UNEQUAL LENGTH FIRST-ORDER LINEAR-PHASE FILTER BANKS FOR EFFICIENT IMAGE CODING

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ABSTRACT

In this paper, we present the structure and design method for a first-order linear-phase filter bank (FOLPFB) which has unequal filter lengths in its synthesis bank (UFLPFB). A FOLPFB is a generalized version of biorthogonal LPFBs regarding their synthesis filter lengths. Ringing artifact is the main disadvantage of image coding based on FOLPFBs. UFLPFBs can reduce the ringing artifacts as well as approximate smooth regions well.

Index Terms— First-order linear-phase filter banks, biorthogonal filter banks, unequal length filter banks, image coding.

1. INTRODUCTION

Image coding has been one of the most significant research topics for a couple of decades. Filter banks (FBs) play an important role in that area. They are used to remove spatial correlation and the subband signals are quantized, coded and stored or transmitted. Several FBs are known for efficient image coding such as the lapped orthogonal transform (LOT) [1], the biorthogonal linear-phase filter banks (BOLPFBs) [2] and the variable-length lapped orthogonal transform (VLLOT) [3]. All of these FBs are based on the motivations for image coding described in [4] as follows:

- The synthesis lowpass filters should be smooth and have (anti)symmetric impulse responses.
- The filter coefficients should decay to zero at both ends.
- The highpass filters should have no DC leakage.
- The analysis FB should be chosen to maximize the coding gain.
- The filters covering low/high-frequency bands should have long/short supports.
- The filters should have good stopband attenuation.

Previous researches achieve some of these properties. For example, the BOLPFB has high coding gain; the regular-BOLPFB [5] has no DC leakage and smoother filters than others; and the VLLOT has long lowpass filters and short highpass filters called *unequal length* filters. All of them also have linear-phase filters which yield (anti)symmetric basis functions. Furthermore, design costs are desired to be as small as possible.

To design a FB, iteration of a nonlinear optimization is usually adopted. The optimization process depends on initial values of the filter coefficients. Having a large number of free parameters

often causes the optimization program to yield a local minimal solution. Hence as the design freedom increases, the optimal solution becomes difficult to obtain. Paraunitary FBs (PUFBs) are suitable for reducing design freedom due to orthogonality. The LOT and the VLLOT belong to this class.

However, none of them has all desired properties. The lack of some properties generates distortions in reconstructed images. Well-known, and particularly noticeable distortions are *blocking* and *ringing* artifacts. The blocking artifacts occur due to short filters for low-frequency regions with no overlap (the main disadvantage of the DCT). The ringing artifacts appear around edges with high contrast because of long basis functions for high-frequency regions. Although PUFBs are good for design costs, biorthogonal solutions are generally better in the viewpoint of coding performance. To obtain a solution which has lower design costs and higher performance is the main objective of this paper.

The authors introduced the possibility to obtain better image coding performance by using simplified first-order (FO) LPFBs [6]. This class of FBs is a generalized version of BOLPFBs where the synthesis filter lengths can be longer than those of analysis filters [7]. Although this property is useful for image compression as mentioned above, the design of FB with the short basis functions for the synthesis highpass filters is still an unsolved problem. In this paper, we propose FOLPFBs which have unequal length filters in their synthesis banks (called UFLPFBs). Our proposed UFLPFB reduces the ringing artifacts significantly as well as retaining high coding gain and good perceptual visual quality in spite of having fewer design parameters.

Notations: The identity matrix is **I**, the reversal matrix is **J**. For simplicity, we omit vector or matrix sizes when they are obvious.

2. REVIEW

2.1. BOLPFBs

Consider an M-channel BOLPFB with filter length KM [2]. Fig. 1 shows a typical structure of a FB and its polyphase representation. Using the lattice structure, the analysis polyphase matrix $\mathbf{E}(z)$ can always be represented as

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\ldots\mathbf{G}_1(z)\mathbf{E}_0. \qquad (1)$$

If perfect reconstruction is achieved, the causal synthesis polyphase matrix $\mathbf{R}(z)$ is given as

$$\mathbf{R}(z) = z^{-(K-1)}\mathbf{E}_0^{-1}\mathbf{G}_1^{-1}(z)\mathbf{G}_2^{-1}(z)\ldots\mathbf{G}_{K-1}^{-1}(z). \qquad (2)$$

When M is even, each matrix in (1) is represented as follows:

$$\mathbf{G}_i(z) = \mathbf{\Phi}_i \mathbf{W} \mathbf{\Lambda}(z) \mathbf{W}, \qquad \mathbf{E}_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{U}_0 & \mathbf{U}_0 \mathbf{J} \\ \mathbf{V}_0 & -\mathbf{V}_0 \mathbf{J} \end{bmatrix} \quad (3)$$

where $\mathbf{\Phi}_i = \mathrm{diag}(\mathbf{U}_i, \mathbf{V}_i)$ and

$$\mathbf{W} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_{M/2} & \mathbf{I}_{M/2} \\ \mathbf{I}_{M/2} & -\mathbf{I}_{M/2} \end{bmatrix}, \quad \mathbf{\Lambda}(z) = \begin{bmatrix} \mathbf{I}_{M/2} & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & z^{-1}\mathbf{I}_{M/2} \end{bmatrix}.$$

If the $M/2 \times M/2$ matrices $\mathbf{U}_i$ and $\mathbf{V}_i$ are nonsingular, the FB is a BOLPFB. Furthermore, $\mathbf{U}_i$ for $i > 0$ can be set to $\mathbf{U}_i \equiv \mathbf{I}$ for simplicity without losing completeness [8].

2.2. FOLPFBs

In [7], the eigenstructure based characterization of M-channel BO-LPFBs whose analysis filter lengths are $2M$ (they are called *first-order*) and their synthesis ones are equal or longer than $2M$ was presented. Its lattice structure of the analysis bank is

$$\mathbf{E}(z) = \mathrm{diag}(\mathbf{A}_1, \mathbf{A}_2)\mathbf{W}' \begin{bmatrix} \mathbf{I}_{M/2}z^{-1} - \mathcal{J}_F & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & \mathcal{J}_F z^{-1} - \mathbf{I}_{M/2} \end{bmatrix}$$
$$\times \mathbf{W} \mathrm{diag}(\mathbf{A}_3, \mathbf{A}_4) \mathbf{W} \mathrm{diag}(\mathbf{I}_{M/2}, \mathbf{J}_{M/2}) \quad (4)$$

where $\mathbf{W}' = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_{M/2} & -\mathbf{I}_{M/2} \\ \mathbf{I}_{M/2} & \mathbf{I}_{M/2} \end{bmatrix}$. Each $\mathbf{A}_i$ ($i = 1, \dots, 4$) is an $M/2 \times M/2$ nonsingular matrix, and $\mathcal{J}_F$ is an $M/2 \times M/2$ block diagonal with Jordan blocks of size b_i ($i = 0, \dots, n$, b_i is non-increasing positive integer and $\sum_{i=0}^{n} b_i = M/2$) with zero eigenvalue, respectively. For example, if $M = 6$ and $\{b_i\} = \{2, 1\}$,
$\mathcal{J}_F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Moreover $\mathbf{R}(z)$ is obtained as follows:

$$\mathbf{R}(z) = z^{-b_0} \mathrm{diag}(\mathbf{I}_{M/2}, \mathbf{J}_{M/2}) \mathbf{W} \mathrm{diag}(\mathbf{A}_3^{-1}, \mathbf{A}_4^{-1}) \mathbf{W}$$
$$\times \begin{bmatrix} \mathbf{I}_{M/2}z + \sum_{i=2}^{b_0} \mathcal{J}_F^{i-1} z^i & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & -\mathbf{I}_{M/2} - \sum_{i=1}^{b_0-1} \mathcal{J}_F^i z^{-i} \end{bmatrix}$$
$$\times \mathbf{W}'^T \mathrm{diag}(\mathbf{A}_1^{-1}, \mathbf{A}_2^{-1}). \quad (5)$$

In this structure, some patterns of the synthesis filter length can be permitted. If $M = 6$, we can design a FOLPFB whose analysis filter length is $2 \times 6 = 12$ and synthesis length is 12 ($b_i = \{1, 1, 1\}$), 24 ($b_i = \{2, 1\}$) or 36 ($b_i = \{3\}$). For further information of this class of FBs, please refer to the article [7]. Obviously, when $b_i = \{1, \dots, 1\}$, the obtained FB is a BOLPFB.
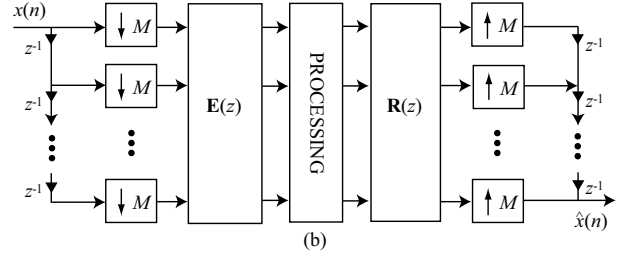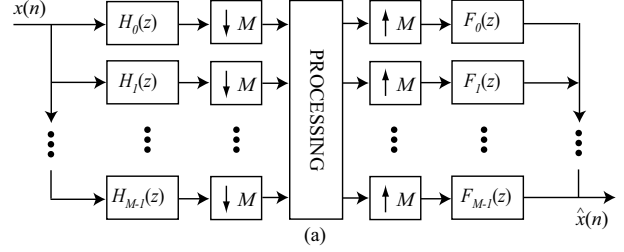
2.3. Simplified FOLPFBs

The authors proposed a simplified lattice structure of FOLPFBs [6] as follows:

$$\mathbf{E}(z) = \mathrm{diag}(\mathbf{A}_{10}, \mathbf{A}_2)\mathbf{W}' \begin{bmatrix} \mathbf{I}_{M/2}z^{-1} - \mathcal{J}_F & \mathbf{0}_{M/2} \\ \mathbf{0}_{M/2} & \mathcal{J}_F z^{-1} - \mathbf{I}_{M/2} \end{bmatrix}$$
$$\times \mathbf{W} \mathrm{diag}(\mathbf{A}_3, \mathbf{A}_4) \mathbf{W} \mathrm{diag}(\mathbf{I}, \mathbf{J}). \quad (6)$$

$\mathbf{A}_{10}$ is the part of the nonsingular matrix factorization $\mathbf{A}_1 = \mathbf{A}_{10}\mathbf{A}_{11}$ where

$$\mathbf{A}_{11} = \begin{bmatrix} \mathbf{S}_{b_0} & \mathbf{T}_{b_0 \times b_1} & \dots \\ \mathbf{T}_{b_1 \times b_0} & \mathbf{S}_{b_1} & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix} \quad (7)$$

Fig. 1. M-channel maximally decimated filter bank. (a) conventional representation. (b) polyphase representation.

and

$$\mathbf{S}_b = \begin{bmatrix} s_0 & s_1 & \dots & s_{b-1} \\ 0 & s_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_1 \\ 0 & \dots & 0 & s_0 \end{bmatrix}$$

$$\mathbf{T}_{l \times m} = \begin{cases} \begin{bmatrix} \mathbf{S}_m \\ \mathbf{0}_{(l-m) \times m} \end{bmatrix} & l \geq m \\ \begin{bmatrix} \mathbf{0}_{l \times (m-l)} & \mathbf{S}_l \end{bmatrix} & m \geq l \end{cases}.$$

For example, in the case of $b_i = \{2, 1, 1\}$, $\mathbf{A}_{10}$ can be represented as

$$\mathbf{A}_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_0 & 1 & 0 & 0 \\ l_1 & 0 & 1 & 0 \\ l_2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha_0 & r_0 & r_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (8)$$

The structure can reduce redundant parameters from traditional FOLPFBs, and can be used despite of various patterns of b_i's. If $b_i = \{1, \dots, 1\}$, $\mathbf{A}_{11}$ has all parameters of $\mathbf{A}_1$. It is the same as a simplified BOLPFB since $\mathbf{A}_{10} = \mathbf{I}$. It implies that the structure covers other works presented in [8].

3. UFLPFBS

In this section, we propose a new FOLPFBs where the synthesis filters have unequal lengths (UFLPFBs). We consider a real-valued, even-channel UFLPFB based on a simplified FOLPFB. To avoid cumbersome calculations, $b_i = \{2, 1, \dots, 1\}$ is only considered in this paper. However, the case is the essential of UFLPFBs, thus it can be easily extended to other b_i's. The synthesis filters are divided into three categories:

1. Long category; N_0 filters with length $2b_0M$
2. Middle category; N_1 filters with length $2M$
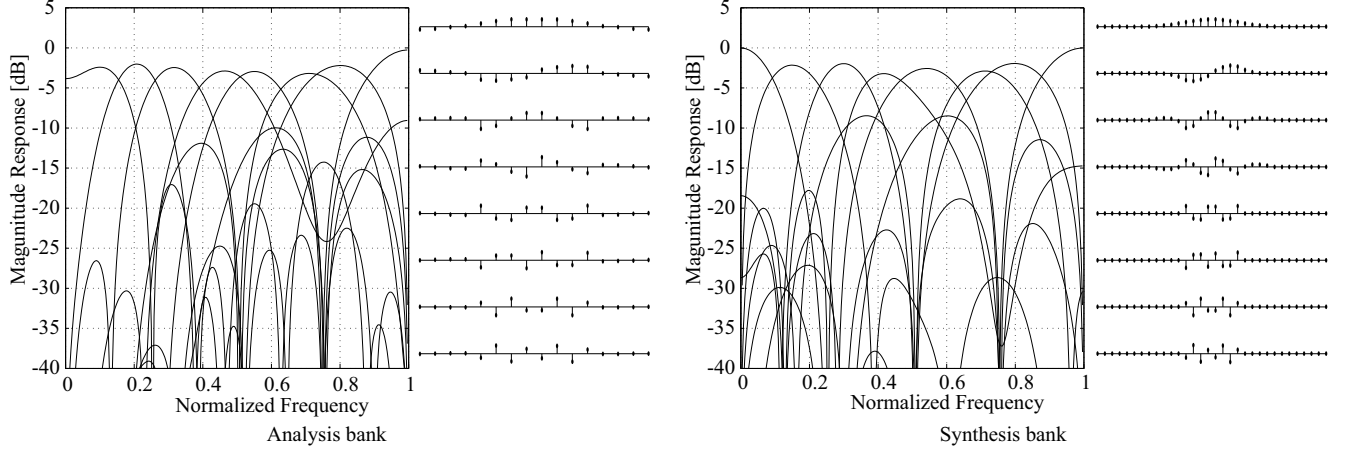
Fig. 2. Design example of an UFLPFB: $M = 8$, $b_i = \{2, 1, 1\}$, the synthesis filter set is 3×32, 1×16 and 4×8.

3. Short category; $(M - N_0 - N_1)$ filters with length M

where $(N_0 + N_1) \leq M$.

3.1. Condition for the long category

First, we consider the condition for the longest synthesis filters. The expanded filter length by $\mathcal{J}_F$ corresponds to the long category. Assume that there are N_{00} symmetric and N_{01} antisymmetric filters where $(N_{00} + N_{01}) = N_0$. For image coding application, $|N_{00} - N_{01}| \leq 1$ is naturally defined since if $|N_{00} - N_{01}| > 1$, it generates shorter filters corresponding to lower frequency and longer filters corresponding to higher frequency. Consequently, the condition is described as follows:

$$\begin{cases} \mathcal{J}_F \mathbf{A}_{10}^{-1} = \begin{bmatrix} \mathbf{X}_{M/2 \times N_{00}} & \mathbf{0}_{M/2 \times (M/2 - N_{00})} \end{bmatrix} \\ \mathcal{J}_F \mathbf{A}_2^{-1} = \begin{bmatrix} \mathbf{X}_{M/2 \times N_{01}} & \mathbf{0}_{M/2 \times (M/2 - N_{01})} \end{bmatrix} \end{cases} \quad (9)$$

where $\mathbf{X}$ denotes coefficients which can be ignored. In the $b_i = \{2, 1, \ldots, 1\}$ case, each of $\mathbf{A}_{10}$ and $\mathbf{A}_2$ has $(M/2 - N_{00})$ and $(M/2 - N_{01})$ restricted parameters, respectively. For example, if we desire an eight-channel UFLPFB with $N_{00} = 2$, both r_0 and r_1 in (8) have to be 0.

3.2. Condition for the middle and short categories

The number of filters in the middle and short categories depends on each other, hence they can be considered jointly. Similar to the previous subsection, we assume that the UFLPFB has N_{10} symmetric and N_{11} antisymmetric *middle* filters with length $2M$. The condition is represented as

$$\begin{cases} (\mathbf{A}_3^{-1} - \mathbf{A}_4^{-1})\mathbf{A}_{10}^{-1} = \begin{bmatrix} \mathbf{X}_{M/2 \times (N_{00} + N_{10})} & \mathbf{0} \end{bmatrix} \\ (\mathbf{A}_3^{-1} - \mathbf{A}_4^{-1})\mathbf{A}_2^{-1} = \begin{bmatrix} \mathbf{X}_{M/2 \times (N_{01} + N_{11})} & \mathbf{0} \end{bmatrix} \end{cases} \quad (10)$$

In this case, $|N_{10} - N_{11}| \leq 1$ and $(N_{00} + N_{10}) = (N_{01} + N_{11}) = N_s$ have to be satisfied from the existence condition described in [3].

The middle category has the following constraints:

$$(\mathbf{A}_3^{-1} - \mathbf{A}_4^{-1}) = \begin{bmatrix} \mathbf{X}_{M/2 \times N_s} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{A}_{10}, \ \mathbf{A}_2 = \begin{bmatrix} \mathbf{X} & \mathbf{0}_{N_s \times (M/2 - N_s)} \\ \mathbf{X} & \mathbf{X} \end{bmatrix}.$$

Note that the *middle/short* condition is independent of the *long* one. Consequently, the overall restriction for UFLPFBs is represented as (Condition for *long* category) $\cup$(Condition for *middle/short* category). The number of eliminable parameters depends on the filter set.

3.3. Particular property of UFLPFBs

Our proposed structure is partly similar to the traditional unequal length structures with an additional property on the filter length. The traditional FBs have to comply with the existence condition; i.e., the number of long and short filters has to be even [3]. However, UFLPFBs could have odd number of filters in the long or middle category since the long category only depends on the structure of $\mathbf{A}_{10}$ or $\mathbf{A}_2$. There are two restrictions for those matrices; unequal length condition (described in (9)) and invertibility. In other words, they are independent of the existence condition proven in [3]. It is the main difference from traditional unequal solutions. However, the condition for UFLPFBs is remaining, i.e., both $(N_0 + N_1)$ and $(M - N_0 - N_1)$ have to be even. The next section presents a design example of an eight-channel UFLPFB whose synthesis filter set is 3×32, 1×16 and 4×8. It implies that $N_0 = 3$, $N_1 = 1$ and $(N_0 + N_1) = (M - N_0 - N_1) = 4$.

4. DESIGN EXAMPLE AND IMAGE CODING APPLICATION

In this section, a design example of an UFLPFB is shown. As previously mentioned, we design the eight-channel UFLPFB with $b_i = \{2, 1, 1\}$ whose analysis length is 16 and synthesis filter set is 3×32, 1×16 and 4×8. The frequency and impulse responses of both banks are shown in Fig. 2. The design objective function is the weighted linear combination of the coding gain, the stopband attenuation and the DC leakage [4]. The number of design parameters is 38. It is

Table 1. Comparison of image coding results (PSNR in Decibels)

Test images	Comp. ratio	Transforms			
		9/7 DWT	8×16 BOLPFB	$8 \times 16/32$ FOLPFB	$8 \times 16/(32, 16, 8)$ UFLPFB
Lena	1:32	31.74	30.07	30.29	30.12
	1:16	35.34	35.28	35.47	35.23
	1:8	38.73	38.87	38.86	38.68
Barbara	1:32	26.42	27.14	27.00	26.58
	1:16	30.11	31.26	31.20	30.77
	1:8	34.72	35.95	35.82	35.46
Yogi	1:32	23.30	23.48	21.88	22.92
	1:16	27.56	27.40	27.06	26.78
	1:8	36.20	34.57	34.63	34.56

Fig. 3. Comparison of reconstructed images (1:32, portions of *Yogi*). From left to right: 9/7 DWT. 8×16 BOLPFB. FOLPFB. UFLPFB.

fewer than a simplified FOLPFB's 54 (for the same b_i) and an 8×16 BOLPFB's 48. However, their coding gains are similar (around 9.60 dB).

The proposed UFLPFB is applied to image coding and compared with the performance of other FBs included the 9/7 wavelet transform (DWT) [4]. We designed another UFLPFB with $b_i = \{2, 1, 1\}$ whose analysis length is 16 and synthesis filter set is 4×32, 2×16 and 2×8 for image coding. Its coding gain is 9.61 dB. We coded each transformed image by the embedded zerotree wavelet image codec presented in [9] for fair comparison. Coding results are summarized in Table 1 and reconstructed images are shown in Fig. 3. In PSNRs, the UFLPFB performs comparable to the traditional FBs. In subjective performance, the DWT yields the "blurred" image and the other traditional FBs generate the ringing artifacts. By contrast, the UFLPFB yields less blurring and smaller ringing artifacts. The fact means the UFLPFB is suitable for image coding.

5. CONCLUSION

In this paper, we presented the structure and design of UFLPFBs for efficient image coding. UFLPFBs can have various filter length sets due to their generalized structure for the synthesis filter lengths. In image coding application, the proposed UFLPFB obtains better perceptual visual quality as well as having fewer design parameters than that in conventional FBs.

6. REFERENCES

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