FAST COMPUTATION OF INVERSE KRAWTCHOUK MOMENT TRANSFORM USING CLENSHAW'S RECURRENCE FORMULA

[1] P.Ananth Raj and [2] A. Venkataramana

[1] Associate Professor, Dept of ECE, College of Engineering, Osmania University, Hyderabad- 500 007, Andhrapradesh, India, email: peruguananth@yahoo.com, Phone no +91 040 27098213, Senior member IEEE, All correspondences may be addressed to this author

[2] Lecturer in ECE, Government Polytechnic College, Mahabubnagar-509 001, Andhrapradesh, India, email:venkat_ramana33@yahoo.com

Abstract: This paper proposes a method for fast computation of inverse Krawtchouk moment transform for signal and image reconstruction using Clenshaw's recurrence formula. It is shown that the proposed approach requires lesser computations than the straightforward method of computation for signal and image reconstruction. In order to verify the proposed approach, simulation results are reported for 1D signal and 2D image reconstructions from the given Krawtchouk moments for signal and image. The proposed approach is suitable for parallel VLSI implementation because the proposed structure is simple, regular and modular.

Keywords: Clenshaw's formula, Krawtchouk moments

1. INTRODUCTION

During the last few years, many moments were suggested as shape descriptors of an image in pattern recognition, image classification, template matching, image watermarking, edge detection etc. The moments used for the above applications are Geometric[1], Legendre[2], Zernike[2] etc. Some of the problems associated with these moments are (i) Numerical approximation of continuous integrals (ii) Large variation in the dynamic range of values and (iii) Coordinate transformations. Hence, in order to solve the above problems, Mukundan et al.[3] proposed Tchebichef moments based on discrete Tchebichef polynomials and they experimentally verified that the reconstruction error is minimum using these moments as compared with other moments like Legendre and Zernike moments. Recently, Pew-Thian Yap et al.[4] proposed Krawtchouk moments based on discrete Krawtchouk polynomials. Their experimental results show that Krawtchouk moments are better in terms of reconstruction error when compared with Zernike, Legendre and Tchebichef moments.

Many algorithms have been developed for fast computation of Legendre moments[5-7]. Recursive algorithms have been found very effective for realization using software and very large scale integrated circuit (VLSI) techniques. Chiang and Liu [8] proposed recursive algorithms for forward and inverse modified discrete cosine transform that are suitable for parallel VLSI implementation. Vladimir Nikolajevic et al.[9] proposed another algorithm for the computation of forward and inverse modified discrete cosine transform using Clenshaw's formula. Algorithm for parallel recursive computation of inverse Legendre moment transform for signal and image reconstruction using Clenshaw's recurrence formula was proposed in reference[10]. Recently, Guobao Wang et al.[11] proposed a recursive algorithm for computation of Tchebichef moment and its inverse transform using Clenshaw's formula. However, less work has been reported for the fast computation of inverse Krawtchouk moment transform.

In this paper, we propose a recursive algorithm for the fast computation of inverse Krawtchouk moment transform for signal and image reconstruction purposes using Clenshaw's recurrence formula. The reconstruction can be effectively implemented using recursive equations. The proposed recursive structure is simple, regular and particularly suitable for low cost VLSI implementation.

This paper is organized into five sections. A brief presentation on Krawtchouk moments is given in section 2. Section 3 presents the details about Clenshaw's recurrence formula. Proposed method of computation of inverse Krawtchouk moments transform for both signal and image reconstruction is presented in section 4. Finally, the last section presents the simulation results on reconstruction and conclusions about the work.

2. A BRIEF PRESENTATION ON KRAWTCHOUK MOMENTS

The p^{th} order Krawtchouk moments [4] Q_p for N point 1D signal $f(x)$ is defined as

$$Q_p = \sum_{x=0}^{N-1} \bar{K}_p(x;\lambda,N-1) f(x) \tag{1}$$

where $\bar{K}_p(x;\lambda,N-1)$ is the p^{th} order weighted Krawtchouk polynomial [4] which is defined as

$$\bar{K}_p(x;\lambda,N-1) = K_p(x;\lambda,N-1)\sqrt{\frac{w(x;\lambda,N-1)}{\rho(p;\lambda,N-1)}} \tag{2}$$

where $K_p(x;\lambda,N-1)$ is the p^{th} order discrete Krawtchouk polynomial [4] defined as

$$K_p(x;\lambda,N-1) = \sum_{k=0}^{p} a_{k,p,\lambda} x^k = {}_2F_1(-p,-x;-(N-1);\frac{1}{\lambda}) \tag{3}$$

for x, p=0,1,2,….N-1, The parameter $\lambda \in (0,1)$, ${}_2F_1$ is the hypergeometric function, defined as

$$ {}_2F_1(a,b;c;z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!} \tag{4} $$

and $(a)_k$ is the Pochhammer symbol given by

$$ (a)_k = a(a+1)....(a+k-1) $$

The weight function $w(x;\lambda, N-1)$ is given by

$$ w(x;\lambda,N-1) = \binom{N-1}{x} \lambda^x (1-\lambda)^{N-1-x} \tag{5} $$

The weight function $w(x;\lambda, N-1)$ can be recursively calculated using

$$ w(x+1;\lambda,N-1) = \left(\frac{N-1-x}{x+1}\right)\frac{\lambda}{1-\lambda} w(x;\lambda,N-1) \tag{6} $$
$$ \textit{with } w(0;\lambda,N-1) = (1-\lambda)^{N-1} $$

and $\rho(p;\lambda, N-1)$ is the squared norm, which is given by

$$ \rho(p;\lambda,N-1) = (-1)^p \left(\frac{1-\lambda}{\lambda}\right)^p \frac{p!}{(-N+1)_p} \tag{7} $$

The three term recursive relation for the weighted Krawtchouk polynomials is given by

$$ \bar{K}_{n+1}(x;\lambda,N-1) = \frac{\sqrt{\frac{\lambda(N-1-n)}{(1-\lambda)(n+1)}}\,((N-1)\lambda - 2n\lambda + n - x)}{\lambda(N-1-n)} \bar{K}_n(x;\lambda,N-1) - \frac{\sqrt{\frac{\lambda^2(N-1-n)(N-n)}{(1-\lambda)^2(n+1)n}}\,(n(1-\lambda))}{\lambda(N-1-n)} \bar{K}_{n-1}(x;\lambda,N-1) $$
$$ \textit{for } n = 1,2,........N-2 \tag{8} $$

$$ \textit{with } \bar{K}_0(x;\lambda,N-1) = \sqrt{w(x;\lambda,N-1)} $$
$$ \textit{and } \bar{K}_1(x;\lambda,N-1) = \left(1 - \frac{x}{\lambda(N-1)}\right)\left(\sqrt{\frac{\lambda(N-1)}{(1-\lambda)}}\right)\sqrt{w(x;\lambda,N-1)} $$

Given a set of Krawtchouk moments Q_p up to order $N_{\max}$ for a digital signal $f(x)$, its reconstruction version from Krawtchouk moments is given by

$$ f(x) = \sum_{p=0}^{N_{\max}} Q_p \bar{K}_p(x;\lambda,N-1) \tag{9} $$

Krawtchouk moments Q_{pq} of order (p,q) for a digital image $f(x,y)$ of size N x M are defined[4] as

$$ Q_{pq} = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} \bar{K}_p(x;\lambda_1,N-1)\bar{K}_q(y;\lambda_2,M-1) f(x,y) \tag{10} $$

A set of Krawtchouk moments up to order (N_{max}, M_{max}) are given, then the inverse moment transform can be computed using

$$ f(x,y) = \sum_{p=0}^{N_{\max}}\sum_{q=0}^{M_{\max}} Q_{pq} \bar{K}_p(x;\lambda_1,N-1)\bar{K}_q(y;\lambda_2,M-1) \tag{11} $$

The evaluation of inverse Krawtchouk moments transform for reconstruction as defined in eq.(9) and eq.(11) can be evaluated by summing the weighted Krawtchouk polynomials weighted with the given coefficients Q. The polynomials can be evaluated using the polynomial recursive formula given in eq.(8). If this straightforward method is applied for computing the inverse Krawtchouk moment transform as given in eq.(9), the weighted Krawtchouk polynomials up to order N_{max} need to be evaluated and it requires accumulating the sum of eq.(9). Hence, it requires more operations for computation which makes evaluation slow. In order to overcome the above problem, we propose an approach for fast computation of inverse Krawtchouk moment transform using Clenshaw's recurrence formula. This method is more effective than the straightforward method and is very much suitable for parallel VLSI implementation.

3. CLENSHAW'S RECURRENCE FORMULA

Since we use Clenshaw's recurrence formula in the proposed approach, a brief description about Clenshaw's recurrence formula is given below. Clenshaw's recurrence formula [12] is an efficient way to evaluate a sum of products of indexed coefficients by functions that obey a recurrence relation. Suppose that the desired sum is

$$ f(x) = \sum_{n=0}^{M} c_n F_n(x) \tag{12} $$

in which $F_n(x)$ obeys the recurrence relation as follows

$$ F_{n+1}(x) = \alpha(n,x)F_n(x) + \beta(n,x)F_{n-1}(x) \tag{13} $$

for some functions $\alpha(n,x)$ and $\beta(n,x)$

Then Clenshaw's recurrence formula states that the sum $f(x)$ can be evaluated by

$$ f(x) = \beta(1,x)F_0(x)\psi_2 + F_1(x)\psi_1 + c_0F_0(x) \tag{14} $$

where the quantities ψ_n can be obtained from the following recurrence:

$$ \begin{aligned} \psi_{M+2} &= \psi_{M+1} = 0 \\ \psi_n &= \alpha(n,x)\psi_{n+1} + \beta(n+1,x)\psi_{n+2} + c_n \end{aligned} \tag{15} $$

for $n = M, M-1,......1$ and solve backward to obtain ψ_2 and ψ_1

4. PROPOSED METHOD

In this section, we propose a method for fast computation of inverse Krawtchouk moments transform for both signal and image reconstruction using Clenshaw's recurrence formula.

4.1. Computation of Inverse Krawtchouk Moment Transform for Signal Reconstruction

In order to evaluate eq.(9) using Clenshaw's recurrence formula, we consider

$$ F_n(x) = \bar{K}_n(x;\lambda,N-1) $$

Comparing eq.(13) with eq.(8), we get

$$ \alpha(n,x) = \frac{\sqrt{\frac{\lambda(N-1-n)}{(1-\lambda)(n+1)}}\,((N-1)\lambda - 2n\lambda + n - x)}{\lambda(N-1-n)} \tag{16} $$

and

$$ \beta(n,x) = -\frac{\sqrt{\frac{\lambda^2(N-1-n)(N-n)}{(1-\lambda)^2(n+1)n}}\,(n(1-\lambda))}{\lambda(N-1-n)} \tag{17} $$

By using the recursive formula defined in (14) with c_0 replaced by Q_0, the reconstruction formula given in (9) can be expressed as

$$ f(x) = \beta(1,x)\bar{K}_0(x;\lambda,N-1)\psi_2 + \bar{K}_1(x;\lambda,N-1)\psi_1 + Q_0\bar{K}_0(x;\lambda,N-1) \tag{18} $$

where ψ_2, ψ_1 and ψ_0 are computed recursively from

$$\psi_{N_{\max}+2} = \psi_{N_{\max}+1} = 0$$

$$\psi_p = \alpha(p,x)\psi_{p+1} + \beta(p+1,x)\psi_{p+2} + Q_p \qquad (19)$$

$$for \;\; p = N_{\max}, N_{\max}-1, \ldots\ldots\ldots 1, 0$$

The value of ψ_0 is given by

$\psi_0 = \alpha(0,x)\psi_1 + \beta(1,x)\psi_2 + Q_0$, from which we have

$Q_0 = \psi_0 - \alpha(0,x)\psi_1 - \beta(1,x)\psi_2$

Substituting this Q_0 in eq.(18)

$$\begin{aligned} f(x) &= \beta(1,x)\bar{K}_0(x;\lambda,N-1)\psi_2 + \bar{K}_1(x;\lambda,N-1)\psi_1 \\ &\quad + (\psi_0 - \alpha(0,x)\psi_1 - \beta(1,x)\psi_2)\bar{K}_0(x;\lambda,N-1) \\ &= \bar{K}_1(x;\lambda,N-1)\psi_1 + \psi_0\bar{K}_0(x;\lambda,N-1) - \alpha(0,x)\bar{K}_0(x;\lambda,N-1)\psi_1 \qquad (20) \end{aligned}$$

By substituting the values of

$\bar{K}_0(x;\lambda,N-1)$, $\bar{K}_1(x;\lambda,N-1)$ *and* $\alpha(0,x)$ in eq.(20), we obtain

$$f(x) = \left(1 - \frac{x}{\lambda(N-1)}\right)\sqrt{\frac{\lambda(N-1)}{(1-\lambda)}}\sqrt{w(x;\lambda,N-1)}\,\psi_1 + \psi_0\sqrt{w(x;\lambda,N-1)} - \frac{\sqrt{\frac{\lambda(N-1)}{(1-\lambda)}}((N-1)\lambda - x)\sqrt{w(x;\lambda,N-1)}}{\lambda(N-1)}\psi_1$$

which on simplification gives

$$f(x) = \psi_0\sqrt{w(x;\lambda,N-1)} \qquad (21)$$

The proposed recursive approach for signal reconstruction can be summarized as follows

$$\begin{aligned} &Initialize \;\; \psi_{N_{\max}+2} = \psi_{N_{\max}+1} = 0 \\ &Do \;\; for \;\; p = N_{\max} : -1 : 0 \\ &\psi_p = \alpha(p,x)\psi_{p+1} + \beta(p+1,x)\psi_{p+2} + Q_p \qquad (22) \\ &end \\ &f(x) = \psi_0\sqrt{w(x;\lambda,N-1)} \end{aligned}$$

In the above presented approach, we recursively generate ψ_p from the input sequence Q_p (p = $N_{\max}$,…..0). At the ($N_{\max}$+1)th step, we obtain ψ_0 which is used to evaluate $f(x)$ as given in (22). Recursive structure for the implementation of 1D inverse Krawtchouk moment transform $f(x)$ according to eq.(22) is shown in Fig.1. The box z^{-1} shown in figure represents delay element.

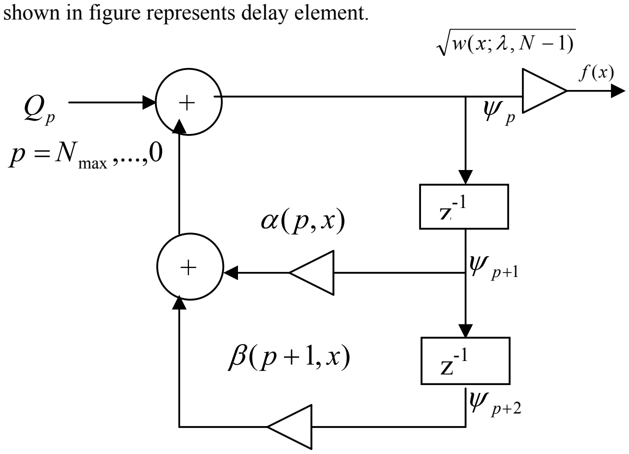

Fig.1. Recursive computation of 1 D inverse Krawtchouk moment transform

In the proposed recursive method for computing the inverse Krawtchouk moment transform, we need 2($N_{\max}$+1)+1 = 2$N_{\max}$+3 multiplications and 2($N_{\max}$+1) = 2$N_{\max}$+2 additions for each value of x. In order to compute the inverse Krawtchouk moment transform as given in (9) by using the straightforward method, we need to calculate the weighted Krawtchouk polynomials upto order $N_{\max}$ using the polynomial recursive formula given in (8) which requires 2($N_{\max}$-1)+($N_{\max}$+1) =3$N_{\max}$ - 1 multiplications including multiplication with square root of weight function and $N_{\max}$-1 additions for each value of x. Evaluation of sum given in (9) requires $N_{\max}$+1 multiplications and $N_{\max}$ additions. Hence, for calculation of inverse Krawtchouk moment transform for a single value of x, requires a total of (3$N_{\max}$-1)+($N_{\max}$+1) = 4$N_{\max}$ multiplications and ($N_{\max}$-1)+$N_{\max}$ = 2$N_{\max}$-1 additions. The same is given in table 1.

Method	Multiplications	Additions
Straightforward method	4$N_{\max}$	2$N_{\max}$-1
Proposed recursive method	2$N_{\max}$+3	2$N_{\max}$+2

Table.1. Comparison of computational complexity for computing a single 1D inverse Krawtchouk moment transform $f(x)$

4.2 Computation of Inverse Krawtchouk Moment Transform for Image Reconstruction

The reconstruction formula for 2D case as given in eq.(11) can be expressed as

$$f(x,y) = \sum_{p=0}^{N_{\max}} r_p(y)\bar{K}_p(x;\lambda_1,N-1) \qquad (23)$$

where $$r_p(y) = \sum_{q=0}^{M_{\max}} Q_{pq}\bar{K}_q(y;\lambda_2,M-1) \qquad (24)$$

and the coefficients $r_p(y)$ defined in (24) are evaluated first in direction 'y' for each value of p ranging from $N_{\max}$ to 0, according to the above presented 1D signal reconstruction method.

$$\begin{aligned} &Initialize \;\; \psi_{M_{\max}+2} = \psi_{M_{\max}+1} = 0 \\ &Do \;\; for \;\; q = M_{\max} : -1 : 0 \\ &\psi_q = \alpha(q,y)\psi_{q+1} + \beta(q+1,y)\psi_{q+2} + Q_{pq} \qquad (25) \\ &end \\ &r_p(y) = \psi_0\sqrt{w(y;\lambda_2,M-1)} \end{aligned}$$

Then $r_p(y)$ are used to evaluate $f(x,y)$ defined in (23) in the 'x' direction.

$$\begin{aligned} &Initialize \;\; \phi_{N_{\max}+2} = \phi_{N_{\max}+1} = 0 \\ &Do \;\; for \;\; p = N_{\max} : -1 : 0 \\ &\phi_p = \alpha(p,x)\phi_{p+1} + \beta(p+1,x)\phi_{p+2} + r_p(y) \qquad (26) \\ &end \\ &f(x,y) = \phi_0\sqrt{w(x;\lambda_1,N-1)} \end{aligned}$$

In eq.(25), the evaluation of $r_p(y)$ requires 2$M_{\max}$+3 multiplications and 2$M_{\max}$+2 additions for each value of 'p'. Since there are $N_{\max}$+1 values of 'p', a total of (2$M_{\max}$+3) ($N_{\max}$+1) = 2$N_{\max}M_{\max}$+2$M_{\max}$+3$N_{\max}$+3 multiplications and (2$M_{\max}$+2)($N_{\max}$+1) = 2$N_{\max}M_{\max}$+2$M_{\max}$+2$N_{\max}$+2 additions are required. Eq.(26) requires 2$N_{\max}$+3 multiplications and 2$N_{\max}$+2 additions . Hence, computation of one $f(x,y)$ using the proposed recursive implementation of (25) and (26) requires a total of (2$N_{\max}M_{\max}$+2$M_{\max}$+3$N_{\max}$+3)+(2$N_{\max}$+3) =

$(2N_{max}M_{max}+2M_{max}+5N_{max}+6)$ multiplications and $(2N_{max}M_{max}+2M_{max}+2N_{max}+2)+(2N_{max}+2)=$ $(2N_{max}M_{max}+2M_{max}+4N_{max}+4)$ additions. The straightforward method for computing one 2D inverse Krawtchouk moment transform is the two stage computation as given in (23) and (24) which requires $4M_{max}(N_{max}+1)+4N_{max} = (4M_{max}N_{max}+4M_{max}+4N_{max})$ multiplications and $(2M_{max}-1)(N_{max}+1)+(2N_{max}-1) = (2M_{max}N_{max}+2M_{max}+N_{max}-2)$ additions. The same is given in table 2.

Method	Multiplications	Additions
Straightforward method	$4M_{max}N_{max}+4M_{max}+4N_{max}$	$2M_{max}N_{max}+2M_{max}+N_{max}-2$
Proposed recursive method	$2N_{max}M_{max}+2M_{max}+5N_{max}+6$	$2N_{max}M_{max}+2M_{max}+4N_{max}+4$

Table.2. Comparison of computational complexity for computing a single 2D inverse Krawtchouk moment transform $f(x,y)$

5. SIMULATION RESULTS AND CONCLUSIONS

The proposed approach was used for reconstructing a 1D signal (30 points), binary image Letter E (40 x 40 pixels) and Lena image of size 256 x 256 from the given Krawtchouk moments of different orders and different values of λ. The reconstructed signal and images are shown in figures 2, 3 and 4 respectively. It is noted from the simulation results that the proposed approach performed well for both 1D and 2D signals. Further, during the binary image reconstruction experiment, the obtained values are not binary. Hence, these values are converted to binary by using a threshold value of 0.5. This paper proposed a simple recursive structure for fast computation of inverse Krawtchouk moment transform using Clenshaw's recurrence formula. The proposed approach requires lesser computations than the straightforward method. This approach is simple and modular in structure, hence suitable for low cost parallel VLSI implementation.

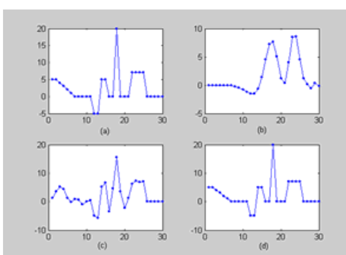

Fig.2. Original and reconstructed 1D signals (a) Original 1D signal (b)-(d) Reconstructed signal using Krawtchouk moments up to order 10, 20, 27 and λ =0.85

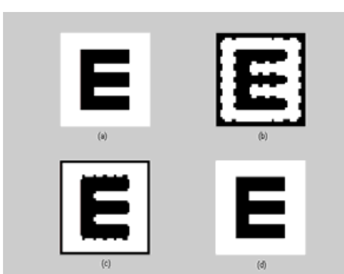

Fig.3. Original and reconstructed images (a) Original binary image (b)-(d) Reconstructed images using moments up to order 10, 15 , 20 and $\lambda_1 = 0.5, \lambda_2 = 0.5$

Fig.4. Original and reconstructed images (a) Original Lena image (b)-(d) Reconstructed images using moments up to order 50, 100, 200 and $\lambda_1 = 0.5, \lambda_2 = 0.5$.

REFERENCES

[1] Ming-Kuin Hu , " Visual Pattern Recognition by Moment Invariants " , *IRE Transaction on Information Theory*, vol.8, pp.179-187, Feb.1962.

[2] M.R.Teague, " Image Analysis Via the General Theory of Moments" , *J. optical soc Am.,* volume 70 ,pp.920-930 , Aug.1980 .

[3] R.Mukundan, S.H.Ong and P.A.Lee, " Image Analysis by Tchebichef Moments", *IEEE Transaction onImage Processing*, vol.10, no.9, pp.1357-1363, Sept.2001.

[4] Pew-Thian Yap, Raveendran Parameshran and Seng-Huat Ong, "Image Analysis by Krawtchouk Moments", *IEEE Transaction on Image Processing*, vol.12, no.11,pp.1367-1377, Nov.2003

[5] R.Mukundan and K.R.Ramakrishnan, "Fast Computation of Legendre and Zernike moments*," Pattern Recognition*, vol.28, no.9, pp.1433-1442,Sept.1995.

[6] H.Z.Shu,L.M.Luo, X.D.Bao, W.X.Yu and G.N.Han, "An Efficient Method for Computation of Legendre Moments," *Graph. Models*,vol.62,no.4,pp.237-262,July2000.

[7] J.D.Zhou, H.Z.Shu, L.M.Luo and W.X.Yu, "Two new algorithms for efficient computation of Legendre moments," *Pattern Recognition*, vol.35,no.5,pp.1143-1152,May2002.

[8] H.C.Chiang and J.C.Liu, "Recursive implementation for the forward and inverse MDCT in MPEG audio coding*," IEEE Signal Processing Lett*., vol.3,pp.116-118,April1996.

[9] Vladimir Nikolajevic and Gerhard Fettweis, " Computation of forward and inverse MDCT using Clenshaw's recurrence formula," *IEEE Transaction on Signal Processing*,vol.51,no.5pp.1439- 1444, May2003.

[10] Guo-Bao Wang and Shi-gang Wang, "Parallel recursive computation of the inverse Legendre moment transforms for signal and image reconstruction" *IEEE Signal Processing Lett*., vol.11,no.12,pp.929-932,Dec.2004.

[11] Guobao Wang and Shigang Wang, "Recursive Computation of Tchebichef Moment and its inverse transform," *Pattern Recognition*, vol.39,pp.47-56,2006

[12] W.Press, S.A.Teukolsky, W.T.Vetterling and B.P.Flannery, Numerical Recipes in C, Cambridge University Press, U.K., 1992.