A NEW CLASS OF FILTERS FOR IMAGE INTERPOLATION AND RESIZING

Amir Said

Hewlett Packard Laboratories, Palo Alto, CA

ABSTRACT

We propose a new set of kernels to simplify the design of filters for image interpolation and resizing. Their properties are defined according to two parameters, specifying the width of the transition band and the height of the first sidelobe. By varying these parameters we can get very good approximations of many commonly-used interpolation kernels. Furthermore, because the Fourier transforms of these kernels have very fast decay, they can also be used for downsampling.

Index Terms— Interpolation, image sampling, multidimensional digital filters

1. INTRODUCTION

Image resizing and interpolation (e.g., for rotation) are two of the most useful image processing operations, and consequently there is a great amount of literature on the subject [1]–[7]. However, many imaging professionals find the task of sorting out and implementing the most appropriate method quite challenging, due to the great number of possibilities, and of conflicting opinions. It is common to settle for some very simple approaches which were once meant to reduce complexity, or adopt one type that was shown to be excellent for one application, without knowing that it may be suboptimal for other applications.

For instance, even in commercial products we find the mistake of using interpolation kernels for downsampling, without the necessary lowpass filtering. A less serious, but also common mistake, is to use for downsampling low-order filters which can be quite good for interpolation, but have much worse properties when time-scaled for resizing.

What is still missing is an approach that is more *convenient* and easy to use, with less emphasis on computational complexity, and that yields high image quality. For that purpose we propose a family of parameterized functions that are simple, and are designed with enough versatility so that well-known kernels can be very closely approximated by simply using the proper parameters. This way it is easy to experiment and identify the parameters that are best suited to a certain type of image, without having to understand and implement several methods.

This paper is organized by first defining the proposed family of functions, and some of their basic properties. Next, we present a set of features that are desirable for interpolation and resizing kernels, and explain how well the proposed functions support those features. Finally, we present some results of finding the approximation to some commonly-used kernels, show how well these kernels are approximated, and discuss the extra features of the proposed kernels.

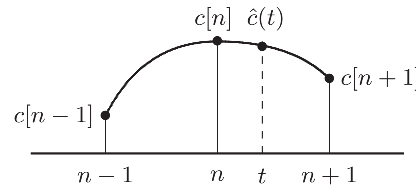

Fig. 1. Notation used for interpolation and resizing.

2. INTERPOLATION AND RESIZING KERNELS

We assume that separable filters are used [3], and to simplify the notation we consider only the one-dimensional case. Fig. 1 shows the basic notation we use: if we have a sequence of pixel values $c[n]$, and a kernel function $h(t)$, then the value at point t is defined as

$$\hat{c}(t) = \sum_{k \in S_h} b \cdot c[k + \lfloor t \rfloor] h(b \cdot [t - k - \lfloor t \rfloor]), \qquad (1)$$

where b is the normalized bandwidth of the signal. When downsampling the image b should be equal or smaller than the reduction factor, which means that $0 < b < 1$. When the image is upsampled or rotated, we have $b = 1$.

The family of functions that we propose for interpolation and resizing has only two parameters, χ and η, and is defined by

$$h_{\chi,\eta}(t) = \mathrm{sinc}(t) \cosh\left(\frac{\sqrt{2\eta}\pi\chi t}{2-\eta}\right) e^{-[\pi\chi t/(2-\eta)]^2} \qquad (2)$$

where $\mathrm{sinc}(0) = 1$ and $\mathrm{sinc}(t) = sin(\pi t)/(\pi t)$, when $t \neq 0$.

The Fourier transforms of these functions are

$$H_{\chi,\eta}(f) = P_\eta\left(\frac{(2f+1)(2-\eta)}{\sqrt{2}\chi}\right) - P_\eta\left(\frac{(2f-1)(2-\eta)}{\sqrt{2}\chi}\right) \qquad (3)$$

where

$$P_\eta(f) = \frac{e^{\eta/2}}{\sqrt{2\pi}} \int_0^f e^{-\phi^2/2} \cos(\sqrt{\eta}\phi)\, d\phi \qquad (4)$$

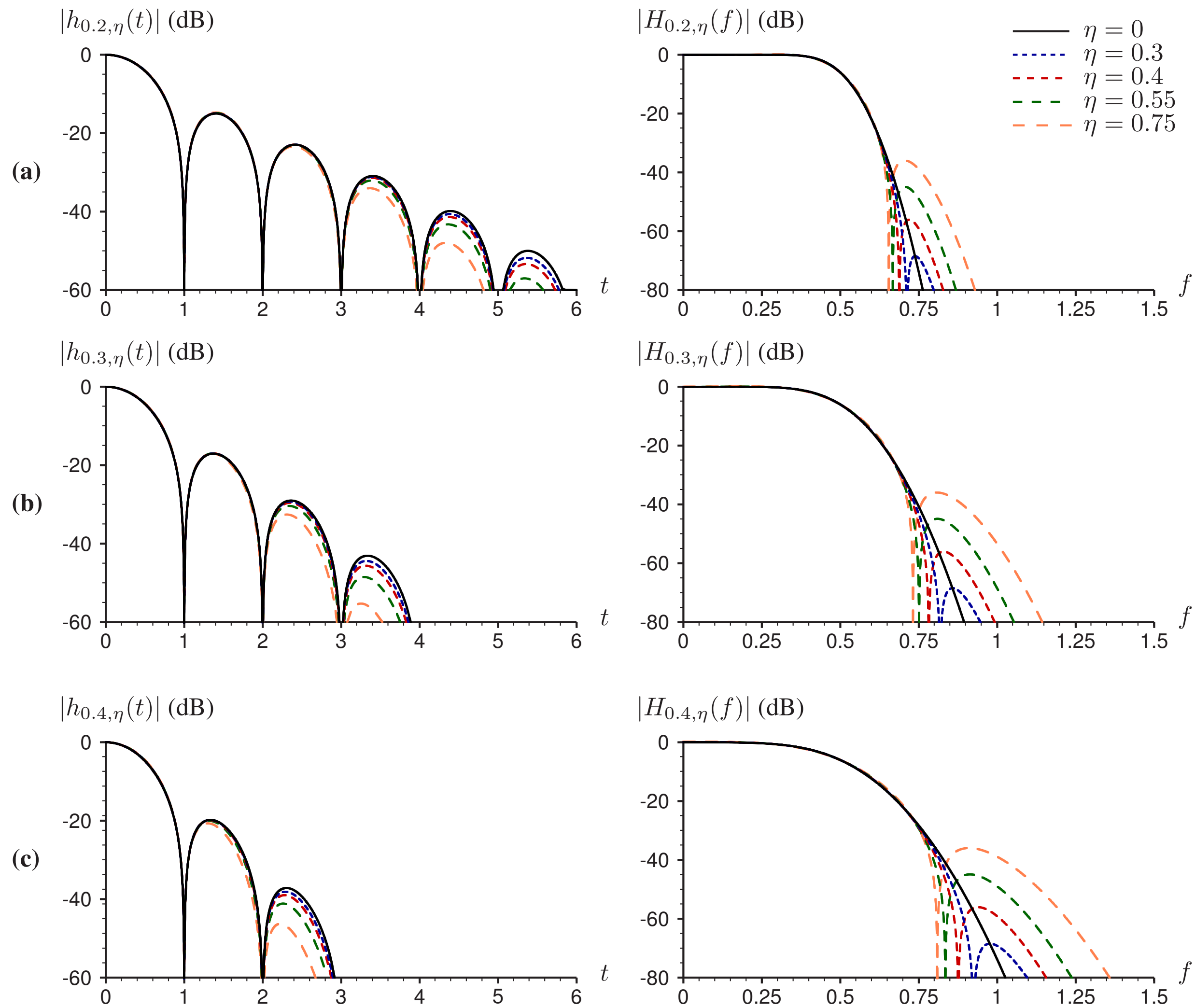

Fig. 2. Impulse and frequency responses for selected parameters: (a) $\chi = 0.2$, (b) $\chi = 0.3$, (c) $\chi = 0.4$.

Fig. 2 shows some examples. Note that all graphs have $|h_{\chi,\eta}(t)|$ and $|H_{\chi,\eta}(f)|$ in decibels. We can observe that χ basically controls the width of the transition band, and η affects the height of the first sidelobe. Also note that $H_{0,\eta}(f)$ is the ideal lowpass filter.

While there is no closed-form expression for integral (4) (related to the complex-valued error function), we used the time-frequency properties of the functions, and found that it can approximated with absolute error smaller than 10^{-16} using

$$P_\eta(f) \approx \begin{cases} 1/2, & f > 8, \\ \dfrac{f}{17} + \displaystyle\sum_{n=1}^{22} \psi_\eta[n] \sin(2\omega n f), & |f| \le 8, \\ -1/2, & f < -8, \end{cases} \tag{5}$$

where $\omega = \pi/17$ and $\psi_\eta[n] = e^{-\omega^2 n^2} \cosh(2\omega n \sqrt{\eta})/(\pi n)$.

3. DESIRABLE FEATURES

There are some important features—not all simultaneously achievable—that are desirable for the functions used for creating the discrete-time filters.

(a) ***Flexibility***

It is necessary to recognize that different types of images (natural, medical, synthetic, etc.) have different requirements. While we have a variety of theoretical tools developed for the analysis and design of interpolation and resizing filters, in most cases it is still essential to experiment several filters, and visually inspect the results.

The proposed kernels are meant to allow imaging professionals to try different filters more easily. In fact, to make their performance and visual quality easier to predict, they can closely approximate other commonly used kernels. Table 1 shows some sets of parameters that can be used for these

Table 1. Parameters that approximate popular interpolation kernels.

Kernel	χ	η
Lanczos, $M = 2$	0.414	0.61
Lanczos, $M = 3$	0.284	0.64
Lanczos, $M = 4$	0.212	0.65
Lanczos, $M = 5$	0.170	0.65
Blackman-Harris, $N = 6$	0.411	0.23
Cubic B-Spline	0.310	0
Mitchell-Netravali, $B = C = 1/3$	0.550	0.32

approximations, and Fig. 3 shows how good the approximation can be. (More details in Section 4.)

(b) ***Intuitive Controls***
While experimenting, it is desirable to be able to finely tune the filter's response. Some kernels provide very little control, being defined only for some discrete parameters, like "order." Others are defined by parameters related to approximation theory, which may have limited relation to the properties of natural images.

We defined the parameters of our kernel in a way that makes its equations somewhat more complicated, but aiming to make them much more intuitive. The parameter χ is defined to be the main control for achieving a compromise between blurring, aliasing and ringing artifacts. If it it too small, we have nearly ideal filters, which create ringing artifacts around edges. If it is too large we certainly have blurring, and some aliasing. The amount of aliasing depends also on the parameter η, which controls the height of the sidelobe. These properties can be seen in Fig. 2.

(c) ***Symmetry and Exact Interpolation***
For imaging applications it is necessary to use linear phase filters, and commonly interpolation functions have even symmetry, i.e., $h(t) = h(-t)$. Under the assumption that pixel values corresponds to samples of a strictly bandlimited signal, we would like to not change the values that are already know, and this is achieved when

$$h(0) = 1, \quad h(n) = 0, \quad n = \pm 1, \pm 2, \ldots \tag{6}$$

Image signals are certainly not strictly bandlimited, but the property is still useful because it implies that for all f we have

$$H_d\left(e^{j2\pi f}\right) = \sum_{n=-\infty}^{\infty} H(f - n) = 1, \tag{7}$$

i.e., we know that the gain for signal plus aliasing always adds to one. Our kernels satisfy this property because they belong to the class of functions created by multiplying sinc(t) with another function.

(d) ***Good Response with Small Spatial Support***
When considering the filtering computational complexity, it is good to use filters with small numbers of taps. Since in imaging applications we need to avoid ringing resulting from lowpass filters with steep transition, good interpolation results had been obtained with very short filters.

Our kernels were chosen such that $h_{\chi,\eta}(t)$ and $H_{\chi,\eta}(f)$ have asymptotic decay as fast as O($e^{-\alpha t^2}$). Thus, while $h_{\chi,\eta}(t)$ strictly has infinite support, the very fast decay makes it easy to find where to truncate the response without significantly changing the filter's performance (cf. Fig. 2). This approach tends to yield somewhat longer filter responses, but it is more convenient for obtaining downsampling filters, which need to be more carefully designed.

(e) ***Good Performance For Both Interpolation and Downsampling***
One of the most natural requirements in interpolation and resizing is that when applied to an image with a constant pixel value, it should always create another image with the same value. This is possible only when we have an exact *partition of unity*:

$$\sum_{k=-\infty}^{\infty} b \cdot h(b \cdot [t - k]) = 1, \quad t \in [0, 1). \tag{8}$$

In the frequency domain this corresponds to

$$H(0) + 2\sum_{n=1}^{\infty} H\left(\frac{n}{b}\right) \cos(2\pi n t) = 1, \quad t \in [0, 1). \tag{9}$$

Interpolation kernels commonly satisfy this property exactly by having $H(0) = 1$, and $H(n) = 0$, $n = 1, 2, \ldots$ Others provide very good approximation with very small values of $|H(0) - 1|$ and $|H(n)|$, $n = 1, 2, \ldots$

The problem of using interpolation functions for downsampling is that while the condition above is satisfied exactly for $b = 1$, it not a good approximation when $b < 1$. For example, the kernel for linear interpolation [5] is clearly inadequate when $b < 1$.

Here we see one of the main advantages of kernel $h_{\chi,\eta}(t)$. With the proper choice of χ and η, the very fast decay guarantees that $|H_{\chi,\eta}(1/b)|$ is very small for $b < 1$, which means that it is also good as a downsampling filter.

4. APPROXIMATION OF OTHER KERNELS

Many of the features of the new kernels can be observed by analyzing versions that have parameters chosen to closely approximate kernels commonly used for interpolation. Table 1 shows some sets of parameters that can be used for these approximations, and Fig. 3 shows comparisons of the corresponding Fourier transforms. In Fig. 3 (a) we can observe that with $\chi = 0.248$ and $\eta = 0.48$ we have a response nearly identical to the Blackman-Harris ($N = 6$) kernel [1, 5]. Choosing $\chi = 0.163$ and $\eta = 1.2$ produces a response very similar to the Lanczos kernel ($M = 2$) [4, § 3] up to its first zero. After that, $H_{\chi,\eta}(f)$ produces a wider sidelobe with roughly

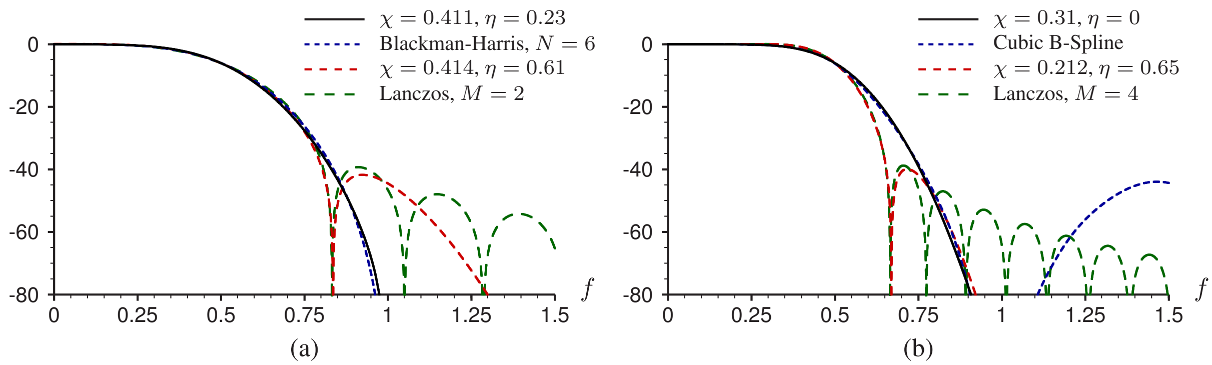

Fig. 3. Fourier transforms of some commonly used kernels and of their approximation with the proposed parameterized kernels.

the same height, but with nearly monotonic decay, instead of several sidelobes.

Fig. 3 (b) shows a comparison with another Lanczos kernel, with similar results. In part, these results are not very surprising, since the Blackman-Harris and Lanczos kernels are based on the sinc(t) function. However, they show that we can get remarkable control of the properties of $H_{\chi,\eta}(f)$ by changing only its two parameters.

Furthermore, the comparison of $H_{\chi,\eta}(f)$ with the cubic b-spline kernel in Fig. 3 (b) shows that we can have very good approximations for other types of kernels too. One important difference, is that $H_{\chi,\eta}(f)$ has no sidelobes above –80 dB when $f > 1$, which means that it has better partition of unity when used for downsampling.

We also tested the Lanczos (LZ4), Blackman-Harris (BH6), and cubic b-spline (CBS) kernels in the 2048×2560 image "Bike" (used in the JPEG2000 tests), chosen because it has many details and test patterns. First, we upsampled all images by factor 2.3. In all cases, the images are visually indistinguishable, so we measured the differences between images. The results, defined as PSNR in dB, are:

Kernel	LZ4	BH6	CBS
LZ4-Approx.	51.3	42.6	49.4
BH6-Approx.	43.4	58.3	48.4
CBS-Approx.	47.9	47.3	58.1

The values in the main diagonal show that the images obtained with the kernels and their approximations are indeed very close. The other values are also large, indicating that all those kernels produce good results, but the differences are also clear.

In the second experiment we downsampled the image by factor 0.3, and obtained similar results.

Kernel	LZ4	BH6	CBS
LZ4-Approx.	51.4	38.9	45.6
BH6-Approx.	39.9	56.1	45.2
CBS-Approx.	45.2	44.0	57.0

5. CONCLUSIONS

We have shown that the proposed kernels for image interpolation and resizing can be easily designed, since their two parameters provide direct control over the most important features, which are the width of the transition band, and the sidelobe height. We also explain that the kernels naturally satisfy many desirable conditions. They yield exact interpolation, both the functions and their Fourier transforms have very fast decays, and thus the same kernels can produce good results for both interpolation and downsampling. We tested the flexibility of the design by presenting sets of parameters that produce kernels that are very good approximations of kernels that are well known for their properties and superior image quality. The differences between the original kernels and the approximations are evaluated by analyzing the frequency response, and also measuring the difference between images resized with those kernels. In conclusion, the new class of kernels provide a very convenient way to test different kernels in order to identify those that produce the best image quality.

6. REFERENCES

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