utilized for the constraint of the POCS algorithm, the accurate reconstruction of the textures, which are missed all over the target image, can be realized. Impressive improvements in both objective and subjective measures have been achieved.

[1] PSNR $= 10\log_{10}\frac{\mathrm{MAX}^2}{\mathrm{MSE}}$, where MAX denotes the maximum value of intensities and MSE is the mean square error between the original image and the reconstructed image.

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