PROCESSING FINE DIGITAL TERRAIN MODELS BY MARKOVIAN REGULARIZATION FROM 3D AIRBORNE LIDAR DATA

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ABSTRACT

Airborne laser scanner systems are based on the emitting / recepting cycle of a laser beam mounted on the nadir of an airplane. They provide 3D point clouds of the topography within an altimetric accuracy of generally less than 10 cm. The divergence of the emitted laser beam allows the recording of at least two distance measurements for each transmitted pulse. It is particularly interesting when surveying forest areas whereon both the top canopy and the ground are recorded at once. The aim of the paper is to describe a methodology for modelling the terrain from sparse laser ground points with a dense altimetric surface, without using classical interpolation algorithms. Our approach is based on the definition of an energy function that manages the evolution of a terrain surface in a Bayesian framework. The energy is designed as a compromise between a data attraction term and a regularization term. The minimum of this energy corresponds to the final terrain surface. We show some conclusive results of the retrieving of a realistic terrain.

***Index Terms*—** Airborne lidar, digital terrain model, markovian regularization, optimization

1. INTRODUCTION

Accuracy in mapping projects have never been as important as today, especially when considering risk management (*i.e.* floods) and surveying. Traditionally, photogrammetric techniques provide Digital Surface Models (DSMs) calculated from couple of aerial/satellite images in a stereoscopic context [1]. Digital Terrain Models (DTMs) can be derived from these photogrammetric DSMs [2]. In case of a natural rural terrain, vegetation often hides the true ground which is not always visible on images. As a result, generated DTMs lack of accuracy and applications using DTMs (*i.e.* flood/tide predictions) provide false conclusions.

Airborne laser systems may sort out these problems under certain conditions. These systems are based on the recording of the time-of-flight distance between an emitted laser pulse and its response after a reflection on the ground (figure 1). They provide sets of tridimensional irregularly distributed points, georeferenced with an integrated GPS/INS system within an altimetric precision less than $10\,cm$[3]. Moreover, these systems can provide multiple returns for a single laser pulse which corresponds to different measured altitudes. It is particularly relevant when surveying vegetated areas where both the altitude of the top of the canopy and the ground can be recorded at once with accuracy. Nevertheless, dense vegetation inhibits laser ground return signal. The terrain is then represented as sparse 3D points as one can notice it on figure 2.

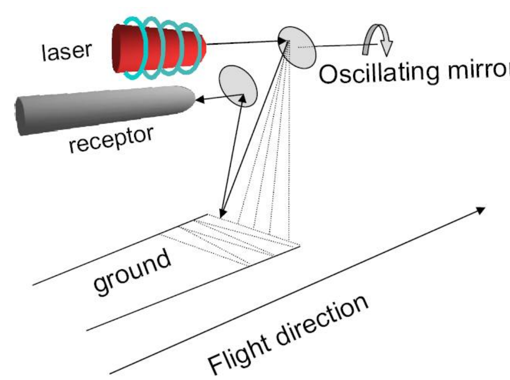


Fig. 1. Principles of Airborne laserscanning systems

We will focus this paper on the description of the terrain from lidar data which have been previously filtered into ground and off-ground points [4]. We used to representing the terrain as an image,a DTM, where the intensity of each pixel is proportional to the height. The problem becomes: how can we get a dense image description of the terrain from a sparse 3D point cloud. Even though the generation of altimetric data can always be considered as interpolation (resampling) processes, the goal of this study is to provide a realistic topographic terrain model without using traditional interpolation algorithms (linear interpolation, kriging, inverse distance weighting ...). These algorithms may generate artefacts or may generalize the terrain relief. Such drawbacks can be avoided considering a deformable surface that takes into account recorded mi-

Fig. 2. Projection of classificated ground lidar points (brown points) onto an otho-rectified image.

crorelieves. From a coarse initial DTM, a very fine DTM will be calculated.The process is ruled by the minimization of an energy. The energy is designed as a compromise between a data attraction term and a regularization term. The minimum of this energy corresponds to the final terrain surface. We therefore propose to use a Bayesian approach which provides the means to incorporate prior knowledge in data analysis.

Having briefly presented the theoretical background of the study, we will describe the algorithm, especially the definition of an energy associated to the Bayesian model. The second part is dedicated to the presentation of some results.

2. METHODOLOGY

2.1. Background

In a probabilistic framework, an image is a set S of sites s where the grey level is a descriptor of each site. An image is considered as a realisation of a random field $X = (X_s)_{s\in\mathbb{N}^2}$, where X_s is a random variable of values in $E^{card(S)}$ [5] . A random field is therefore a measurable mapping $X : \Omega \longrightarrow E^{card(S)}$ associated to a complete measurable space $(\Omega, \mathcal{F}, \mathbf{P})$. This model is described by the probability law $\mathbf{P}(X = x)$ the event x to be a realisation of X. As usual in image processing, we will consider the particular Markov Random Fields (MRF). In a MRF, the value of a site only depends of its local environment through a neighbouring system $\mathbb{V}$ defined as [6]:

$$\begin{cases} s \notin \mathbb{V}(s) \\ \forall r \in S\backslash\{s\}, s \in \mathbb{V}(r) \Leftrightarrow r \in \mathbb{V}(s). \end{cases} \tag{1}$$

In case of a MRF, we have

$$\begin{aligned}\forall x \in \Omega, \forall s \in S, &\mathbf{P}(X_s = x_s | X_r = x_r, r \in S\backslash\{s\}) \\ =&\mathbf{P}(X_s = x_s | X_r = x_r, r \in \mathbb{V}_s)\end{aligned} \tag{2}$$

The Hammersley-Clifford theorem argues the equivalence between a MRF and a Gibbs field. The *a priori* probability of a random variable X can therefore be explicitly written as:

$$\mathbf{P}(X = x) = \frac{1}{Z} e^{-\mathcal{U}(x)} \tag{3}$$

where $Z = \sum_{x\in\Omega} e^{-\mathcal{U}(x)}$ is a normalization constant and $\mathcal{U}$ an energy function (which has the properties to be decomposed into local energies) defined as:

$$\begin{aligned} \mathcal{U} : \Omega &\longrightarrow \mathbb{R} \\ x &\longrightarrow \mathcal{U}(x) = \sum_{c\in\mathcal{C}} \mathcal{U}_c(x) \end{aligned} \tag{4}$$

We propose a Bayesian model for regularizing the terrain surface in order to introduce *a priori* knowledge on the model. The Bayes's law, which relates *a priori* and conditional probability is defined as

$$\mathbf{P}(X|D) = \frac{\mathbf{P}(D|X)\mathbf{P}(X)}{\mathbf{P}(D)} \propto \mathbf{P}(D|X)\mathbf{P}(X) \tag{5}$$

The Bayesian model is related to the inverse problem of how retrieving the best configuration $\hat{x}$ knowing observations D. We look for the maximum a posteriori (MAP) defined as:

$$\hat{x}_{MAP} = \arg\max_{x\in\Omega} \mathbf{P}(X = x|D) \tag{6}$$

that can be written as

$$\hat{x}_{MAP} = \arg\min_{x\in\Omega} \Big(-log\big(\mathbf{P}(D|X = x)\big) - log\big(\mathbf{P}(X = x)\big)\Big) \tag{7}$$

Under the markovian hypothesis, solving equation 7 is equivalent to globally minimize an energy $\mathcal{E}$, sum of a data term $\mathcal{E}_d$ and of a regularization term $\mathcal{E}_r$ $\mu \in \mathbb{R}$

$$\hat{x}_{MAP} = \arg\min_{x\in\Omega} \underbrace{(\mathcal{E}_d + \mu\mathcal{E}_r)}_{\mathcal{E}} \tag{8}$$

2.2. Definition of the energy

2.2.1. The data term

The data term depends on the distance between the terrain surface and the data $D = \{d_s / s \in S'\}$ ($S' \subset S$). D is precalculated by projecting "terrain" lidar points onto a regular grid of the same resolution as the final DTM. This distance has to be minimal so that the final DTM should be as near as possible from laser measurements of the ground. We therefore define $\mathcal{E}_d$ as:

$$\mathcal{E}_d(D = d_s | X = x_s) = \begin{cases} (d_s - x_s)^2 & \text{if } s \in S', \\ 0 & \text{if not.} \end{cases} \tag{9}$$

2.2.2. The regularization term

The regularization term aims to compensate the effect of the data term so that the final surface should not be too noisy. This term depends on the intrinsic geometry of the surface [7].
Let h be the surface defined as:

$$\begin{aligned} h : \mathbb{R}^2 &\longrightarrow \mathbb{R}^3 \\ (x, y) &\longrightarrow (x, y, z = h(x, y)) \end{aligned}$$

A second order Taylor developpement at point $\mathbf{u_0} = (x_0, y_0)$ with $\mathbf{u} = [x\ y]^T$ can be written

$$\begin{aligned} h(\mathbf{u}) = &\underbrace{h(\mathbf{u_0}) + (\mathbf{u} - \mathbf{u_0}).\nabla_{\mathbf{u_0}} h}_{\Pi_0(\mathbf{u})} \\ &+ \frac{1}{2}(\mathbf{u} - \mathbf{u_0})\mathbf{H}(\mathbf{u} - \mathbf{u_0})^T + o(||(\mathbf{u} - \mathbf{u_0})||^2) \end{aligned} \tag{10}$$

where Π_0 is the tangent plane to h in $\mathbf{u_0}$ and $\mathbf{H} = \begin{pmatrix} \frac{\partial^2 h}{\partial x^2} & \frac{\partial^2 h}{\partial x \partial y} \\ \frac{\partial^2 h}{\partial x \partial y} & \frac{\partial^2 h}{\partial y^2} \end{pmatrix}$ the Hessian matrix of h. This matrix describes the local properties of the surface curvature.
We define the regularization term as a function of the trace and the determinant of the Hessian matrix. The trace describes the local convexity of the surface while the determinant is linked to the shape of the surface with regard to its tangent plane (parabolic, elliptic, hyperbolic). We therefore define $\mathcal{E}_r$ as:

$$\begin{aligned} \mathcal{E}_r &= \alpha_1 tr(H)^2 - \alpha_2 \det(\mathbf{H}) \quad \alpha_1, \alpha_2 \in \mathbb{R}^+ & (11) \\ &= \frac{\alpha_2}{2}\left(\left(\frac{\partial^2 h}{\partial x^2}\right)^2 + \left(\frac{\partial^2 h}{\partial y^2}\right)^2\right) + \alpha_2\left(\frac{\partial^2 h}{\partial x \partial y}\right)^2 \\ &\quad + (\alpha_1 - \frac{\alpha_2}{2})\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right)^2 & (12) \end{aligned}$$

This energy is designed so that its convexity should be managed for optimization purposes. Indeed, in case of estimating a fine terrain surface, there are not any forbidden natural shapes. A repulsive term is therefore not appropriate. If the constrains

$$\alpha_2 \geq 0 \text{ and } \alpha_1 \geq \frac{\alpha_2}{2} \tag{13}$$

are applied, the energy becomes convex. Derivatives are calculated using the finite difference approximation. A steepest gradient algorithm has been used to solve the optimization problem.

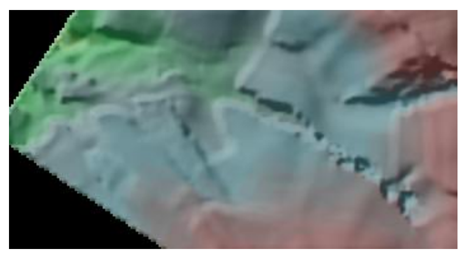
3. RESULTS

3.1. Practical remarks

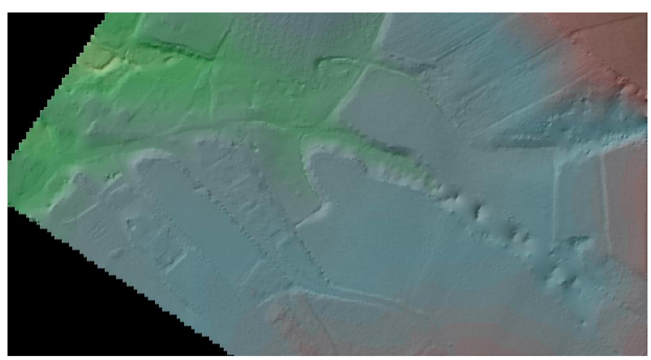
The lidar point cloud used in this study has a spatial density of $\sim 25\,pt/m^2$. Last lidar pulses have been processed since terrain points were only of interest.
An initial coarse resolution DTM ($3\,m$) is first calculated from the classification algorithm [4]. The algorithm presented here works at any resolution and provides fine resolution terrain surfaces. As a result, the coarse DTM is resampled with a PPV interpolator. The final resolution depends both on the mean point density and on the spatial distribution of ground points. A compromise has to be found between the regularity of the terrain and the description of microrelieves. Here, we set the final resolution to $0.5\,m$. Finally, the resampled DTM is smoothed with a Gaussian filter to produce a differentiable surface.
Depending on the surveying configurations, real terrain does not always fit a dense square image representation. Empty pixels (black pixels in figure 3(b)) have no altimetric information. We therefore produce a mask of eligible sites (DTM points) whereon the regularization is applied.

3.2. Discussion

The algorithm we have developed provides successful results with regard to the aim of the study. As one can notice on figure 3(a), the coarse DTM represents low frequencies of the terrain. After applying the regularization algorithm onto a resampled surface, micro-relieves appear in the description of the terrain (figure 3(b)).



(a) Initial $3\,m$ resolution DTM.



(b) Final $0.5\,m$ resolution DTM.

Fig. 3. DTM before and after the regularization step. Images are coded from green to brown with increasing altitudes. The area is approximated $540\,m \times 300\,m$.

This approach clearly takes benefit of the potential of lidar scanning data for deriving very fine DTMs. The Bayesian formulation is flexible enough to integrate external constraints. We think of integrating break lines that are often underestimated. Indeed, the defined energy has been constructed based on geometrical considerations and has been tested on different landscapes with success. Other formulations may be derived based on a morphological knowledge of the real terrain. We have noticed that, under the constrained equation 13, parameters α_1 and α_2 have a very low influence on the final shape.

As to the optimization step, a steepest gradient method appears sufficient since the energy has been designed to be convex. The convergence is fast considering the set of eligible sites whereon the regularization step is fully defined.

We show on figure 4 some profiles of the surface terrain evolution. The initial coarse DTM (light grey curve) generalizes the terrain. Nevertheless, the description of the terrain becomes progressively detailed until fitting laser points (black dots). The regularization is performed in 2D that can explain the final shape (bump) around at $22.5\,m$.

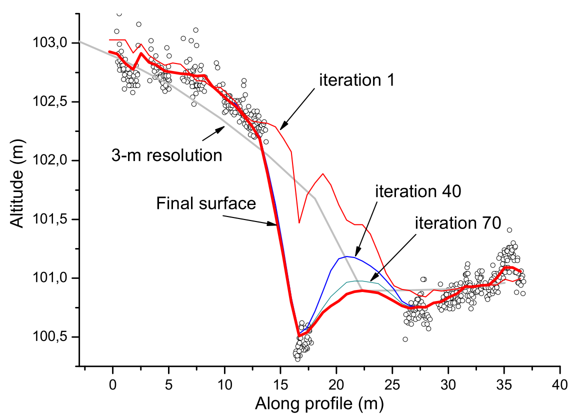


Fig. 4. Different profiles of a DTM at different iteration steps of the regularization algorithm. Laser ground points are represented as black dots.

In this study, the final DTM has not been validated by external measurements. It is admitted that laser scanning data are globally accurate within $0.10\,m$ in altimetry and $< 0.40\,m$ in planimetry [8]. Nevertheless, it appears, in some local areas, that underground points which are not natural microrelieves, remain. These outliers come from erroneous time measurements of the lidar system. The surface is then attracted by these points. We planed to filter these points out. The future work consists in validating the DTM with some field measurments and to compare our approach to other interpolators.

4. CONCLUSION AND FUTURE WORK

We have presented in this article a methodology for deriving accurate DTMs from 3D topographic lidar data. The originality of this study lies in the ability of calculating a dense and realistic topographic surface without using classical interpolation approaches. Based on a regularization process that involves terrain points in a Bayesian framework, we showed that we are able to retrieve accurate micro-relieves recorded by the lidar system.

5. REFERENCES

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