PI-LINE BASED FAN-BEAM LAMBDA IMAGING WITHOUT SINGULARITIES

Lingjian Chen[1], *Jianhua Ma*[2], *Wufan Chen*[*]

School of Biomedical Engineering of the Southern Medical University, Guangzhou, China

ABSTRACT

Since the ionizing radiation may induce cancers and genetic damages in the patient, it is highly desirable to minimize the X-ray dose during a CT scan. As one of the local imaging techniques, the Lambda imaging reduces the X-ray dose and imaging time. But the existence of the singular values results in the low quality of the image. The broad applications of the PI-lines proof that it can deal with the truncated projections effectively. In this work, we propose a new exact Lambda imaging algorithm based on Wang G's local imaging method and PI-lines segment to reconstruct an image with utilizing a Gaussian kernel function convoluting the projection data. We also analyze how to choose the parameters of the Gaussian kernel function. Numerical simulations support our new reconstruction algorithm with high quality reconstruction image.

***Index Terms*—**fan-beam; lambda tomography; PI-line; truncations; singularities

1. INTRODUCTION

A fan-beam scanning configuration is widely used in current clinical computed tomography (CT) systems for data acquisition. Even though future generation of CT is expected to adopt a helical cone-beam configuration for rapid volumetric coverage and for efficient usage of the X-ray source, it can also be used in a fan-beam mode, which may still find useful applications in many imaging protocols. Recently, the proposals of super-short-scan reconstruction [1] and reconstruction in regions-of-interest (ROI) from truncated projections [2, 8] were the significant breakthroughs in fan-beam scan imaging, whose main ideas were similar to the 3D spiral CT [3, 9]. In 1982, Parker [11] proposed conception of the short scan in fan-beam CT based on circle locus. The range of the scanned angles is $\pi + 2\gamma_m$, $2\gamma_m$ is the open angle. In 2002, Kudo-Noo [10] proposed the first fan-beam CT super-short-scan reconstruction algorithm which possessed a smaller scanned angular range than that in the short scan. Also in the year, Noo-Defrise [1] investigated the relationship between the fan-beam scanned angular range and the exact reconstruction of the ROI by using the fan-beam rebinnig data transformed from the parallel-beam projection data and proposed the exact fan-beam super-short-scan reconstruction algorithm. Zou-Pan [8] developed a new algorithm for exact image reconstruction within ROI from reduced fan-beam scan data containing truncations. All the existing algorithms didn't take the affection of the X-ray to the human body into account since the ionizing radiation may induce cancers and damages in the patient, it is highly desirable to minimize the X-ray dose during a CT scan. For that purpose, region-of-interest (ROI) based tomography has been extensively proposed and studied, which reconstructs a local image from truncated projection data but suffers from image cupping and intensity shifting artifacts whose results different to the original data.

Traditional global CT scanned the whole slice for collecting the projection data even we just reconstructed a small local area within the slice. Local reconstruction, which was investigated a lot in recent years, just scanned a region slightly larger than the region-of-interest for projection data and reconstructed the ROI. It is clearly that the local reconstruction algorithm reduced the data collection time and image reconstruction time, also reduced the X-ray dose during a CT scan. Over the past decade a number of methods to localize the reconstruction have been proposed. These range from methods for region-of-interest tomography which use integrals over all lines passing through a region slightly larger than the ROI, to strictly local methods where reconstruction at a point x only requires integrals over lines very close to x. The wavelet-based multiresolution local tomography [4] belongs to the former. The Lambda tomography (LT) including the fan-beam Lambda tomography [5] and the cone-beam Lambda tomography [6], which is strictly local, belongs to the latter.

In this work, we derive a new algorithm for image reconstruction from reduced-scan fan-beam data based on the PI-line segments. Unlike the existing algorithms, the proposed method reconstructs the related function $Lf = \Lambda f + \mu \Lambda^{-1} f$ instead of reconstructing the density function f , where $\Lambda = \sqrt{-\Delta}$, and Δ denotes the Laplacian operator. Combining the PI-line-based algorithm proposed in [8] and the fan-beam-scan-based Lambda tomography, we exactly reconstruct the truncated projections within an ROI. It must be pointed out that the new idea solves the main two problems including the singularities and the truncations.

2. BACKGROUND

In a helical scan (3D), each point within the helix cylinder belongs to one and only one PI-line segment. However, in a fan-beam scan, each point within the circular trajectory can belong to an infinite number of PI-line segments. This is because, as [8] indicates, for two given parameters x and y, one needs to solve for three parameters λ_1, λ_2 and t. In this article, we decide to use the sets of PI-line segments that do not intersect with each other and can completely fill the object or an ROI within the object.

2.1. Lambda tomography (LT)

Traditionally, here not the function f itself but the related function $Lf = \Lambda f + \mu \Lambda^{-1} f$ is reconstructed. Computing $\Lambda^{-1} f(x)$ consists to taking the average of all integrals over lines passing through x. However, since Λ^{-1}, the inverse of Λ, is given by convolution with the Riesz kernel R_1,

$$\Lambda^{-1} f = R_1 * f,\ R_1(x) = (\pi \left| S^{n-2} \right|)^{-1} \left| x \right|^{1-n}, \qquad (1)$$

the result is a very blurry image of f which is beneficial to reduce the noises. S^{n-2} denotes the n-dimensional unit sphere.

In this paper, we are motivated to design theoretically exact, computationally efficient and practically flexible LT algorithm to reconstruct $\Lambda f(x)$ from fan-beam or cone-beam data.

2.2. Analysis of the classical algorithm

Let S represent the unit circle in $\mathbb{R}^2$. Assume that $\Gamma \subset \mathbb{R}^2$ is a differentiable curve parameterized by $\vec{r}(\lambda)$, $\lambda \in \mathbb{R}$, f an bounded function with a compact support $\Omega \subset \mathbb{R}^2 \setminus \Gamma$, and a fan-beam projection of f along a scanning trajectory Γ

$$D_f(\vec{r},\vec{\theta}) = \int_0^\infty ds f(\vec{r} + s\vec{\theta}),\ (\vec{r},\vec{\theta}) \in \Gamma \times S. \qquad (2)$$

A chord L is defined as a line-segment with two endpoints $\vec{r}(\lambda_1)$ and $\vec{r}(\lambda_2)$ on trajectory Γ, and the unit vector along L

$$\vec{e}(\lambda_1,\lambda_2) = \frac{\vec{r}(\lambda_2) - \vec{r}(\lambda_1)}{\left\| \vec{r}(\lambda_2) - \vec{r}(\lambda_1) \right\|}. \qquad (3)$$

For any point $\mathrm{x} \in L$, the unit vector $\vec{\theta}(\mathrm{x},\lambda)$ defined as

$$\vec{\theta}(\mathrm{x},\lambda) = \frac{\mathrm{x} - \vec{r}(\lambda)}{\left\| \mathrm{x} - \vec{r}(\lambda) \right\|}. \qquad (4)$$

Let $(\bullet)$ represent the inner product, and $\vec{\theta}^{\perp}(\mathrm{x},\lambda)$ be a vector perpendicular to $\vec{\theta}(\mathrm{x},\lambda)$. Clearly, $\vec{\theta}^{\perp}(\mathrm{x},\lambda)$ is uniquely determined by $\vec{\theta}(\mathrm{x},\lambda)$ in the 2D space up to a direction flip. For every x inside an ROI, reconstruct $\Lambda f(\mathrm{x})$ as follows:

$$\Lambda f = -\frac{1}{2\pi} PV \int_{\lambda_1}^{\lambda_2} d\lambda \frac{\mathrm{sgn}(\vec{e} \bullet \vec{\theta}^{\perp})}{\left\| \mathrm{x} - \vec{r}(\lambda) \right\| \cdot (\vec{r}\,'(\lambda) \bullet \vec{\theta}^{\perp})} \times \left(\frac{\partial^2}{\partial q^2} (D_f(\vec{r}(q),\vec{\theta}))\big|_{q=\lambda} - \frac{(\vec{r}\,''(\lambda) \bullet \vec{\theta}^{\perp})}{(\vec{r}\,'(\lambda) \bullet \vec{\theta}^{\perp})} \frac{\partial}{\partial q} (D_f(\vec{r}(q),\vec{\theta}))\big|_{q=\lambda} \right), \qquad (5)$$

where $\mathrm{x} \in L$ and $\mathrm{x} \notin \Gamma$, Γ is a differentiable general curve, $\vec{e} = \vec{e}(\lambda_1,\lambda_2)$, $\vec{\theta} = \vec{\theta}(\mathrm{x},\lambda)$, $\vec{\theta}^{\perp} = \vec{\theta}^{\perp}(\mathrm{x},\lambda)$, $\vec{r}\,'(\lambda) = \frac{d\vec{r}(\lambda)}{d\lambda}$, $\vec{r}\,''(\lambda) = \frac{d^2\vec{r}(\lambda)}{d\lambda^2}$ and "PV" represents the principle value integral. Note that $\vec{\theta}^{\perp}(\mathrm{x},\lambda) = (-\theta_2,\theta_1)$ or $(\theta_2,-\theta_1)$ if $\vec{\theta}(\mathrm{x},\lambda) = (\theta_1,\theta_2)$.

3. THE NEW PI-LINE BASED LT RECONSTRUCTION WITHOUT SINGULARITIES

In the classical LT algorithm, the scanned range of one single point was determined by the chord passing through it and the origin of the Cartesian coordinate. That is means, suppose the scanning trajectory is a circle, we need semicircular projection data to reconstruct one single point. Otherwise, most of the CT reconstruction methods based on ROI contained truncations. Considering all the factors mentioned above, we decide to use the PI-line segment to solve problems since it can shorten the reconstruction time, reduce the X-ray dose and reconstruct the truncations efficiently. According to the conditions of the data acquisition proposed by Noo-Defrise [1], complete (non-truncated) fan-beam projections provide sufficient information for exact reconstruction when every line passing through the ROI intersects the vertex path in a non-tangential way. Unfortunately, their method reconstructs only one single point within every scan and can not work with truncations any more. Zou-Pan [8] proved that the PI-line-based algorithm can exactly reconstruct the fan-beam projection data containing truncations and generalized the reconstruction from single point to entire PI-line segment. Since the projection $P(\lambda,u)$ used to reconstruct the entire PI-line segment are the same, the projection value u_d of every point passing through the PI-line can be calculated from the same angle. The whole PI-line segment can be exactly reconstructed as long as the projections are within the range of $[u_{d1},u_{d2}]$ even if the imaged object is larger than the FOV (field of view) or truncations exist. The detailed proofs were investigated in [8].

There is another intractable problem which is the reconstruction of the singularities existing in LT. Indeed, since Λ is an invertible elliptic pseudo-differential operator [7], f and Λf have precisely the same singular set. As shown in Fig. 1, the similarity between the image of f and Λf is at first glance surprising. However, we see that Λf is cupped where f is constant, and the singularities are amplified in Λf. The image of $\Lambda^{-1} f$ by itself seems less useful, but it provides a countercup for the cup in Λf. Thus, the image of Lf shows less cupping and looks even more similar to f than the image of Λf. For example, the image of Lf indicates that the density just inside the boundary of the object is larger than the density outside the object, while this can not be clearly seen from the image of Λf.

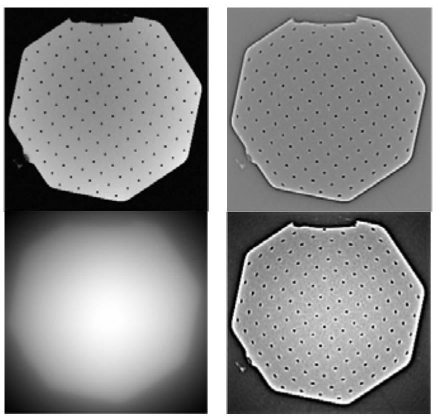

Fig. 1. Top left: Global reconstruction of density $f(x)$ of calibration object. Top right: Reconstruction of Λf. Bottom left: Reconstruction of $\Lambda^{-1} f$. Bottom right: Reconstruction of Lf.

We found that most of the existing CT reconstruction algorithms didn't robust to the singularities and eventually decreased the qualities of the reconstructed images. For that reason, we considered that if the integral over the projections can be avoided efficiently, the problems of reconstructing the singularities and improving the resolution of the reconstructed image may be resolved. As a result, we proposed the idea using a differential kernel function convoluting with the projection data. Consequently, we transform the projection data into differential kernel function as the convoluted object and realize the first derivatives and second derivatives at singularities.

In this article, following the idea that our method was fitted for other 2D scanning trajectories, we discuss the new algorithm for image reconstruction from fan-beam geometry with a circular trajectory. First, we introduce the differential Gaussian kernel function $G(\lambda, u)$, defined as

$$G(\lambda,u)=\frac{1}{(\sqrt{2\pi})^2\sigma_\lambda\sigma_u}\exp(-\frac{\lambda^2}{2\sigma_\lambda^2}-\frac{u^2}{2\sigma_u^2}), \quad (6)$$

where σ_λ and σ_u denotes the variance on its own direction, respectively. For the convenience of theoretical analysis, we introduce a new function $q(\lambda,u)$, defined as

$$q(\lambda,u):=p(\lambda,u)*G(\lambda,u)=\iint_{\mathbb{R}^2}p(x,y)G(x-\lambda,y-u)dxdy, \quad (7)$$

where $*$ denotes the convolution.

As mentioned above, if we differentiate the result which is received by the Gaussian kernel function convoluting with the projection data, we can transform the convoluted object theoretically.

$$\frac{\partial}{\partial\lambda}q(\lambda,u)=\frac{\partial}{\partial\lambda}\left[p(\lambda,u)*G(\lambda,u)\right]$$

$$=\iint_{\mathbb{R}^2}p(x,y)\frac{\partial}{\partial\lambda}G(x-\lambda,y-u)dxdy=p(\lambda,u)*\frac{\partial}{\partial\lambda}G(\lambda,u), \quad (8)$$

$$\frac{\partial^2}{\partial\lambda^2}q(\lambda,u)=\frac{\partial}{\partial\lambda}\left(\frac{\partial}{\partial\lambda}\left[p(\lambda,u)*G(\lambda,u)\right]\right)$$

$$=\iint_{\mathbb{R}^2}\frac{\partial}{\partial\lambda}\left(p(x,y)\frac{\partial}{\partial\lambda}G(x-\lambda,y-u)\right)dxdy=p(\lambda,u)*\frac{\partial^2}{\partial\lambda^2}G(\lambda,u). \quad (9)$$

According to the equation (12) to (17), the new reconstruction formula is

$$\Lambda f=-\frac{1}{2\pi}PV\int_{\lambda_1}^{\lambda_2}d\lambda\frac{\mathrm{sgn}(\vec{e}\bullet\vec{\theta}^\perp)}{\|\mathrm{x}-\vec{r}(\lambda)\|\cdot(\vec{r}\,'(\lambda)\bullet\vec{\theta}^\perp)}\times Q, \quad (10)$$

where

$$Q(\lambda,u)=p(\lambda,u)*\frac{\partial^2}{\partial\lambda^2}G(\lambda,u)-\frac{(\vec{r}\,''(\lambda)\bullet\vec{\theta}^\perp)}{(\vec{r}\,'(\lambda)\bullet\vec{\theta}^\perp)}p(\lambda,u)*\frac{\partial}{\partial\lambda}G(\lambda,u).$$

We can implement our LT algorithm in the following steps.

1. Select a differential kernel function. In this article, we choose the Gaussian function;
2. Define the scanning curve and the ROI. We use a circular scanning trajectory in our method;
3. Build a new function formed by the projection data convoluting with the Gaussian kernel function;
4. For every λ, compute $\frac{dp(\lambda,u)}{d\lambda}$ and $\frac{d^2p(\lambda,u)}{d\lambda^2}$ according to 8) and (9);
5. For every x inside the ROI, determine a pair of parameters (λ_1,λ_2) [8] that x, $\vec{r}(\lambda_1)$ and $\vec{r}(\lambda_2)$ are collinear. That is the so called PI-line. Through the data projected by the PI-line, we reconstruct $\Lambda f(\mathrm{x})$ according to (10);

6. Calculate $\Lambda^{-1}f(\mathrm{x})$ through equation (1);

7. Utilize $Lf(x) = \Lambda f(x) + \mu\Lambda^{-1}f(x)$ to calculate the final result, where μ is a weighted coefficient computed by a series of experiments and errors.

4. NUMERICAL STUDIES

We have performed a computer-simulation study to demonstrate and verify quantitatively the proposed fan-beam algorithm. In the numerical study, we selected the 2D differentiable Shepp-Logan phantom image and the clock image to generate projection data based on the circular trajectory. The locus has a radius of R = 847.7mm and the (virtual) one-dimensional (1D) detector array is at the centre of rotation. In this case, $D_d = R =$ 847.7mm. The 1D detector array consists of 1440 square-shape elements each of which has a size of 0.02mm. The covering angular ranges of the scanning trajectory is $[0, 2\pi]$. Also, we assumed that the detector was always centered at the system origin. Since there were numerous chords through any fixed point x, we selected the one through the origin o and x.

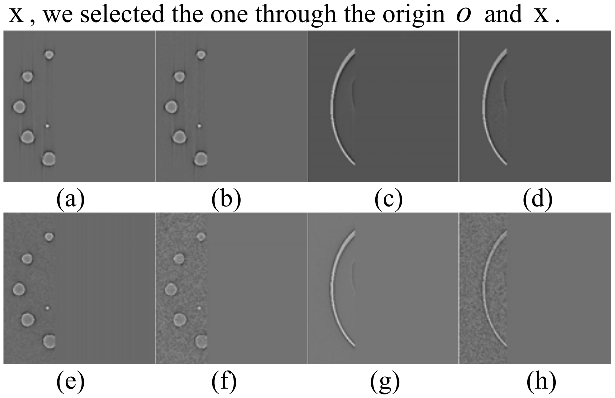

Fig. 2. All the projections are containing truncations. (a) Reconstruction of the clock using the improved method. (b) Reconstruction of the clock with 3 times random noise using the improved method. (c) Reconstruction of the phantom using the improved method. (d) Reconstruction of the phantom with 3 times random noise using the improved method. (e), (f), (g), (h) Reconstruction results using the classical algorithm under the corresponding conditions mentioned above, respectively.

The two reconstruction results displayed in Fig. 2(a) and 2(b) seems similar except the contrast. That is because we compute a linear combination of Λf and $\Lambda^{-1}f$ which enhances the image edges and increases the image resolution. Fig. 2(b) and 2(f) are the reconstructed images with random noises adding into the projection data, respectively. We can see that the reconstruction is destroyed completely displayed in Fig. 2(f). That proves the fact that the proposed algorithm is more robust in the ability of antinoise than that of the classical algorithm. Similarly, Fig. 2(c), (c), (e), (f) changes the experimental object into Shepp-Logan phantom image. Also, the simulation proves the same result that the resolution and the ability of antinoise of our method are better than that of the classical method.

5. CONCLUSION

In conclusion, we have proved an exact and efficient general fan-beam LT formula based on the PI-line segments. The numerical simulation has verified the correctness of the formulation. The new method eliminates the inexistence of the derivatives at singularities and reconstructs images within ROI from truncated projections. The experimental results show that the resolution and contrast are enhanced especially when we add in the term of $\Lambda^{-1}f$. Furthermore, it would be worthwhile to generalize the PI-line based fan-beam LT algorithm to address the ROI-reconstruction problem from truncated data acquired with a circular cone-beam configuration.

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