ROBUST TARGET DETECTION BY SPATIAL/SPECTRAL RESTORATION BASED ON TENSOR MODELLING

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ABSTRACT

Target detection in hyperspectral images (HSI) in one of the most common applications. But the classical detection algorithms are sensitive to noise. It is crucial to well restore the spectral signature in order to decrease the noise dependence of the detection algorithm. In this paper, we propose a restoration method which takes advantage of spatial and spectral information in order to estimate the spectral signature without impair the discriminate power. Our method is based on tensor decomposition where all ways are processed simultaneously. By considering the cross-dependency of spatial and spectral information for the filtering, we improve the probability of detection. Our optimization criterion is the minimization of the mean square error between the estimated and the desired tensors. This minimization leads to estimate the n-mode filter for each way and are jointly estimated by using an Alternating Least Squares (ALS) algorithm. Comparative studies with the classical bidimensional restoration methods show that our algorithm exhibits better detection probability in noisy situation. Indeed, the detection probability obtained after our algorithm is higher than 0.7 until a signal to noise ratio equal to -3 dB.

Index Terms— Detection, tensor signal, hyperspectral images, multilinear algebra, multiway filtering.

1. INTRODUCTION

The noise corrupting HSI depends not only on the performance of sensors but also on the conditions during the data acquisition including illumination and atmospheric effects. Under such conditions, noise reduction (NR) is a necessary preprocessing step to increase the SNR in order to improve the detection or classification processing by both decreasing the target spectral variability and spatially smoothing homogeneous areas. The HSI is a multiway and multicomponent data, represented by a datacube with several hundreds of spectral channels. Filtering this multiway data is far from being trivial. A basic restoration scheme processes all channels separately, considering each one as an independent signal. This NR method does not take advantage of inter-channel relationships which is one of the principal HSI characteristics. In order to make use of this inter-channel information, Hunt and Kubler propose in [1] a Karhunen-Loeve domain orthogonalization that decorrelates the channels. Actually, the most common NR method when dealing with multiway data is to perform an hybrid filter which consists first in making a principal component analysis (PCA) transform and then in removing noise with one independent spatial restoration for each decorrelated channel. But those classical techniques consist in splitting the data set into matrices or vectors and operate in the spatial and spectral domains independently. The splitting reduces considerably the information quantity related to the whole data : the possibility of studying the relations between components of different ways is lost.

In this paper, data are modelled as a tensor where each mode (way) is associated with a physical quantity. The originality of this model is to keep the intact multi-way structure during the processing and to jointly process the data spatially and spectrally. Tensor models are used in a large range of fields such as data analysis or signal and color image processing [2, 3]. These methods are based on multilinear algebra and on Tucker3 tensor decomposition [4]. Hence, we adapted a tensor decomposition based filtering for the HSI tensor model. In addition to [3] we propose a detection criterion to estimate the signal subspace dimension of each mode. The aim of this paper is to illustrate the improvement of target detection when spatial and spectral information are taken into account simultaneously during restoration.

The rest of the paper is organized as follows. Section 2 introduces the tensor formulation of the classical noise-removing problem. Section 3 summarizes the concept of multiway filtering. Section 4 presents and discusses some comparative detection results. Finally, concluding remarks are given in Section 5.

2. HSI TENSOR MODEL

2.1. Tensor modelling

HSI can be modelled by a three-order tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ where I_1 is the number of rows, I_2 the number of columns, and I_3 the number of spectral channels. Each way of the tensor is called n-mode where n refers to the n^{th} index. Let us define $E^{(n)}$, the n-mode vector space of dimension I_n, associated with the n-mode of tensor $\mathcal{X}$.

Unfolding a tensor $\mathcal{X}$ along a specific vector space $E^{(n)}$ allows to study the data properties in a given n-mode. An illustration of the n-mode unfolding of a tensor, denoted by $\mathbf{X}_n$ is represented in Fig. 1. In each unfolding matrix of the tensor, the whole information is present, data are rearranged along each n-mode vector space $E^{(n)}$.

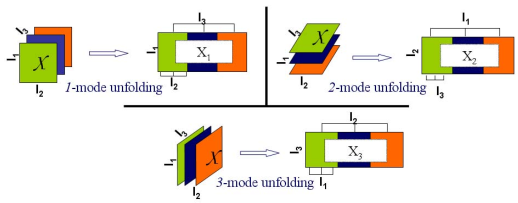

Fig. 1. n-mode unfolding of tensor $\mathcal{X}$.

This modelling naturally implies the use of multilinear algebraic tools and especially tensor decomposition and approximation methods. A commonly used is the Tucker3 [4] decomposition, which is illustrated in Fig. 2 and expressed for a 3-order tensor by :

$$\mathcal{X} = \mathcal{C} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times_3 \mathbf{U}^{(3)}, \qquad (1)$$

where, $\mathcal{C}$ is the core tensor, $\mathbf{U}^{(n)}$ is the matrix of eigenvectors associated with the K_n largest singular values along the n-mode unfolding of the tensor $\mathcal{X}$. If $K_n = I_n$, the Tucker3 decomposition is called Higher Order SVD (HOSVD) and if $K_n < I_n$ it is called Lower Rank-(K_1, K_2, K_3) Tensor Approximation (LRTA-((K_1, K_2, K_3)) [5].

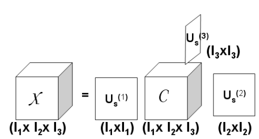

Fig. 2. Tucker3 decomposition model of a 3 order tensor $\mathcal{X}$.

2.2. A tensor decomposition based restoration

We assume that the hyperspectral tensor $\mathcal{R}$ is the sum of the desired information $\mathcal{X}$ and an additive white Gaussian noise $\mathcal{N}$:

$$\mathcal{R} = \mathcal{X} + \mathcal{N}. \qquad (2)$$

When noise $\mathcal{N}$ is assumed to be independent from signal $\mathcal{X}$ and provided the classical multidimensional signal processing assumptions [3] are fulfilled, $E^{(n)}$ is the superposition of two orthogonal subspaces: the signal subspace $E^{(n)}_{ss}$ of dimension K_n, and the noise subspace $E^{(n)}_{ns}$ of dimension $I_n - K_n$, such that $E^{(n)} = E^{(n)}_{ss} \oplus E^{(n)}_{ns}$.

The LRTA-(K_1, K_2, K_3) [5] uses the K_n singularvectors associated with the K_n singularvalues to obtain the lower rank tensor approximation $\widehat{\mathcal{X}}$ from tensor $\mathcal{R}$:

$$\widehat{\mathcal{X}} = \mathcal{R} \times_1 \mathbf{U}_s^{(1)}\mathbf{U}_s^{(1)T} \times_2 \mathbf{U}_s^{(2)}\mathbf{U}_s^{(2)T} \times_3 \mathbf{U}_s^{(3)}\mathbf{U}_s^{(3)T} \qquad (3)$$

where $\mathbf{U}_s^{(n)}$ is the signal subspace spanned by K_n singular vectors.

In a filtering framework, LRTA does not necessarily provide the best n-mode filters such that it is a approximation method which minimizes the square error between the estimated tensor $\widehat{\mathcal{X}}$ with the noised tensor $\mathcal{R}$. Our aim is to estimate the desired $\mathcal{X}$ thanks to a multidimensional filtering of the data:

$$\widehat{\mathcal{X}} = \mathcal{R} \times_1 \mathbf{H}^{(1)} \times_2 \mathbf{H}^{(2)} \times_3 \mathbf{H}^{(3)} \qquad (4)$$

From a signal processing point of view, the n-mode product is a n-mode filtering of data tensor $\mathcal{R}$ by n-mode filter $\mathbf{H}^{(n)}$.

The optimization criterion chosen to determine the optimal n-mode filters $\{H^{(n)},\ n = 1, 2, 3\}$ is the minimization of the mean square error between the estimated HSI tensor $\widehat{\mathcal{X}}$ and the expected HSI tensor $\mathcal{X}$:

$$e(\mathbf{H}^{(1)}, \mathbf{H}^{(2)}, \mathbf{H}^{(3)}) = \mathrm{E}\left[\left\| \mathcal{X} - \mathcal{R} \times_1 \mathbf{H}^{(1)} \times_2 \mathbf{H}^{(2)} \times_3 \mathbf{H}^{(3)} \right\|^2\right] \qquad (5)$$

This filtering modelling permits to process jointly all modes and to keep intact the multidimensional structure of the data. In the next section we express $\mathbf{H}^{(n)}$, we propose a detection criterion to estimate K_n and finally we define the multiway filtering from the n-mode filters $\mathbf{H}^{(n)}$.

3. MULTIWAY FILTERING

3.1. Expression of n-mode filters

Following [3] by developing the squared norm of equation (5), the expression of $\mathbf{H}^{(n)}$ becomes :

$$\mathbf{H}^{(n)} = E[\mathbf{X}_n \mathbf{q}^{(n)} \mathbf{R}_n^T] E[\mathbf{R}_n \mathbf{Q}^{(n)} \mathbf{R}_n^T]^{-1} \qquad (6)$$

with

$$\mathbf{q}^{(n)} = \mathbf{H}^{(m)} \otimes \mathbf{H}^{(p)}, \qquad (7)$$

$$\mathbf{Q}^{(n)} = \mathbf{H}^{(m)T}\mathbf{H}^{(m)} \otimes \mathbf{H}^{(p)T}\mathbf{H}^{(p)} \qquad (8)$$

where m $\neq$ n and p $\neq$ n and where the symbol $\otimes$ defines the Kronecker product.

From the criterion which is minimized, the n-mode filters $\mathbf{H}^{(n)}$ can be called n-mode Wiener filters. In opposite to the result obtained for 2D Wiener its covariance matrices are weighted by the others n-mode filters according to equations (7) and (8). Thus, each n-mode filter is expressed by :

$$\mathbf{H}^{(n)} = \mathbf{U}_s^{(n)} \Lambda_s^{(n)} \mathbf{U}_s^{(n)^T} \qquad (9)$$

where,

$$\Lambda_s^{(n)} = diag\left\{\frac{\lambda_1^\gamma - \sigma_\gamma^{(n)^2}}{\lambda_1^\Gamma}, ..., \frac{\lambda_{K_n}^\gamma - \sigma_\gamma^{(n)^2}}{\lambda_{K_n}^\Gamma}\right\} \qquad (10)$$

in which $\{\lambda_i^\gamma, \forall i = 1, \ldots, K_n\}$ and $\{\lambda_i^\Gamma, \forall i = 1, \ldots, K_n\}$ are the K_n largest eigenvalues, of matrices $\mathrm{E}\left[\mathbf{X}_n \mathbf{q}^{(n)} \mathbf{R}_n^T\right]$ and $\mathrm{E}\left[\mathbf{R}_n \mathbf{Q}^{(n)} \mathbf{R}_n^T\right]$ respectively . Also, $\sigma_\gamma^{(n)^2}$ can be estimated by determining the mean of $I_n - K_n$ smallest eigenvalues of $\gamma_{RR}^{(n)}$:

$$\sigma_\gamma^{(n)^2} = \frac{1}{I_n - K_n} \sum_{i=K_n+1}^{I_n} \lambda_i^\gamma. \qquad (11)$$

Note that all expressions require the knowledge of the signal subspace dimension K_n. We propose in the next section to estimate it by an information criterion.

3.2. Estimation of the signal subspace dimension

K_n is the dimension of the useful n-mode signal subspace of each n-mode. When filtering is performed, if K_n is too low, information is lost if K_n is too elevated, noise is included in restoration. In these two cases, the necessary number of eigenvalues is not well approximated and the estimated tensor is not *optimal*.

For this purpose, we adapt a well-know detection Akaike information criterion (AIC) [6]. We estimate K_n for each n-mode by minimizing AIC criterion. Consequently, for each n-mode unfolding of $\mathcal{R}$, the AIC criterion can be expressed as

$$AIC(k) = -2M \sum_{i=k+1}^{i=I_n} \log \lambda_i$$

$$+M(I_n - k) \log \left(\frac{1}{I_n - k} \sum_{i=k+1}^{i=I_n} \lambda_i\right) + 2k(2I_n - k) \qquad (12)$$

where $(\lambda_i)_{1 \leq i \leq I_n}$ are the I_n eigenvalues of the covariance matrix of the n-mode unfolding matrix of $\mathcal{R}$: $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{K_n} > \lambda_{K_n+1} = \lambda_{K_n+2} = \ldots = \lambda_{I_n} = \sigma^2$, and M is the number of columns of the n-mode unfolding matrix of $\mathcal{R}$.

3.3. ALS algorithm

An alternating least Square algorithm is needed to jointly find $\mathbf{H}^{(n)}$ n-mode Wiener filters that permit to reach the global minimum of mean square error $e(\mathbf{H}^{(1)}, \mathbf{H}^{(2)}, \mathbf{H}^{(3)})$ given by (5). One ALS algorithm can be summarized in the following steps, with $H^{(n),0} = I_{In}$ for all $n = 1, 2, 3$:

ALS loop: while $\left\|\mathcal{X} - \mathcal{R}^k\right\|^2 > \epsilon$ *a priori* fixed threshold for $n = 1, 2, 3$:

1. **Signal subspace dimension estimation**:

   $K_n = \underset{k}{argmin}\ AIC(k)$, $k = 1, \cdots, I_n$, see equation (12)

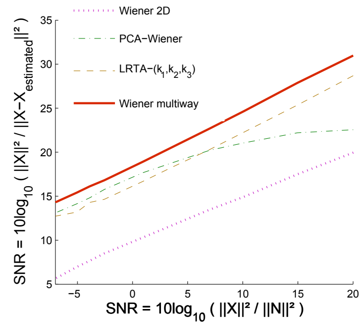

Fig. 3. SNR improvement with respect to the injected SNR varying from -8 dB to 20 dB

2. **n-mode filter estimation**:

   - $\mathcal{R}_n^k = \mathcal{R} \times_m \mathbf{H}^{(m),k} \times_p \mathbf{H}^{(p),k}$
   - $\mathbf{H}^{(n),k+1} = \underset{\mathbf{Z}^{(n)}}{argmin} \left\|\mathcal{X} - \mathcal{R}_n^k \times_n \mathbf{Z}^{(n)}\right\|^2$, $\mathbf{Z}^{(n)} \in \mathbb{R}^{I_n \times I_n}$

3. **Multiway filtering**, $\mathcal{R}^{k+1} = \mathcal{R} \times_1 \mathbf{H}^{(1),k+1} \times_2 \mathbf{H}^{(2),k+1} \times_3 \mathbf{H}^{(3),k+1}$, $k \leftarrow k + 1$.

The ALS algorithm permits to jointly determine the n-mode filters. Therefore all ways are processed simultaneously. The n-mode filters are iteratively fitted to each others in order to provide an *optimal* multiway filtering in the sense of mean square error.

4. IMPROVEMENT OF TARGET DETECTION

A high spatial resolution HYperspectral Digital Imagery Collection Experiment (HYDICE) is considered in all our experiments. This HSI can be modelled by a 3-order tensor $\mathcal{R} \in \mathbb{R}^{150 \times 150 \times 157}$. We compare our multiway filtering denoted by *Wiener multiway* with 3 classical methods. The two first methods permit to highlight the advantages of use of the tensor model. The first one consists in a consecutive Wiener filtering of each spectral channel, which takes advantage of spatial information, denoted by *2D-Wiener*. The second one, an hybrid filter, consists in a preprocessing that decorrelates the channels before applying Wiener filtering, this filter denoted by *PCA-2D Wiener* takes advantage of spatial and spectral informations but independently. The third comparative NR method, a tensor decomposition based method takes advantage of whole information jointly and simultaneously but by minimizing different criterion. It is the *LRTA*-(K_1, K_2, K_3).

Our noise is a Gaussian distribution. Figure 3 shows the SNR improvement of the different NR methods with respect to a injected SNR varying from -8 dB to 20 dB. In the sense of SNR the multiway filtering gives the best restoration.

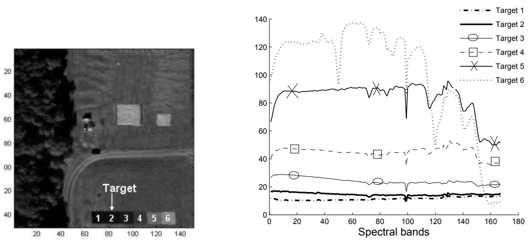

Fig. 4. Targets from the HYDICE image (left) and their spectral signature (right).

To perform the target detection we use the adaptative coherence / cosine estimator (ACE) detector [7] a well-known constant false alarm rate (CFAR). The ACE can be expressed as :

$$\mathcal{D}_{ACE} = \frac{\mathbf{x}^T \hat{\Gamma} \mathbf{s} (\mathbf{s}^T \hat{\Gamma}^{-1} \mathbf{s})^{-1} \mathbf{s}^T \hat{\Gamma}^{-1} \mathbf{x}}{\mathbf{x}^T \hat{\Gamma}^{-1} \mathbf{x}} \tag{13}$$

where $\hat{\Gamma}$ is the estimated covariance matrix of the 3-mode unfolding matrix, $\widehat{\mathbf{X}}_3$, $\mathbf{s}$ and $\mathbf{x}$ are respectively the target and test spectra. $\mathbf{s}$ is assumed *a priori* known from a supervised method directly on the initial HSI. Figure 4 shows the tests targets in the initial HSI and their spectral signature. So when,

$$\begin{cases} \mathcal{D}_{ACE} > \eta_{ACE}, & \text{the target is present;} \\ \mathcal{D}_{ACE} < \eta_{ACE}, & \text{the target is absent.} \end{cases} \tag{14}$$

Where η_{ACE} is a detection threshold which allows the probability of detection and of false alarm estimation.

Figure 5 represents the probability of detection with respect to the SNR varying from -3 to 30 dB and with a probability of false alarm fixed at 10^{-3}. In regard to the probability obtained without processing we notice that the ACE detector is sensitive to noise power. The two tensor decomposition based methods give the best improvement of the detection probability compared to the classical bidimensionnal methods. The use of tensor model is highlighted. Whatever noise power our multiway restoration permits a significant improvement of the target detection. This can be explained by its good spectral signature estimation which does not impair the discrimination power of the targets thanks to the joint spatial-spectral processing. The ACE detector gives probability of detection upper to 0.7 until a SNR equal to 20 dB without processing, 14 dB after the band by band restoration, 8 dB after the hybrid filter, 3 dB after the LRTA-(K_1, K_2, K_3) and -3 dB after our multiway filter (Fig. 5). Our method permits to provide a target detector which is robust to noise.

5. CONCLUSION

In this paper, we have described a new algorithm for multidimensional and multicomponent restoration in order to improve target detection. We proposed a tensor model to consider the whole data. Both spatial and spectral information are jointly taken into account in the processing thanks to the ALS algorithm. The extension of the well-known AIC criterion enables to estimate the signal subspace dimension for each mode which can not be chosen empirically due to the large HSI dimension. The importance of the non-separability of both spatial and spectral information is highlighted and its impact on target detection was demonstrated using experimental data. We conclude that multiway filtering realizes valuable target detection of HSI by restoring the spectral signature.

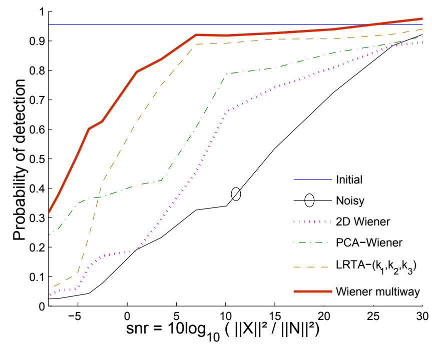

Fig. 5. Probability of detection with respect to the SNR and with a fixed probability of false alarm equal to 10^{-3}

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