FEATURE-ADAPTED FAST SLANT STACK

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ABSTRACT

This paper presents a new method for computing the Feature-adapted Radon and Beamlet transforms [1] in a fast and accurate way. These two transforms can be used for detecting features running along lines or piecewise constant curves. The main contribution of this paper is to unify the Fast Slant Stack method, introduced in [2], with linear filtering technique in order to define what we call the Feature-adapted Fast Slant Stack. If the desired feature is chosen to belong to the class of steerable filters, our method can be achieved in $O(N \log(N))$, where $N = n^2$ is the number of pixels. This new method leads to an efficient implementation of both Feature-adapted Radon and Beamlet transforms, that outperforms our previous works [1]. Our method has been developed in the context of biological imaging to detect image features lying along curves like edges or ridges as well as any kind of features that can be designed by *a priori* knowledge.

Index Terms— Fast slant stack, radon transform, beamlet transform, steerable filters, features detection.

1. INTRODUCTION

The problem of detecting curvilinear objects in images arises in various areas of image processing and computer vision, since such kind of objects occur in every natural and synthetic images, like contours of objects, roads in aerial imaging or DNA filaments in biological microscopy.

Commonly, curvilinear objects are considered as 1-dimensional manifolds that have a specific profile running along a smooth curve. The shape of this profile may be an edge- or a ridge-like feature. It can also be represented by more complex designed features. For example, in the context of DNA filament analysis in fluorescent microscopy, it is acceptable to consider the transverse dimension of a filament to be small relative to the PSF width of the microscope. Hence, the shape of the profile may be accurately approximate by a PSF model. A recent study of such models for various types of microscopes can be found in [3].

One way to detect curvilinear objects is to track locally the feature of the curve-profile; linear filtering or template matched filtering are well-known techniques for doing so. The classical Canny edge detector [4] and more recently detectors designed in [5] are based on such linear filtering techniques. They involve the computation of inner-products with shifted and/or rotated version of the feature template at every point in the image. High response at a given position in the image means that the considered area has a similarity with the feature template. Filtering is usually followed by a non-maxima suppression and a thresholding step in order to extract the objects. The major drawbacks of such approaches come from the fact that linear filtering is based on local operators: it is highly sensitive to noise but not sensitive to the underlying smoothness of the curve, which is a typical non-local property of curvilinear objects.

Alternativaly, the Radon transform is a powerful tool which may be used for line detection. Also known as the Hough transform in the case of discrete binary images, it performs a mapping from the image space into a line parameter space by computing line integrals. Formally, given an image f defined on a sub-space of $\mathbb{R}^2$, for every line parameter (t, θ), with $t \in \mathbb{R}$ and $\theta \in [0, \pi)$, the continuous Radon transform is defined by

$$R[f](t, \theta) = \int f(x, y)\delta(t - x\cos(\theta) + y\sin(\theta))dxdy. \quad (1)$$

Much attention has been given over the last twenty years to adapt this transform to digital arrays, *i.e.* when f is represented by a discrete array $I = I(u, v) : -n/2 \leq u, v < n/2$. There are two distinct approaches to compute equation (1) efficiently on digital arrays. The first one is the multiscale approach. This approach splits the image domain by recursive partitioning into dyadic squares. Radon coefficients are then computed by recombining those which have been already computed on smaller squares. The major drawback of this approach in the discrete case is the recombination process on digital grids that introduces approximations. The second kind of approach is the Fourier-based approach. It exploits the projection-slice theorem which says that the 1-dimensional constant θ-slice of the Radon transform $(R[f](t, \theta) : -\infty < t < \infty)$ and the 1-dimensional radial slice of the Fourier transform make a 1-dimensional Fourier transform pair. The major drawback of this method is again the discretization, where points of a radial slice do not intersect the cartesian grid points of the Fourier space and hence involves interpolations.

Recently, a novel Fourier-based approach has been proposed, the Fast Slant Stack methodology [2]. It avoids the drawback mentioned above by introducing a discrete definition of equation (1) and by carefully choosing a set of *grid-friendliness* lines. It yields to a fast, geometrically and algebraically exact method to compute the Radon transform in $O(N\log(N))$, where $N = n^2$ is the number of pixels.

The main objective of this article is to compute $(R[f * h^\theta](t,\theta) : -\infty < t < \infty)$, for a given set of angles that fully span the range of orientations. h is a feature template representing a specific 2-dimensional line profile, which can be an edge, a ridge or a more complex image feature. Our main result is that, thanks to the Fast Slant Stack technique presented above, for a certain class of filters h, our objective can be fulfilled efficiently in $O(N\log(N))$. Section 2 gives some key elements to understand the underlying mechanisms of the Fast Slant Stack methodology. Section 3 presents our contribution while section 4 presents an extension of our contribution to the Beamlet transform [6], which can be viewed as a multiscale Radon transform.

2. FAST SLANT STACK

Our methodology being based on the Fast Slant Stack approach [2], we recall here some of its key elements and refer the reader to [2] for more details. A *basically horizontal* line is a line of the form $y = \tan(\theta)x + t$, where the slope $|\tan(\theta)| \leq 1$. Notice that the methodology described in the sequel of this paper could be applied directly to the complementary set of *basically vertical* lines of form $x = \tan(\theta)y + t$. The Radon transform associated with this set of line is

$$R[I](t,\theta) = \sum_u \tilde{I}(u, \tan(\theta)u + t). \tag{2}$$

where $\tilde{I}(u,y)$ is an interpolant, that takes discrete values in the first argument and continous values in the second argument. The 1-dimensional interpolation is realized thanks to a Dirichlet kernel (see [2] for complete details). The parametrization of the Radon space is chosen as follows: one considers only the lines having an intercept $-n \leq t < n$ and the set of angles $\theta = \arctan(2l/n), -n/2 \leq l < n/2$. According to this set of angles, the fundamental property of equation (2) is driven by the following result:

Theorem 1 *(Projection-Slice Theorem) Define the 2-dimensional Fourier transform of the array I via:*

$$\hat{I}(k_1,k_2) = \sum_{u,v} I(u,v)\exp\{-i\frac{\pi}{n}(uk_1 + vk_2)\},$$

where $-n \leq k_1, k_2 < n$*. Then, for each fixed* $\theta = \arctan(2l/n)$*,* $-n/2 \leq l < n/2$*, the 2n numbers*

$$R[I](t,\theta), \qquad -n \leq t < n,$$

are a 1-dimensional discrete Fourier transform pair with the 2n numbers

$$\hat{I}(\pi\frac{k}{n}\tan(\theta), \pi\frac{k}{n}), \qquad -n \leq k < n.$$

The key point of the Fast Slant Stack method is the special nature of the angles θ chosen above. Using Theorem (1), one has a connection between Radon values and a set of spatial frequencies $\xi_{l,k} = (\pi\frac{k}{n}\frac{2l}{n}, \pi\frac{k}{n})$ with $-n \leq k < n$ and $-n/2 \leq l < n/2$. This is a special non-Cartesian pointset in frequency domain which has been known as the Pseudopolar grid, and is illustrated in Fig 1.

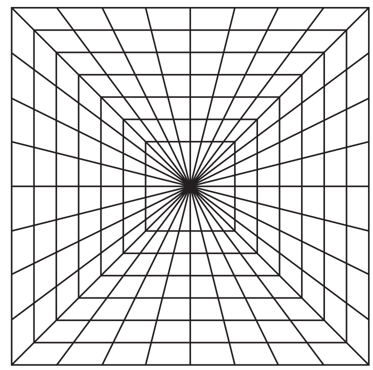

Fig. 1. The Pseudopolar Grid for $n = 8$

This pointset can be efficiently computed according to the Pseudo-Polar Fast Fourier transform [2], noted $\mathcal{P}$. $\mathcal{P}[I]$ is an array of $2n$ rows by n columns where the l^{th} column refers to the values $\hat{I}(\pi\frac{k}{n}\frac{2l}{n}, \pi\frac{k}{n})$, $-n \leq k < n$. Hence, thanks to theorem (1), the Discrete Radon transform of equation (2) is reduced to $R = \mathcal{F}_{-1} \circ \mathcal{P}$, where $\mathcal{F}_{-1}$ denotes the 1-dimensional inverse Fourier transform performed on each column of $\mathcal{P}[I]$.

3. FEATURE-ADAPTED FAST SLANT STACK

Consider a filter h representing a 2-dimensional line-profile. Let h^θ be a rotated version of h in the direction θ:

$$h^\theta(x,y) = h(\mathbf{R}_\theta(\mathbf{x},\mathbf{y})), \tag{3}$$

where $\mathbf{R}_\theta$ is the 2-dimensional rotation matrix of angle θ. In a first step, we filter the image I with h^θ before computing equation (2). For a fixed θ, we have

$$R[I * h^\theta](t,\theta) = \sum_u \widetilde{I * h^\theta}(u, \tan(\theta)u + t). \tag{4}$$

A high coefficient means that the local feature runs significantly along the line $y = \tan(\theta)x + t$. Equation (4) can be obtained efficiently by Theorem (1). We call this transform the Feature-adapted Fast Slant Stack. In general, the computation of all these coefficients is not achievable, since it requires to convolve the image and to perform Pseudo-Polar Fourier transform as many times as the number of θ's, *i.e.* $2n$ times. For the special case where h is selected to be within the class of steerable filters [7], we can write h^θ as a linear combination of basis filters:

$$h^\theta(x,y) = \sum_{j=1}^{M} \phi_j(\theta) h^{\theta_j}(x,y), \tag{5}$$

where ϕ_j's are interpolation functions that only depend on θ and the basis filters h^{θ_j}'s are independent of θ. A convolution of an image with a steerable filter of arbitrary orientation is then equal to a finite weighted sum of convolution of the same image with the basis filters. As a result, we state the following result:

Proposition 1 *For each fixed* $\theta = \arctan(2l/n)$, $-n/2 \leq l < n/2$, *the 2n numbers*

$$R[I * h^\theta](t,\theta), \qquad -n \leq t < n,$$

are a 1-dimensional discrete Fourier transform pair with the 2n numbers

$$\sum_{j=1}^{M} \phi_j(\theta) \widehat{I * h^{\theta_j}}(\pi \frac{k}{n} \tan(\theta), \pi \frac{k}{n}), \qquad -n \leq k < n.$$

The proof is given in Appendix A. Thanks to this result, we compute equation (4) for every angle, as follows: we first convolve the image as many times as the number of basis filters composing our filter h. This number is typically very small (< 10). On each filtered image, we compute the Pseudo-Polar Fourier transform and then, for each angle $\theta = \arctan(2l/n)$, we extract the l^{th} column of each transforms and combine them thanks to Proposition (1). Finally, we perform 1-dimensional inverse Fourier transforms on each resulting series. All these steps can be performed in $O(N\log(N))$. A graphical representation of the implementation is given in Fig 2.

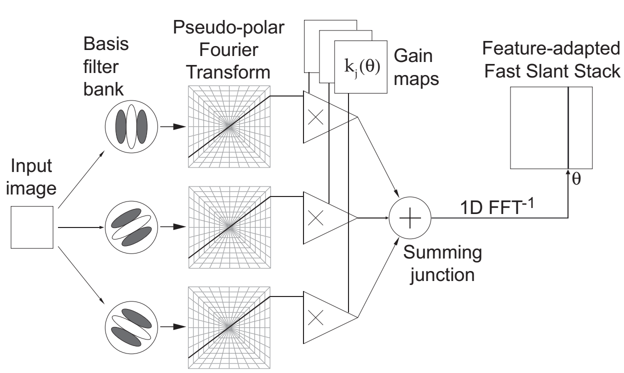

Fig. 2. Feature-adapted Fast Slant Stack diagram.

4. EXTENSION TO BEAMLET TRANSFORM

Beamlet transform [6] can be viewed as a multiscale Radon transform. It defines a set of dyadically organized line segments occupying a range of dyadic locations and scales, and spanning a full range of orientations. This system of line segments, called beamlets, have both their end-points lying on dyadic squares that are obtained by recursive partitionning of the image domain (see [6] for complete details). The collection of beamlets has a $O(N^2\log(N))$ cardinality. The underlying idea of the Beamlet transform is to compute line integrals only on this smaller set, which is an efficient substitute of the entire set of segments for it can approximate any segment by a finite chain of beamlets. Beamlet chaining technique also provides an easy way to approximate piecewise constant curves.

In [1], we used a technique akin to the one presented here, in that we embedded a profile that is represented by a steerable filter into the Beamlet transform. A two-scale recursion technique in order to compute beamlet coefficients was used, where beamlets at a given scale can be obtained by the combination of beamlets coefficients computed at smaller scales. This strategy is quite fast at the expense of a significant memory load and leads to numerical approximations. Here we propose that the Feature-adapted Beamlet transform can be computed thanks to the method presented in section 3, since the set of orientations implicitly defined by the beamlet set exactly matches the $\theta's$ defined throughout this paper. Hence, the Feature-adapted Fast Slant Stack can be applied on every dyadic square that partitions the image domain to compute the Feature-adapted Beamlet transform. Applications for curvilinear objects detection in noisy images can be found in [1]; our method presented here can be used instead.

5. CONCLUSION

In this paper, we have presented a method for computing the Feature-adapted Radon and Beamlet transforms in a fast and accurate way. These two transforms can be used for detecting features running along lines or piecewise constant curves. Our contribution unifies the Fast Slant Stack method with the steerable filtering technique. It leads to an original and efficient implementation of the Feature-adapted Radon and Beamlet transforms. This method is very general for representing curves carrying any kind of features designed by *a priori* knowledge, under the hypothesis that this feature is selected within the class of steerable filters. This work is a first step towards a more in-depth investigation of the method. We point out that precise statistical analysis of the coefficients can be easily performed due to the accuracy of the Fast Slant Stack method. This is a crucial advantage since it makes it possible to control the number of false alarms in a detection application [1].

6. ACKNOWLEDGMENTS

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APPENDIX A

In order to prove Proposition (1), a detailed presentation of the Pseudo-Polar Fourier transform is needed. The Pseudo-Polar Fourier transform proceeds in two stages: first it computes the regular Fourier transform of the image on the cartesian grid $(\frac{\pi}{n}k_1, \frac{\pi}{n}k_2)$, for $-n \leq k_1, k_2 < n$. Second, for each rows k, it applies the following operator:

$$G_{n,k} = \frac{1}{m}.\mathcal{F}_\alpha \circ \mathcal{F}_{-1},$$

where $\mathcal{F}_\alpha$ is the 1-dimensional Fractional Fourier transform [8] with $\alpha = 2k/n$. This operator takes a serie of $2n$ values and provides n values with correspond to the k^{th} row of the Pseudo-Polar Fourier transform (see [2] for complete details). Then, for a given angle θ, we first compute the 2-dimensional Fourier transform of $I * h^\theta$:

$$\widehat{I * h^\theta}(k_1, k_2) = \sum_{u,v} I * h^\theta(u, v) \exp\{-i\frac{\pi}{n}(uk_1 + vk_2)\},$$

with $-n \leq k_1, k_2 < n$. We extract the k^{th} row and apply $G_{n,k}$, which results in the following series of n numbers:

$$\mathcal{F}_\alpha[\mathcal{F}_1[I * h^\theta(i, .)](\frac{\pi k}{n})], \qquad -n \leq i < n,$$

where $(i, .)$ stands for the i^{th} column and $\mathcal{F}_1$ is the 1-dimensional Fourier transform applied on columns. If h is selected to be within the class of steerable filters, we can rewrite the previous equation using equation (5):

$$\mathcal{F}_\alpha[\mathcal{F}_1[\sum_{j=1}^{M} \phi_j(\theta) I * h^{\theta_j}(i, .)](\frac{\pi k}{n})], \qquad -n \leq i < n.$$

Due to the linearity of the operators $\mathcal{F}_1$ and $\mathcal{F}_\alpha$, it yields to

$$\sum_{j=1}^{M} \phi_j(\theta) \underbrace{\mathcal{F}_\alpha[\mathcal{F}_1[I * h^{\theta_j}(i, .)](\frac{\pi k}{n})]}_{\mathcal{P}[I * h^\theta](.,k)}, \qquad -n \leq i < n.$$

where $\mathcal{P}[I * h^{\theta_j}](., k)$ is the k^{th} row of the Pseudo-polar Fourier transform computed on $I * h^{\theta_j}$. This completes the proof.

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