3D SEISMIC DATA FUSION AND FILTERING USING A PDE-BASED APPROACH

Sorin Pop[1,2], *Romulus Terebes*[2], *Monica Borda*[2], *Sebastien Guillon*[3], *Naamen Keskes*[3], *Pierre Baylou*[1], *Olivier Lavialle*[1]

[1]LASIS, UMR 5218 IMS, 351, Cours de la Liberation, 33405 Talence, France
[2] Technical University of Cluj-Napoca, 26-28 Baritiu Street, 400027, Cluj-Napoca, Romania
[3] SISMAGE team, TOTAL, CSTJF Avenue Larribau, 64000 Pau, France

ABSTRACT

In order to aid the interpretation of seismic data, we present a new method for the denoising and fusion of multiple 3D registered blocks of the same area of subsoil. We propose to combine low-level fusion and diffusion processes through the use of a unique model based on Partial Differential Equations (PDE). The denoising process is driven by the Seismic Fault Enhancing Diffusion equation. Meanwhile, relevant information (as seismic faults) is injected in the fused blocks by an inverse diffusion process. One of the advantages of such an original approach is to improve the quality of the results in case of noisy inputs, frequently occurring in seismic unprocessed data. Finally, two examples will demonstrate the efficiency of our method on synthetic and real seismic data.

***Index Terms*—** seismic data, multi-azimuth acquisition, 3-D filtering, anisotropic diffusion, fusion

1. INTRODUCTION

The interest of multi-azimuth acquisition of 3-D reflection seismic data has been proved during the last few years. Multi-azimuth acquisition is particularly attractive in case of small Signal-to-Noise Ratio. In these cases, the multi-azimuth data must yield to improvements in fault resolution.

The combination of the information provided by the different sources (i.e. acquisition for different azimuths) can be achieved by means of classical fusion techniques. The aim is to obtain a final result including the relevant information about seismic events in general and faults in particular.

Image fusion is a process consisting in combining different sources to increase the quality of the resulting images. In case of pixel-level fusion, the value of the pixels in the fused image is determined from a set of pixels in each source image.

In order to obtain output images which contain better information, the fusion algorithms must fulfil some requirements: the algorithm must not discard the relevant information contained in the input images. Additionally, it must not create any inconsistencies in the output images.

In the last decade, a lot of works were dedicated to image-level fusion methods [1]. Among the classical methods, we can notice the well known methods based on pyramid decompositions [2,3], wavelet transform [4], or different weighted combinations [5]. These techniques have been applied in a wide variety of application fields including remote sensing, medical imagery or defect detection.

The most popular fusion methods are based on a multiscale decomposition. These approaches consist in performing a multiscale transform on each source image to obtain a composite multiscale representation. Then, by defining a selective scheme, the fused image is obtained through the use of an inverse multiscale transform.

In this paper, we propose an original low-level approach based on the use of Partial Differential Equations. The PDE formulation is inspired by the works dedicated to the non-linear diffusion filters.

The simplest diffusion process is the linear and isotropic diffusion that is equivalent to a convolution with a Gaussian kernel. The similarity between such a convolution and the heat equation was proved by Koenderink[6]:

$$\frac{\partial U}{\partial t} = div(c(x, y, t)\nabla U) \tag{1}$$

In this PDE, U represents the intensity function of the data; c is a constant which, together with the scale of observation t, governs the amount of isotropic smoothing. Setting c=1, (1) is equivalent to convolving the image with a Gaussian kernel of width $\sqrt{2t}$. Nevertheless, the application of this linear filter over an image produces undesirable results, such as edge and relevant details blurring.

To overcome these drawbacks Perona and Malik [7] proposed the first non-linear filter by replacing the constant c with a decreasing function of gradient ∇U . Practical implementations of the P-M filter give impressive results, noise is eliminated and edges are kept or even enhanced provided that their gradient value is greater than a threshold.

Shock filters constitute another successful class of PDE-based filters. In order to sharpen an image, these filters, initially proposed by Osher and Rudin[8] employ an inverse diffusion equation. The well-known stability problem of the inverse heat equation is solved for the discrete domain by the mean of *minmod* function [8].

Other important theoretical and practical contributions were brought by Weickert [9-10]. The proposed EED (Edge Enhancing Diffusion) and CED (Coherence Enhancing Diffusion) models are anisotropic diffusion methods or often called tensor based diffusion.

The general equation is written in PDE form, as:

$$\frac{\partial U}{\partial t} = div(D\nabla U) \tag{2}$$

with D some square 2*2 matrix for 2-D images and two additional boundary and initial conditions.

The purpose of a tensor based approach is to steer the smoothing process according to the directional information contained in the image structure. In the CED, the diffusion matrix D is created based on the tensor structure. This tensor is a powerful tool for analyzing coherence structures:

$$J_\rho(\nabla U_\sigma) = K_\rho * (\nabla U_\sigma \otimes \nabla U_\sigma) \tag{3}$$

Each component of the resulted matrix of the tensor product ($\otimes$), is convolved with a Gaussian kernel (K_ρ). The eigenvectors of J_ρ, represent the average orientation of the gradient vectors and the structure orientation, at scale ρ. The diffusion matrix D in (2) has the same eigenvectors as J_ρ, but its eigenvalues are chosen according to a coherence measure. The diffusion process acts mainly along the structure direction and becomes stronger when the coherence increases.

Recently, specific PDE-based approaches were devoted to the seismic 3D filtering [11-14].

In the next section, we will introduce a PDE formulation for fusion-diffusion process. In section 3, we will show some results obtained by our fusion-diffusion approach both on noisy-blurred synthetic blocks and real seismic data. Conclusions and perspectives are given in section 4.

2. PDE-BASED FUSION-DIFFUSION PROCESS

In low-level fusion, we consider that each source data provide a part of the relevant information we want to obtain in the output.

We propose to apply a PDE-based evolution process for each seismic source. At each step of the process, we are interested in keeping the relevant information contained in the current source and in adding the information from the others sources.

To achieve this task, we propose a PDE process involving both a direct seismic oriented diffusion and an inverse diffusion process. The general continuous evolution equation of a source data can be formalised as:

$$\frac{\partial U_i}{\partial t} = div(D_i \nabla U_i) - \beta_i div\left(g\left(|\nabla U|_{\max}\right)\nabla U_{\max}\right) \tag{4}$$

where i represents the current source, *max* denotes the source corresponding to the maximum of gradient absolute value and β_i is a positive weight parameter:

$$\beta_i = \begin{cases} 0 & if\ i = \max \\ \beta \in [0;1] & otherwise \end{cases} \tag{5}$$

The weight parameter (β) sets the importance of fusion term with respect to diffusion term.

Even if equation (4) describes the evolution of a single block (i), the principle of our approach is to perform the process on each of the input blocks. The blocks are updated in parallel at each time step. In the next sub-sections, we will describe precisely the two terms of the equation.

2.1 Diffusion term

As diffusion process, we adopt a dedicated seismic diffusion: Seismic Fault Preserving Diffusion (SFPD) [13] [14], but other models can be considered as well. SFPD is a 3D extended model based on Weikert's 2D CED diffusion.

The diffusion 3*3 matrix D_i, specific at each source, has the same eigenvectors as the structure tensor J_ρ (equation 3), but its eigenvalues are chosen according to a seismic confidence measure C_{fault}, introduced by Bakker [15]. The objective of C_{fault} is to discriminate between fault neighborhoods and non-broken horizons:

$$C_{fault} = \frac{\mu_2 - \mu_3}{\mu_2 + \mu_3}\left(1 - \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right) \tag{6}$$

Wwhere $\mu_1 > \mu_2 > \mu_3$ are the eigenvalues of J_ρ. In order to denoise and preserve the seismic faults, we employ this measure in the system of choosing the matrix D eigenvalues:

$$\begin{aligned} &\lambda_1 = \alpha \\ &\lambda_2 = \lambda_3 - (\lambda_3 - \lambda_1) h_\tau(C_{fault}) \\ &\lambda_3 = \begin{cases} \alpha & if\ k = 0 \\ \alpha + (1-\alpha)\exp\left(\frac{-C}{k}\right) & else \end{cases} \end{aligned} \tag{7}$$

where α, C are constants and k is the coherence measure proposed by Weickert. $h_\tau()$ is an increasing sigmoid function described in [16] taking values in [0 ; 1] which allows to parameterize the influence of the confidence measure.The originality of our system lies in the second eigenvalue. λ_2 takes values between λ_1 and λ_3 and depends continuously on the measure C_{fault}.

This system allows to diffuse only in one orientation in a fault neighborhood and to perform a diffusion process guided by two orientations along the layers otherwise. Thus we are able to denoise the seismic blocks while the faults are preserving. Also, this approach exempts from the creation of false anisotropic structures, artifacts typically observable in images processed with the classical tensorial models.

2.2 Fusion term

The second term of equation 4 is an inverse diffusion term. The aim of such a term is to inject in the current block the seismic relevant information from the other sources. In our seismic case, we consider that the relevant information is provided by the block corresponding to the maximum absolute value of the gradient.

Looking for the maximum of the absolute gradient value leads to detecting the strong discontinuities, like faults. We search the maximum of gradient for each voxel. When the maximum gradient occurs in the current block, the equation is reduced to the diffusion term. Otherwise, if the maximum is detected in another source block, we inject the difference observed by inversing the diffusion process.

The quantity of the fusion is modulated by a function g of absolute gradient value. In this paper we adopt the constant positive function ($g()$=1), which will provide an isotropic behavior for the fusion process.

Thus, the fusion process is a linear inverse diffusion process, which is similar to a Gaussian de-convolution. The major drawbacks of this type of process are the instability, noise amplification and oscillations [17]. We avoid these undesirable effects by imposing the limits of gray level variation for each voxel (gray level constraint). The limits are fixed considering maximum and minimum values through all sources. In order to reduce the impact of noise present in the original blocks, these boundaries are evaluated at each time step. Using this first constraint, no new local extrema are created.

The aim of the fusion term is to inject high frequency signals like edges or faults. When the difference concerns low frequency information, the process tends to create new discontinuities not present in the input data. To avoid this problem, we impose a second constraint by forcing the difference between two neighboring voxels to be limited by the maximum of the difference observed in the input blocks (neighborhood constraint). Using this second constraint, no new local edge is created.

Contrary to the classical fusion methods, our algorithm provides with one output for each source block. Obviously, the aim is to obtain similar outputs while the relevant information is preserved. In practice, we can observe a convergence of the process: the distance (i.e. RMSE) between the fused blocks decreases in time. The stopping time, like in pure diffusion case, is chosen by the operator; nevertheless a criterion based on a distance measure or a quality factor calculation can be proposed.

As numerical scheme, we adopt an explicit time scheme and the forward and backward approximations for spatial derivatives. The maximum gradient absolute value is evaluated for the nearest neighborhood (6 voxels).

3. RESULTS

This section illustrates the efficiency of our approach on both synthetic and real seismic blocks.

Since it is much easier to judge the efficiency of the fusion process on a synthetic image, we propose to use 3-D synthetic blocks composed by a stack of layers with a sinusoidal profile. One of them is clearly broken by a fault. In the second the fault was smoothed using a frequency filtering. Both of them are corrupted with additive Gaussian white noise (σ=25). Figure 1(a,b) shows a front section of the noisy blocks.

The fusion-diffusion process applied to the noisy blocks leads respectively to the blocks shown in Figure 1 (c,d). The results are obtaining after 30 iterations with a time step dt=0.1, a weight parameter β=0.5 and only with the gray level constraint. In addition, parameters specific to SFPD diffusion are set to α=10^{-6}, σ=0.4, ρ=0.8 and τ=0.05 as threshold for sigmoid function.

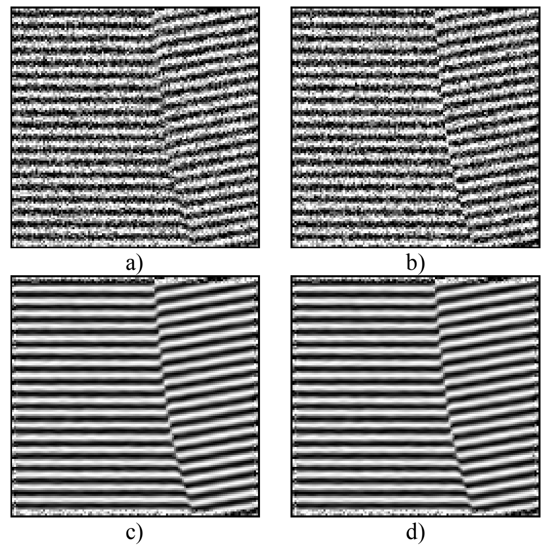

Fig. 1 a,b) The noisy synthetic blocks; c,d) fused-diffused blocks

The efficiency of the diffusion process is clearly illustrated: the two blocks are filtered and the fault is preserved in the block at right. The fusion process allowed to inject the fault in the block at left. The effectiveness of the fusion is proved through the computation of the RMSE (Root Mean Squared Error) between the blocks: the RMSE between the input blocks is equal to 36.11and it is equal to 0.056 between the output blocks. Thus, the output blocks are quite similar.

Figure 2 shows results generated from real seismic data. These results illustrate that our approach is adapted to remove the noise while preserving and injecting the faults.

4. CONCLUSIONS AND PERSPECTIVES

In this paper we propose a new method for fusion and filtering seismic 3D data. The method is based on a single PDE equation including both fusion and diffusion terms. The advantage of such an approach is that it can deal with noisy data. We can consider our approach as a very general framework.

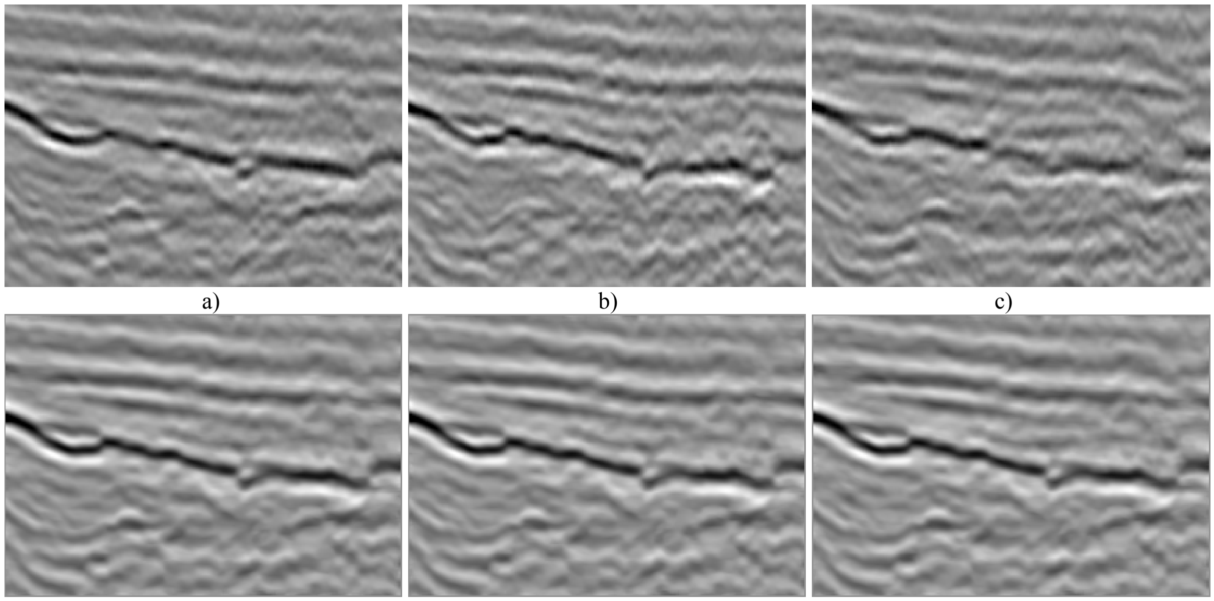

Fig. 2 a,b,c) Real seismic blocks (front section) ; d,e,f) Fused-Diffused blocks corresponding to real blocks

The diffusion and the fusion terms can be adapted or improved to deal with different types of fusion problems for 2D and 3D applications.

In the further works we will focus on finding an optimal stop criterion for our method and on integrating the constraints described in sub-section 2.2 in one powerful anisotropic function (g).

5. REFERENCES

[1] R.S. Blum, Z. Xue and Z. Zhang, "An overview of image fusion", *in: R.S. Blum, Z. Liu (Eds), Multi-Sensor Image Fusion and Its Applications, Signal and Image Processing Series*, Marcel Dekker/CRC Press, 2005.

[2] P.J. Burt and R.J. Kolczynski, "Enhanced image capture through fusion", *In Fourth International Conference on Computer Vision*, Berlin, Germany, pp. 173-182, 1993.

[3] G. Piella, "A general framework for multiresolution image fusion: from pixels to regions", *Information Fusion,* Vol. 9, pp. 259-280, 2003.

[4] O. Rockinger, "Image Sequence Fusion Using a Shift-Invariant Wavelet Transform", *International Conference on Image Processing ICIP 1997*, Washington DC, Vol. III, pp. 288-292, 1997.

[5] O. Rockinger and T. Fechner, "Pixel-level image fusion: the case of image sequences" *Proc. SPIE*, vol. 3374, pp. 378-388, 1998.

[6] J. Koenderink, "The structure of images", *Biological Cybernetics*, Vol.50, pp. 363-370, 1984.

[7] P. Perona and J. Malik, "Scale space and edge detection using anisotropic diffusion", *IEEE Transactions PAMI*, vol.12, no.7, pp. 629-639, 1990.

[8] S. Osher and L. Rudin, "Feature-oriented image enhancement with shock filters", *SIAM Journal on Numerical Analysis*, vol.27, no.3, pp. 919-940, 1990.

[9] J. Weickert, "Multiscale texture enhancement", *In: Hlavac V., Sara R.(Eds.) Computer analysis of images and patterns*, Springer, Berlin, pp.230-237, 1995.

[10] J. Weickert, "Coherence enhancing diffusion filtering", *International Journal of Computer Vision*, no.31, pp. 111-127, 1999.

[11] R. Dargent, O. Lavialle, S. Guillon and P. Baylou, "Sector-based Diffusion Filter", *Proceedings of the IEEE International Conference on Pattern Recognition*, Cambridge,UK, pp. 679-682, 2004.

[12] S. Pop, R. Terebes, M. Borda, O.Lavialle, I. Voicu and P. Baylou, "3D Directional Diffusion", *The International Conference on "Computer as a tool"- EUROCON 2005*, Belgrade, Serbia & Montenegro, 2005.

[13] O. Lavialle, S. Pop, C. Germain, M. Donias, S. Guillon, N. Keskes and Y. Berthoumieu, "Seismic Fault Preserving Diffusion", *Journal of Applied Geophysics*, Volume 61, Issue 2, pp. 132-141, February 2007.

[14] O. Lavialle, S. Pop, R. Dargent, S.Guillon, N. Keskes "A tensor based diffusion process to enhance faults in seismic blocks*", The second IEEE-EURASIP International Symposium on Control, Communications, and Signal Processing*, Special session: Geophysical Signal Processing, Marrakech, Morocco, 13-15 Mars 2006.

[15] P. Bakker, P.W. Verbeek and L.J. Van Vliet, "Confidence and curvature estimation of curvilinear structures in 3-D" *Proceedings of the Eighth International Conference On Computer Vision*, Vancuver, vol.II, pp. 139-144, 2001.

[16] R. Terebes, M. Borda, O. Lavialle and P. Baylou, "Flow Coherence Diffusion, Linear and Nonlinear case", *Lectures Notes in Computer Science*, vol. 3708, pp.316-323, 2005.

[17] G. Gilboa, N. Sochen and Y. Zeevi, "Forward-and-Backward Diffusion Processes for Adaptive Image Enhancement and Denoising", *IEEE Trans. Image Processing*, Vol. 11, No. 7, pp. 689-703, July 2002.