REGISTRATION OF GEOMETRIC DEFORMATIONS IN THE PRESENCE OF VARYING ILLUMINATION

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ABSTRACT

We address the problem of object registration when the observation differs from the object both geometrically and radiometrically. The geometric deformations being considered are affine. The radiometric deformations are due to the *a-priori* lack of knowledge regarding the locations and intensities of the light sources. Hence, to solve the registration problem, a joint solution for the radiometric and the geometric deformations must be offered. A direct approach for solving the joint registration problem as an optimization problem leads to a high-dimensional non-convex search problem. In this paper, we treat the images as vector valued measurements, such that each element of the vector provides the intensity at a specific spectral (color) band. By applying a set of operators, derived in the paper, to the vector valued data the original high-dimensional search problem is replaced by an *equivalent* problem, expressed in terms of two systems of *linear* equations. Their solution provides an exact solution to the joint problem.

Index Terms— Image registration, Image recognition, Parameter estimation, Nonlinear estimation, Multidimensional signal processing

1. INTRODUCTION

This paper deals with the problem of deformation estimation and object registration when two observations on an object differ both geometrically and radiometrically. The geometric deformations we consider are affine, and the radiometric changes are due to variations in the illumination on the object. Thus, the problem we face is that of jointly estimating the geometric and the radiometric changes that deforms one observation into the other. More specifically, in many problems the illumination on the observed object is varying from observation to observation due to changes in the locations and intensities of the light sources, and their position relative to the object. This variability results in variations in the measured intensities across different observations. Solutions to the image registration problem, that aim to find for each point in one observation its corresponding point in the other by exploiting the rich intensity information in order to achieve high accuracy in estimating the geometric deformation, must therefore take into account the radiometric variability of the possible observations. Hence, one must jointly solve the illumination and geometric registration problems.

In this paper we study the case where the observed three-dimensional rigid object undergoes an affine transformation. The position of the light source(s) illuminating the object is unknown. Hence the observed image may change with the varying position of the light source relative to the object. To simplify the derivation it is assumed however that the camera is at a "large" distance from the object so that the geometric deformation of the observed rigid object is affine.

In order to solve the registration problem completely, the effects of the possible illumination variations between the two images to be registered must be taken into account, and the solution for estimating the geometric deformation must be designed to be invariant to the radiometric changes. This can be achieved either by making the deformation estimation intrinsically invariant to the radiometric changes, or by solving jointly the problems of estimating the radiometric and the geometric deformations. We intend to address the problem of jointly estimating the geometric deformation $\mathbf{x} \rightarrow \mathbf{A}\mathbf{x} + \mathbf{k}$ and the illumination variation. To do so, we begin by introducing a model of the intensity variations caused by illumination variations.

2. THE ILLUMINATION MODEL

Many pieces of work attest to the fact that the set of images of an object under all possible illumination conditions lies on or near a low-dimensional cone in the space of images. In the case of Lambertian reflectance and a convex object (or more generally, no cast shadows), it is well known (see *e.g.*, [2]) that any image of the object under an arbitrary point light source at infinity can be generated from three other such images, provided that in these images all points of the object are illuminated (in the following we'll use the super script c to denote a single channel of a vector valued measurement):

$$g^c(\mathbf{x}) = max_0(\sum_{i=1}^{3} a_i g_i^c(\mathbf{x})) \qquad (1)$$

where $\max_0$ takes the maximum of 0 and its argument. This equation is a result of the fact that

$$g^c(\mathbf{x}) = \max_0(n(\mathbf{x}) \cdot s) \qquad (2)$$

where in the inner product expression $s \in R^3$ represents the light source magnitude and direction (assuming a point light source at infinity, so that all the light rays reach the surface in parallel and at same intensity, allows us to use s and not $s(\mathbf{x})$), and $n(\mathbf{x}) \in R^3$ represents the object normal direction and albedo at point $\mathbf{x}$. Let $\{s_i\}_{i=1..3}$ be the light sources for the three images, assumed linearly independent. Any other light source can be represented as a linear combination of these three vectors:

$$s = \sum_{i=1}^{3} a_i s_i \qquad (3)$$

Substituting (3) in (2), and noting that by the assumption that all points of the object are illuminated by the three sources s_i, $n(\mathbf{x}) \cdot s_i \geq 0$, for every $\mathbf{x}$, and thus $n(\mathbf{x}) \cdot s_i = \max_0(n(\mathbf{x}) \cdot s_i) = g_i^c(\mathbf{x})$, we obtain (1).

If we now include the effect of a geometric affine distortion in (1), we find the following generalization of the basic problem $h^c(\mathbf{x}) = g^c(\mathbf{Ax} + \mathbf{k})$:

$$h^c(\mathbf{x}) = max_0(\sum_{i=1}^{3} a_i g_i^c(\mathbf{Ax} + \mathbf{k})) \qquad (4)$$

where now we have to estimate both the geometric transformation parameters $\mathbf{A}, \mathbf{k}$ and the radiometric parameters a_i.

A generalization of the above result [1] recognizes that under multiple point light sources, the images formed by each light source independently simply add. Thus under M light sources, s_m, an arbitrary image can be written as

$$g^c(\mathbf{x}) = \sum_{m=1}^{M} max_0(\sum_i a_{m,i} g_i^c(\mathbf{x})) \qquad (5)$$

Again we can include the effect of a geometric distortion and create a further generalization of our basic model, in which now we must also estimate the number of light sources, if unknown.

The above analysis has a straightforward extension to the case where the data (both templates and observations) are vector valued images: Let $\mathbf{h} : R^n \rightarrow R^l$ be a vector valued function defined on some compact support subset of R^n. (If, for example, we analyze color images, $n = 2$, and the color space is the span of the color coordinate system RGB, *i.e.*, $\ell = 3$). In the following it is assumed that all light sources are point light sources located at infinity, and are ideal white so that every change of the direction, intensity or number of such light sources uniformly affects all spectral bands.

Assuming the existence of M light sources and that the deformation is affine, the image obtained in each spectral band obeys the above model. Hence, the vector valued observation is given by

$$\mathbf{h}(\mathbf{x}) = [h^1(\mathbf{x}), \ldots, h^\ell(\mathbf{x})]^T \text{such that}$$
$$h^c(\mathbf{x}) = \sum_{m=1}^{M} max_0(\sum_{i=1}^{3} a_{m,i} g_i^c(\mathbf{Ax} + \mathbf{k})), c \in \{1, \ldots \ell\} \qquad (6)$$

In the following we make two assumptions:

1. Assuming that a light source illuminates the entire surface, means mathematically that $n(x) \cdot s \geq 0$. Therefore, narrowing our solution to the case where all light sources (both the ones used for creating the templates and the one creating the observation) illuminate the entire surface (*i.e.*, a line that connects a light source to each point of the surface will not intersect with any other point of the surface and will first meet the surface on its front side), the $\max_0$ operation in (4) becomes redundant.

2. Single light source: Dealing with light sources such that all of them view the whole surface (as explained above) enables an easy extension to the case of a finite number of light sources: omitting the $\max_0$ from (5), we can change the summation order, and treat the problem as if there was a single light source which is equivalent to the M existing ones:

$$\begin{aligned} g^c(\mathbf{x}) &= \sum_{\nu=1}^{M}\sum_{i=1}^{3} a_{\nu,i} g_i^c(\mathbf{x}) = \sum_{i=1}^{3}\sum_{\nu=1}^{M} a_{\nu,i} g_i^c(\mathbf{x}) \\ &= \sum_{i=1}^{3}(\sum_{\nu=1}^{M} a_{\nu,i}) g_i^c(\mathbf{x}) \end{aligned} \qquad (7)$$

Since $(\forall \nu \in [1..M], n(x) \cdot s_\nu \geq 0) \Rightarrow n(x) \cdot (\sum_{\nu=1}^{M} s_\nu) \geq 0$ there exists an "equivalent" light source, given by the following linear combination of the three independent light source: $s = \sum_{i=1}^{3} \tilde{a}_i s_i$ where $\tilde{a}_i = \sum_{\nu=1}^{M} a_{\nu,i}$.

Under these assumptions, (6) can be compactly rewritten in a vector form

$$\mathbf{h}(\mathbf{x}) = \sum_{i=1}^{3} a_i \mathbf{g}_i(\mathbf{Ax} + \mathbf{k}) \qquad (8)$$

3. PROBLEM DEFINITION

Denote by $\mathbf{A}, \mathbf{k}$ the parameters of the affine deformation (we'll further assume that the determinant of the matrix $\mathbf{A}$ is positive), and by $\{a_i\}_{i=1..3}$ the illumination parameters. Given non-negative, bounded, Lebesgue measurable, compactly supported functions with no affine symmetry $\mathbf{h}(\mathbf{x}), \{\mathbf{g}_i(\mathbf{x})\}_{i=1..3} \in M_{Aff}(R^2, R^\ell)$ (see [3] for a rigorous definition), representing vector-valued images as explained earlier, such that (8) holds, the problem is to *find the matrix* $\mathbf{A}$*, the translation vector* $\mathbf{k}$ *and the coefficients* $\{a_i\}, i = 1..3$. In the following we will refer to $\{\mathbf{g}_i(\mathbf{x})\}_{i=1..3}$ as the set of three available templates, respectively obtained by illuminating the object using three linearly independent point sources, while $\mathbf{h}(\mathbf{x})$ is the deformed observation.

4. AN ALGORITHMIC SOLUTION

Let μ denote the Lebesgue measure on R^2, and let $I_{(0,\infty)}(x)$ denote the characteristic function of the interval $(0, \infty) \in R$. Thus, for example, $I_{(0,\infty)}(\mathbf{h}(\mathbf{x}))$ is one for all the points $\mathbf{x}$ in the support of $\mathbf{h}(\mathbf{x})$. Applying the operator $I_{(0,\infty)}(x)$ to both sides of (8) we have:

$$\int I_{(0,\infty)}(\mathbf{h}(\mathbf{x}))d\mathbf{x} = \int I_{(0,\infty)}(\sum_{i=1}^{3} a_i \mathbf{g}_i(\mathbf{A}\mathbf{x} + \mathbf{k}))d\mathbf{x}$$

$$= |\mathbf{A}|^{-1} \int I_{(0,\infty)}(\sum_i a_i \mathbf{g}_i(\mathbf{y}))d\mathbf{y}$$

$$= |\mathbf{A}|^{-1} \int I_{(0,\infty)}(\sum_i \mathbf{g}_i(\mathbf{y}))d\mathbf{y}$$

$$\Rightarrow \ |\mathbf{A}| = \frac{\mu\{\text{supp}[\sum_i \mathbf{g}_i(\mathbf{x})\}}{\mu\{\text{supp}[\mathbf{h}(\mathbf{x})]\}} = \frac{\mu\{\bigcup_i \text{supp}[\mathbf{g}_i(\mathbf{x})]\}}{\mu\{\text{supp}[\mathbf{h}(\mathbf{x})]\}} \qquad (9)$$

Therefore, $|\mathbf{A}|$ can be estimated by the ratio between the supports of $\mathbf{h}(\mathbf{x})$ and the templates. Having estimated $|\mathbf{A}|$, we can launch the process of estimating$\{a_i\}, i = 1, 2, 3$.

Rewriting (8) as a set of ℓ equations we have

$$h^c(\mathbf{x}) = \sum_{i=1}^{3} a_i g_i^c(\mathbf{A}\mathbf{x} + \mathbf{k}), c = 1, \dots \ell \qquad (10)$$

Following [3], in order to convert (10) into a linear system of equations we integrate both sides to get

$$\int_{R^n} h^c(\mathbf{x})d\mathbf{x} = \int_{R^n} \sum_{i=1}^{3} a_i g_i^c(\mathbf{A}\mathbf{x} + \mathbf{k})d\mathbf{x}$$

$$= \left|\mathbf{A}^{-1}\right| \int_{R^n} \sum_{i=1}^{3} a_i g_i^c(\mathbf{y})d\mathbf{y}, \quad c = 1, \dots, \ell \qquad (11)$$

From which we obtain

$$|\mathbf{A}| \int_{R^n} h^c(\mathbf{x})d\mathbf{x} = \sum_{i=1}^{3} a_i \int_{R^n} g_i^c(\mathbf{y})d\mathbf{y} \quad c = 1, \dots \ell$$

or in a matrix form :

$$\begin{bmatrix} \int g_1^1(\mathbf{y})d\mathbf{y} & \int g_2^1(\mathbf{y})d\mathbf{y} & \int g_3^1(\mathbf{y})d\mathbf{y} \\ \vdots & & \vdots \\ \int g_1^\ell(\mathbf{y})d\mathbf{y} & \int g_2^\ell(\mathbf{y})d\mathbf{y} & \int g_3^\ell(\mathbf{y})d\mathbf{y} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = |\mathbf{A}| \begin{bmatrix} \int h^1(\mathbf{x})d\mathbf{x} \\ \vdots \\ \int h^\ell(\mathbf{x})d\mathbf{x} \end{bmatrix} \qquad (12)$$

Where in the above, the only unknowns are $a_i, i = 1..3$. Solving this equation system (a least squares solution if $\ell > 3$) yields the solution for the a_i's.

Having evaluated $\{a_i\}_{i=1,2,3}$, we compose a new *equivalent* template $\mathbf{g}(\mathbf{x})$:

$$\mathbf{g}(\mathbf{x}) = \sum_{i=1}^{3} a_i \mathbf{g}_i(\mathbf{x})$$

This templates differs from the observation only geometrically:

$$\mathbf{h}(\mathbf{x}) = \mathbf{g}(\mathbf{A}\mathbf{x} + \mathbf{k})$$

Following the approach in [3] and extending it to the case of vector valued measurements, we evaluate first order moments of the different "layers" , $c = 1, \dots, \ell$ to calculate the affine deformation matrix $\mathbf{A}$. Let

$$\mathbf{A}^{-1} = \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix}$$

$$\mathbf{k} = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}, \tilde{\mathbf{k}} = -\mathbf{A}^{-1}\mathbf{k} = \begin{bmatrix} \tilde{k}_1 \\ \vdots \\ \tilde{k}_n \end{bmatrix}$$

Let $(\mathbf{A}^{-1})_j$ denote the jth row of $\mathbf{A}^{-1}$. We then have

$$\int_{R^n} x_j h^c(\mathbf{x})d\mathbf{x} = \int_{R^n} x_j g^c(\mathbf{A}\mathbf{x} + \mathbf{k})d\mathbf{x} \qquad (13)$$

$$= \left|\mathbf{A}^{-1}\right| \int_{R^n} ((\mathbf{A}^{-1})_j\mathbf{y} + \tilde{\mathbf{k}}_j)g^c(\mathbf{y})d\mathbf{y}$$

$$= \left|\mathbf{A}^{-1}\right| \{\sum_{i=1}^{n} q_{ji} \int_{R^n} y_i g^c(\mathbf{y})d\mathbf{y} + \tilde{\mathbf{k}}_j \int_{R^n} g^c(\mathbf{y})d\mathbf{y}\}$$

$$c = 1, \dots, \ell$$

Thus for the case where $n + 1 \leq \ell$ we can solve for $\{q_{ji}\}_{i=1}^n$ and $\tilde{\mathbf{k}}_j$. Similar system of equations is solved for each j to obtain the entire matrix $\mathbf{A}^{-1}$ and the vector $\mathbf{k}$.

Therefore, in the absence of noise and under the conditions of the model, we accurately calculate the different parameters of the deformations. Due to the linearity of the solution, when additive zero-mean noise is in presence, a L.S. solution gives us a highly accurate estimation.

5. NUMERICAL EXAMPLE

Fig. 1. The three templates

We start with a colored image and simulate different illumination conditions to create the three templates. See Figure 1. With these three templates we create the observation (see, top image in Figure 2), which is both radiometrically and geometrically deformed such that

$$h(\mathbf{x}) = (0.35g_1 + 0.05g_2 + 0.6g_3)\left[\begin{pmatrix} 0.7 & 0.4 \\ -1.1 & 1.2 \end{pmatrix}\mathbf{x}\right]$$

Using the proposed procedure, we first estimate the Jacobian of the deformation ($\hat{\Delta} = 1.2801$) and the coefficients of the illumination model ($\hat{\mathbf{a}} = [0.3509 , 0.0503 , 0.6014]$). Having these at hand, we create the "equivalent" template (middle image in Figure 2), by linearly combining the templates based on the estimated coefficients in $\hat{\mathbf{a}}$. The calculation of $\mathbf{A}$ yields

$$\mathbf{A} = \begin{bmatrix} 0.7030 & 0.4023 \\ -1.1024 & 1.2033 \end{bmatrix}$$

From which we can reconstruct the observation (bottom image in Figure 2). The difference between the observation and the estimated observation has a mean equal to 2.0526e-004 and variance equal to 0.0092.

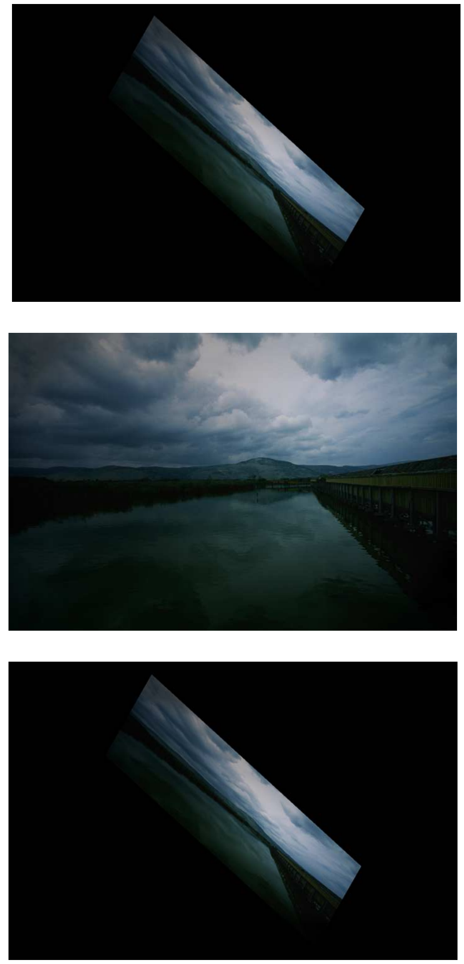

Fig. 2. Top to bottom: Observation; The new template - radiometrically registered; Estimated observation

6. REFERENCES

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