UNSCENTED KALMAN FILTER FOR IMAGE ESTIMATION IN FILM-GRAIN NOISE

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ABSTRACT

This paper presents a novel approach based on the unscented Kalman filter (UKF) for image estimation in film-grain noise. The image prior is modeled as non-Gaussian and is incorporated within the UKF frame work using importance sampling. A small carefully chosen deterministic set of sigma points is used to capture the prior and is propagated through film-grain nonlinearity to compute image statistics. Experimental results are given to demonstrate the efficacy of the proposed method.

***Index Terms*—** Film-grain noise, Unscented Kalman filter, Markov random fields, Importance sampling.

1. INTRODUCTION

Image estimation refers to estimating an original image from its degraded observation. When the observation is linearly related to state, and the modeling errors are Gaussian, then the Kalman filter provides an optimal estimate of the state. But in practice, an image sensor can possess nonlinear characteristics. One such example is the photographic film in which the film density is related linearly to the logarithm of the exposure (given by the D - log E curve of the film). Film-grain noise manifests itself as multiplicative non-Gaussian noise in the exposure domain. Among earlier works on film-grain noise removal, Andrews et al. [1] expanded the nonlinear observation model into a Taylor series expansion about the mean of the observed image and derived an approximate filter for recovering the original image. Naderi and Sawchuk [2] proposed a signal-dependent model for film-grain noise and develop an adaptive Wiener filter based on the non-stationary first-order statistics of the image. Tekalp et. al. [3] proposed a modified Wiener filter (MWF), assuming the noise to be wide sense stationary and incorporating the sensor nonlinearity into the restoration procedure. They demonstrated improvements over the linear Wiener filter. Ibrahim and Rajagopalan [4] recently proposed a particle filter (PF) based approach for image restoration in film-grain noise. Because it works by propagating particles, the PF is computationally quite intensive.

In this paper, we propose an importance sampling based unscented Kalman filter (UKF) for film-grain noise removal which is not only computationally efficient but also performs very well. Image estimation based on the traditional autoregressive (AR) model often results in smoothed edges. We adopt a discontinuity adaptive markov random field model to encode the statistical dependence among the neighboring pixels but with edge preserving capability. We employ importance sampling to estimate the statistics of this non-Gaussian prior and propagate them to the update stage of the UKF.

1.1. Problem formulation

Photographic film is a widely used image recording medium. In addition to grainy appearance, film-grain noise in photographic films poses a serious limitation for compressing old archival movies. In this paper, we pose the problem of image recovery in film-grain noise as an image estimation problem. In the density domain, the degraded image can be modeled as a logarithmic nonlinearity of the original image corrupted by additive white Gaussian noise. i.e.,

$$r_d(m,n) = \alpha \log_{10}(s(m,n)) + \beta + v(m,n) \qquad (1)$$

where s is the original image, r_d is the degraded observation in density domain, noise v is additive white Gaussian with zero mean and variance σ_v^2 while parameters α and β are the slope and offset of the $D - \log E$ curve of the film. Alternatively, we can write an equivalent linear model with the noise being multiplicative and non-Gaussian in the exposure domain. i.e.,

$$r_e(m,n) = s(m,n)(10^{v(m,n)/\alpha}) \qquad (2)$$

Here, r_e is the degraded observation in the exposure domain, and r_d and r_e are related as $r_d = \alpha \log_{10}(r_e) + \beta$. The problem of image estimation is to compute s given r_d or r_e.

2. UNSCENTED KALMAN FILTER

In the recent past, there has been excellent success in 1-D nonlinear filtering by employing the UKF [5, 6, 7]. The UKF proposed by Julier and Uhlmann [5] not only outperforms the extended Kalman filter (EKF) in implementation ease and accuracy but has also been observed to be more stable. The UKF is a straight forward application of the scaled unscented transformation to recursive minimum mean-square-error estimation [6].

2.1. Unscented Transformation

The Unscented transformation (UT) is founded on the intuition that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation [5, 6]. The UT is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation. Consider propagating a n_x dimensional random variable $\mathbf{x}$ through an arbitrary nonlinear function $\mathbf{g} : R^{n_x} \rightarrow R^{n_y}$ to generate $\mathbf{y}$, i.e., $\mathbf{y} = \mathbf{g}(\mathbf{x})$. Assume $\mathbf{x}$ has mean $\bar{\mathbf{x}}$ and covariance $\mathbf{P}_x$. To calculate the statistics (first two moments) of $\mathbf{y}$ using the scaled UT, we proceed as follows: First, a set of $2n_x + 1$ weighted samples or sigma points $\mathbf{S}_i = \{\mathbf{W}_i, \mathbf{X}_i\}$ are deterministically chosen so that they completely capture the true mean and covariance of the prior random variable $\mathbf{x}$. A selection scheme that satisfies this requirement is given in [6, 7]. Each sigma point is now propagated through the true nonlinear function $\boldsymbol{Y}_i = \mathbf{g}(\boldsymbol{X}_i), \quad i = 0, 1, ..., 2n_x$ and the estimated mean and covariance of $\mathbf{y}$ are computed as $\bar{\mathbf{y}} = \sum_{i=0}^{2n_x} \mathbf{W}_i^{(m)} \mathbf{Y}_i, \quad \mathbf{P}_y = \sum_{i=0}^{2n_x} \mathbf{W}_i^{(c)} (\mathbf{Y}_i - \bar{\mathbf{y}})(\mathbf{Y}_i - \bar{\mathbf{y}})^T$. The estimates of the mean and covariance are accurate to second order (third order for Gaussian priors) [5] of the Taylor series expansion of $\mathbf{g}(\mathbf{x})$ for any nonlinear function.

2.2. UKF for film-grain noise removal

It is well known that an image can be modeled as a 2-D autoregressive (AR) process. The corresponding AR equation can be written as a state transition equation in the form

$$\mathbf{x}_{(m,n)} = \mathbf{F}\mathbf{x}_{(m,n-1)} + \mathbf{u}_{(m,n)} \tag{3}$$

where $\mathbf{x}_{(m,n)}$ is the state vector and $\mathbf{F}$ is the state transition matrix. In the Kalman filtering frame work, there are two occurrences where we want to propagate the state variable through the transformation. One is in predicting the new state from the past and the other is while obtaining the observation from the predicted state. If either the state or the measurement model is non linear, we can make use of the unscented transformation which leads to the UKF [5, 6, 7].

Film-grain noise is multiplicative and non-Gaussian in the exposure domain. We first propose a UKF based image estimation algorithm. For non-Gaussian/multiplicative noise, the state random variable (RV) must be augmented with the noise variables: $\mathbf{x}^a_{(m,n)} = [\mathbf{x}^T_{(m,n)} \mathbf{u}^T_{(m,n)} \mathbf{v}^T_{(m,n)}]^T$ (having dimension n_a). The scaled UT sigma point selection scheme [6] is applied to this new augmented state RV (assuming some initial mean and covariance for the state and known noise statistics) to calculate the corresponding sigma matrix $\boldsymbol{X}^a = [(\boldsymbol{X}^x)^T \ (\boldsymbol{X}^u)^T \ (\boldsymbol{X}^v)^T]^T$. These sigma points are propagated according to the state equation[1] $\boldsymbol{X}^x_{n/n-1} = \mathbf{f}(\boldsymbol{X}^x_{(m,n-1)}, \boldsymbol{X}^u_{(m,n-1)})$ and the measurement equation $\boldsymbol{Y}_{n/n-1} = \mathbf{h}(\boldsymbol{X}^x_{n/n-1}, \boldsymbol{X}^v_{(m,n-1)})$ where
$\mathbf{f}(\boldsymbol{X}^x, \boldsymbol{X}^u) = \mathbf{F}\boldsymbol{X}^x + \begin{bmatrix}\boldsymbol{X}^{uT} & \mathbf{0}^T & \mathbf{0}^T\end{bmatrix}^T$ and $\mathbf{h}(\boldsymbol{X}^x, \boldsymbol{X}^v) = \mathbf{H}\boldsymbol{X}^x .* 10^{\cdot(\boldsymbol{X}^v/\alpha)}$ Here '.' denotes point-wise operation. Since we do not assume any blurring, $\mathbf{H} = [1 \ \ 0 \ \ 0]$. The required means and covariances can be computed from the sigma points in each recursive filtering step [6, 7]. The UKF algorithm that updates the mean and covariance of the Gaussian approximation to the posterior distribution of the states [6, 7] can be employed to estimate the image.

Though, typically an AR model is used to model the image, it cannot account for sudden changes in the image such as edges. Also, an accurate identification of the AR parameters is difficult. Moreover, it cannot incorporate contextual constraints effectively. We now improve the filter by incorporating a non-Gaussian prior within the UKF framework.

3. NON-GAUSSIAN PRIOR

As against the linear dependence in AR models, Markov random fields (MRF) [8] provide better flexibility in incorporating statistical dependence along with edge preservation. Consider a 1-D signal e that is to be estimated. Let $e^{(n)}$ denote the n^{th} derivative of e. A potential function $y(e^{(n)}(x))$ quantifies the penalty against the irregularity in $e^{(n-1)}(x)$ and corresponds to prior clique (a set of connected pixels) potentials in MRF models [8]. Let $\eta = e'(x)$. The magnitude $|y'(\eta)| = |2\eta t(\eta)|$ is the strength with which the regularizer performs smoothing, where t is the interaction function. A necessary condition for any regularization model to be adaptive to discontinuities [8] is

$$\lim_{\eta \to \infty} |y'(\eta)| = \lim_{\eta \to \infty} |2\eta t(\eta)| = A \tag{4}$$

where $A \geq 0$ is a constant. The above condition with $A = 0$ completely prohibits smoothing at discontinuities as $\eta \to \infty$ whereas with $A > 0$ it allows limited (bounded) smoothing.

For a standard quadratic regularizer, $y(\eta) = \eta^2$ and the state PDF is $\exp(-\eta^2)$. An MRF with such a quadratic regularizer is referred to as Gaussian MRF (GMRF). For this regularizer, the smoothness strength increases linearly with η which inevitably leads to over-smoothing of edges. A better way to handle the situation is to use a non-Gaussian conditional density function to model the original image.

Following [8, 9], we propose to use the interaction function $t_\gamma(\eta) = \frac{1}{1+\frac{\eta^2}{\gamma}}$. For this choice of $t_\gamma(\eta)$, the smoothing strength $|\eta t_\gamma(\eta)|$ increases monotonically as η increases

[1] In the following $n/n-1$ is used to denote $(m,n)/(m,n-1)$ for convenience of presentation

within a band $B_\gamma = (-\sqrt{\gamma}, \sqrt{\gamma})$. Outside the band, smoothing decreases and becomes zero as $\eta \rightarrow \infty$. Since this enables to preserve image discontinuities, it is called a discontinuity adaptive MRF. The resulting state conditional density is non-Gaussian and is of the form $p(z) = \exp(-y_\gamma(\eta))$ where $y_\gamma(\eta) = \gamma \log(1 + \frac{\eta^2}{\gamma})$ is the corresponding potential function. For a first-order non-symmetric half plane (NSHP) support, $\eta^2(z) = ((z - z_1)^2 + (z - z_2)^2 + (z - z_3)^2 + (z - z_4)^2)/2\rho^2$ where z corresponds to the pixel to be estimated, pixels z_1, z_2, z_3, z_4 have been previously estimated in the NSHP support, and ρ controls the variation between neighboring pixel values.

3.1. Importance sampling

To incorporate a non-Gaussian prior within the UKF framework, we need to estimate the moments under the non-Gaussian PDF $p(z)$. For this we use a Monte Carlo approach known as importance sampling [10]. The main idea of importance sampling can be briefly explained as follows:

When it is difficult to draw samples from the target PDF p, we draw L samples, $\{z^{(l)}\}_{l=1}^{L}$ from another PDF q that roughly approximates p, and is known upto a multiplication constant. We use samples from the sampler PDF q to determine any estimates under p. In the regions where q is greater than p, the estimates are over-represented. In the regions where q is less than p, they are under-represented. To account for this, we use correction weights $w^l = \frac{p(z^{(l)})}{q(z^{(l)})}$ in determining the estimates under p. For example, to find the mean of the distribution p we use $\widehat{\mu}_p = \frac{\sum_{l=1}^{L} w^l z^{(l)}}{\sum_{l=1}^{L} w^l}$. As $L \rightarrow \infty$ the estimate $\widehat{\mu}_p$ tends to the actual mean value of p.

4. THE PROPOSED FILTER

In this section, we present a recursive algorithm for film-grain noise removal by extending the basic structure of the UKF discussed in section 2 to include a non-Gaussian prior.

1. At each pixel, construct the state conditional PDF using the past pixels in the NSHP support, and the values of ρ and γ in the MRF model (as discussed in section 3).
$$P\left(s(m,n)/\widehat{s}(m-i,n-j)\right) = \exp\left(-\gamma \log\left(1 + \frac{\eta^2(s(m,n))}{\gamma}\right)\right)$$
where $0 \leq i,j \leq M$ and M is order of the NSHP support.

2. Obtain the mean and covariance of the above PDF using importance sampling (Section 3). Draw samples $\{z^l\}$, $l = 1, 2, ..., L$ from a Cauchy sampler[2] q. The samples are weighted by $w^l = \frac{p(z^l)}{q(z^l)}$. The mean $\widehat{\mu}_p$ and variance $\widehat{\sigma}_p^2$ of p are computed as
$$\widehat{\mu}_p = \frac{\sum_{l=1}^{L} w_l z^{(l)}}{\sum_{l=1}^{L} w_l} \quad \text{and} \quad \widehat{\sigma}_p^2 = \frac{\sum_{l=1}^{L} w_l (z^{(l)} - \widehat{\mu}_p)^2}{\sum_{l=1}^{L} w_l}$$

3. These estimates are directly used to obtain the one-step ahead predicted sigma points as follows:
$$\overline{\mathbf{x}}_{n/n-1} = \widehat{\mu}_p \quad \text{and} \quad \boldsymbol{P}_{n/n-1} = \widehat{\sigma}_p^2.$$
$$\overline{\mathbf{x}}^a_{n/n-1} = [\overline{\mathbf{x}}^T_{n/n-1} \quad \mathbf{0}]^T, \quad \boldsymbol{P}^a_{n/n-1} = \begin{bmatrix} \boldsymbol{P}_{n/n-1} & \mathbf{0} \\ \mathbf{0} & \sigma_v^2 \end{bmatrix}$$

   - Calculate sigma points:
$$\boldsymbol{X}^a_{n/n-1} = [\overline{\mathbf{x}}^a_{n/n-1}, \quad \overline{\mathbf{x}}^a_{n/n-1} \pm \sqrt{(n_a + \lambda)\boldsymbol{P}^a_{n/n-1}}]$$
$$\boldsymbol{Y}_{n/n-1} = \mathbf{h}(\boldsymbol{X}^x_{n/n-1}, \boldsymbol{X}^v_{(m,n-1)})$$
$$\overline{\mathbf{y}}_{n/n-1} = \sum_{i=0}^{2n_a} \boldsymbol{W}_i^{(m)} \boldsymbol{Y}_{i,n/n-1}$$

   - Measurement update is the same as in UKF [6, 7]
$$\widehat{s}(m,n) = \overline{\mathbf{x}}_{(m,n)}$$

Here, $(\sqrt{(n_a + \lambda)\boldsymbol{P}_x})$ is the matrix square root, λ is a scaling parameter, $\boldsymbol{X}^a = [(\boldsymbol{X}^x)^T \; (\boldsymbol{X}^v)^T]^T$ and $\mathbf{h}(\boldsymbol{X}^x, \boldsymbol{X}^v) = \boldsymbol{X}^x .* 10^{\cdot(\boldsymbol{X}^v/\alpha)}$. Observe that the mean and variance obtained by importance sampling of the non-Gaussian prior are used to generate the sigma points. They are propagated through the true non-linearity and are taken to the update step of the unscented Kalman filter to determine image estimates.

5. EXPERIMENTAL RESULTS

In this section, we present results obtained using the proposed filter and compare them with MWF and PF. In synthetic images, for a quantitative comparison, we use improvement-in-signal-to-noise-ratio (ISNR) which is defined as $ISNR = 10 \log_{10} \left(\frac{\sum_{m,n} (s_{deg}(m,n) - s(m,n))^2}{\sum_{m,n} (\widehat{s}(m,n) - s(m,n))^2} \right) dB$. Here, (m,n) are over the entire image, $s_{deg}(m,n)$ is the exposure domain degraded image pixel and $\widehat{s}(m,n)$ represents the estimated image pixel.

Fig. 1(a) shows the Peppers image. It is degraded by film-grain noise (Eq. 1) and is shown in the exposure domain (Fig. 1(b)).The image recovered by the MWF is shown in Fig. 1(c). We note that a considerable amount of noise is left unfiltered (visible on uniform gray backgrounds). The PF improves noise reduction significantly but is slightly blurred (Fig. 1(d)). The proposed method performs the best both in terms of noise reduction and preservation of edges (Fig. 1(e)). This is also reflected in its high ISNR value.

Fig. 2(a) shows an image with real film-grain noise. The image obtained by MWF (Fig. 2(b)) leaves residual noise on the face. The image obtained by PF (Fig. 2(d)) blurs finer details such as the lips, the ear and the hair. The image estimated by the proposed filter is shown in Fig. 2(f). It has very sharp edges, has almost no grains and all the facial details are preserved. Also, the image obtained with the proposed method has the best visual and natural appearance. On a Pentium 4 PC with 256 MB RAM for a 200 × 200 image PF with 200 samples requires 150 seconds where as the proposed ISUKF executes in 20 seconds.

[2] Cauchy sampler is employed to benefit from its heavy tailed distribution [10]

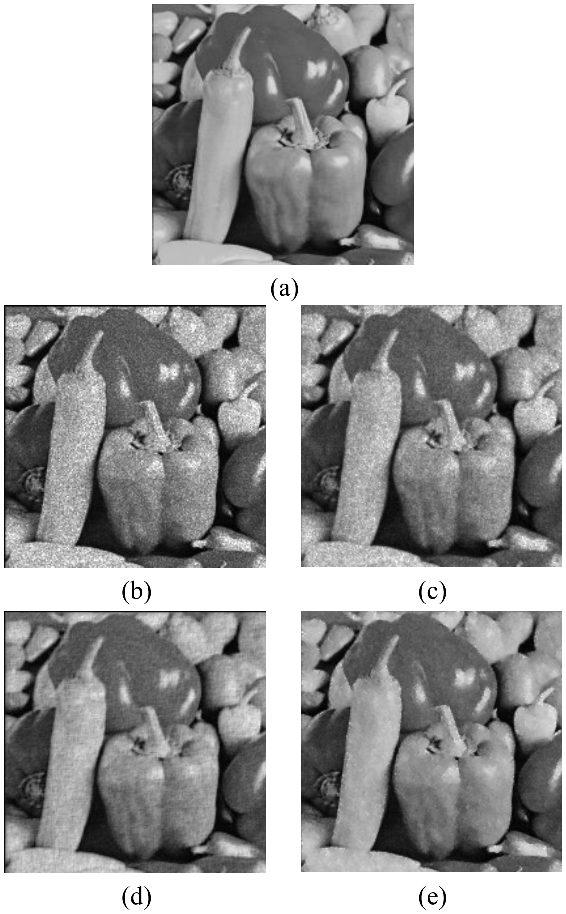

Fig. 1. Peppers image (a) Original. (b) Degraded ($\sigma_v^2 = 0.1$). Estimated image using (c) MWF ($ISNR = 2.98$ dB), (d) PF ($ISNR = 4.12$ dB) and (e) Proposed ($ISNR = 4.92$ dB).

6. CONCLUSIONS

In this paper, we have proposed a discontinuity adaptive unscented Kalman filter for film-grain noise removal. The exact exposure domain relation is used as the observation model for the UKF. This is further improved by incorporating a non-Gaussian prior through importance sampling. The improvement obtained over existing filters is demonstrated with both synthetic and real examples.

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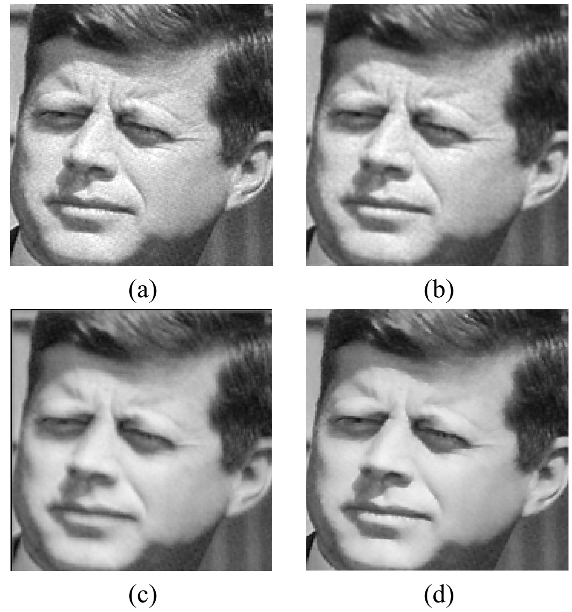

Fig. 2. (a) Face image with real film-grain noise ($\sigma_v^2 = 0.05$). Image estimated using (b) MWF, (c) PF, and (d) Proposed.

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