CONDITIONS FOR COLOR MISREGISTRATION SENSITIVITY IN CLUSTERED-DOT HALFTONES

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ABSTRACT

Misregistration between the color separations of a printed image, which is often inevitable, can cause objectionable color shifts in average color. We analyze the impact of inter-separation misregistration on clustered-dot halftones using Fourier analysis in a lattice framework. Our analysis provides a complete characterization of the conditions under which the average color is invariant to displacement misregistration. In addition to known conditions on colorant spectra and periodicity of the halftones, the work reveals that invariance can also be obtained when these conditions are violated for suitable dot shapes and displacements. Examples for these conditions are included, as is the consideration of traditional halftone configurations.

Index Terms— Color halftoning, clustered-dot halftones, inter-separation misregistration, lattice theory

1. INTRODUCTION

Color halftone printing typically employs four color separations, *viz.* Cyan (C), Magenta (M), Yellow (Y) and Black (K). In several printing systems, these C, M, Y, K halftone separations are sequentially printed on the paper substrate to produce the color hardcopy output. Due to variations in the operations within the printing process, for example, in mechanical paper transport, paper shrinkage, and imager alignment; some misregistration between halftone image separations is unavoidable. This misregistration can cause objectionable color shifts from print to print. In this paper, we consider color shifts induced by inter-separation displacement misregistration in *clustered-dot halftones* [1], which constitute the primary method for halftoning for lithography and xerography- the two main technologies for high volume printing. Specifically, we provide a full mathematical characterization of the conditions for sensitivity to inter-separation displacement misregistration.

Color shifts due to inter-separation misregistration are known to depend on the spectral characteristics of the colorants [2] as well as the joint spatial characteristics of the halftone separations [3, 4, 5, 6, 7]. Conventionally, it is believed that misregistration insensitivity is achieved if either the colorants have non-overlapping absorption bands [2], or if the halftone periodicities meet a "non-singularity" condition [5]. Our analysis reveals additional situations under which insensitivity to misregistration is achievable, despite these conditions being violated. In addition, the analysis provides a comprehensive framework under which the conditions for insensitivity to displacement misregistration maybe fully understood. The analytic model presented here complements the experimental and simulation results presented earlier [6, 7].

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2. COLOR MISREGISTRATION ANALYSIS

Conventional clustered-dot halftones are amplitude modulated (AM) signals in the sense that different gray levels are reproduced by varying the size of halftone spots while keeping the periodicity of spots constant. Color printing is accomplished by halftoning the K individual color separations, where $K = 4$ for the typical CMYK scenario, and printing these in overlay. For our analysis, we model individual colorant halftones in terms of a lattice that represents their periodicity and a spot function that represents the shape of the halftone dots. The average color is obtained from the spectral Neugebauer model [8, 9] that computes the average reflectance as the weighted average of the reflectances of all possible overlays of the colorants on the substrate - known as the Neugebauer primaries. The weights in the model correspond to the fractional areas of the Neugebauer primaries. A change in these fractional areas is the primary source of color shifts when the colorants are non-ideal. We therefore obtain our model using these elements and obtain expressions for the average reflectance spectrum and the fractional areas, which we use in turn to characterize the conditions under which the average spectrum is invariant to displacement misregistration.

2.1. Individual Colorant Halftones

A halftone image $h_k(\mathbf{x})$ generated for the k^{th} colorant plane of a constant gray-level contone image can be modeled as the convolution of a planar lattice denoted by $\mathbf{\Lambda}_k$ and a binary halftone spot function denoted by $s_k(\mathbf{x})$ [10], where $\mathbf{x} = [x, y]^T$ represents the spatial coordinates. $\mathbf{\Lambda}_k$ represents the 2-D periodicity of the k^{th} halftone separation and is mathematically defined as [11]

$$\mathbf{\Lambda}_k = \{\mathbf{V}_k \mathbf{n}_k \mid \mathbf{n}_k \in \mathbb{Z}^2\}, \tag{1}$$

where $\mathbb{Z}$ denotes the set of integers and $\mathbf{V}_k = [\mathbf{v}_1^k \,\vdots\, \mathbf{v}_2^k]$ is a 2×2 real-valued matrix, with two 2×1 *linearly independent* vectors $\mathbf{v}_1^k$ and $\mathbf{v}_2^k$ as its columns. The vectors $\mathbf{v}_1^k$ and $\mathbf{v}_2^k$ represent a *basis* for the lattice $\mathbf{\Lambda}_k$ and for any point $\mathbf{V}_k\mathbf{n}_k$ in the lattice, the vector $\mathbf{n}_k = [n_{kx}, n_{ky}]^T$ is the representation of the point in the lattice with respect to the basis $\mathbf{V}_k$.

The halftone spot function $s_k(\mathbf{x})$ is defined within a unit cell of $\mathbf{\Lambda}_k$, denoted by $\mathcal{U}_k$, and takes values 1 and 0 corresponding to the situation that ink k is, or is not deposited at the position $\mathbf{x} = [x, y]^T$. Displacement misregistration of the k^{th} separation by the vector $\mathbf{d}_k = [\Delta x_k, \Delta y_k]^T$ is readily incorporated in this representation by replacing $s_k(\mathbf{x})$ with $s_k(\mathbf{x} - \mathbf{d}_k)$. The halftone separation $h_k(\mathbf{x})$ can accordingly be written as

$$h_k^{(\mathbf{d}_k)}(\mathbf{x}) = s_k(\mathbf{x} - \mathbf{d}_k) * \sum_{\mathbf{n}_k} \delta(\mathbf{x} - \mathbf{V}_k\mathbf{n}_k). \tag{2}$$

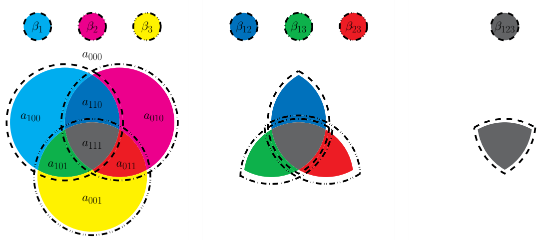
2.2. Spectral Neugebauer Model

On a color print, these multiple halftone image separations are overlaid, typically producing all possible 2^K overlay of the K colorants. The colors associated with each of the 2^K primaries are referred to as the *Neugebauer primaries*. Using the Yule-Nielsen-modified Neugebauer model [8], the average spectrum of the printed halftone is

$$R_{avg}(\lambda) = \left(\sum_{i=1}^{2^K} a_i R_i^{\frac{1}{\gamma}}(\lambda) \right)^{\gamma}, \qquad (3)$$

where a_i and $R_i(\lambda)$ are the fractional area coverage and the spectral reflectance of the i^{th} Neugebauer primary, respectively, and γ is the empirical Yule-Nielsen correction factor.

In order to obtain expressions for the Neugebauer primary areas, we first represent these areas in terms of an alternative but equivalent (in the sense that either is obtainable from the other) set of areas. For notational convenience, in this process, we index each of the 2^K possible combinations of the K colorants by a K-bit binary index string $\mathbf{c} = c_1 \ldots c_K$, where $c_k = 1$ indicates the presence of the k^{th} colorant and $c_k = 0$ its absence in the combination. We then denote by $\beta_{\mathbf{k}(\mathbf{c})}$ the total fractional area covered by all of the colorants present in $\mathbf{c}$, with $\mathbf{k}(\mathbf{c}) = k_1 \ldots k_m$ where the indices k_j denote the separation indices (arranged, say in ascending order for uniqueness). These are distinct from the Neugebauer primaries since the total fractional areas include areas that are also covered by additional colorants. Then it can be seen that [7] the (re-indexed) Neugebauer primary areas $\{a_{c_1 \ldots c_K}\}_{c_j \in \{0,1\}}$ are equivalent to the areas $\beta_{\mathbf{k}(\mathbf{c})}$. Fig. 1 illustrates the $a_{\mathbf{c}}$ and $\beta_{\mathbf{k}(\mathbf{c})}$ terms for the $K = 3$ case with CMY colorants in order to clarify the definitions of these terms.



(a) (b) (c)

Fig. 1. Relation between Neugebauer primary areas $a_{\mathbf{c}}$ and colorant overlap areas $\beta_{\mathbf{k}(\mathbf{c})}$. (a) shows $a_{\mathbf{c}}$ and the corresponding β_k terms, (b), (c) shows the $\beta_{\mathbf{k}(\mathbf{c})}$ terms inscribed by the outlined colors.

Due to the equivalence of the Neugebauer primary fractional areas in (3) to the areas $\beta_{\mathbf{k}(\mathbf{c})}$, invariance properties with respect to displacement misregistration established for one are applicable to the other. We therefore proceed by obtaining expressions for the areas $\beta_{\mathbf{k}(\mathbf{c})}$.

2.3. Fractional Areas of Colorant Combinations

In order to compute $\beta_{\mathbf{k}(\mathbf{c})}$, consider the overlay of the halftones that constitute $\mathbf{k}(\mathbf{c})$. The function

$$h_{\mathbf{k}(\mathbf{c})}(\mathbf{x}; \mathbf{d}_{\mathbf{k}(\mathbf{c})}) = \prod_{k \in \mathbf{k}(\mathbf{c})} h_k^{(\mathbf{d}_k)}(\mathbf{x}) \qquad (4)$$

indicates the spatial locations covered by the colorants $\mathbf{k}(\mathbf{c})$, taking a value 1 if $\mathbf{x}$ is covered by the colorants $\mathbf{k}(\mathbf{c})$ and 0 otherwise, where $\mathbf{d}_{\mathbf{k}(\mathbf{c})} = [\mathbf{d}_{k_1}, \ldots, \mathbf{d}_{k_m}]$ are the displacement vectors of the individual separations that constitutes $\mathbf{k}(\mathbf{c})$. Thus, $\beta_{\mathbf{k}(\mathbf{c})}$ is the *spatial average* of $h_{\mathbf{k}(\mathbf{c})}(\mathbf{x}; \mathbf{d}_{\mathbf{k}(\mathbf{c})})$.

If each of the matrices $\mathbf{V}_{k_i}^{-1}\mathbf{V}_{k_l}$ has only rational numbers as their elements $\forall\ k_i, k_l \in \mathbf{k}(\mathbf{c})$ the intersection of the lattices $\mathbf{\Lambda}_{\mathbf{k}(\mathbf{c})} = \bigcap_{k \in \mathbf{k}(\mathbf{c})} \mathbf{\Lambda}_k$, is a two-dimensional lattice [12]. The overlay $h_{\mathbf{k}(\mathbf{c})}(\mathbf{x}; \mathbf{d}_{\mathbf{k}(\mathbf{c})})$ of the constituent halftones in $\mathbf{k}(\mathbf{c})$ is then periodic over this lattice and thus the spatial average is obtained as

$$\beta_{\mathbf{k}(\mathbf{c})} = \frac{1}{|\mathcal{U}_{\mathbf{k}(\mathbf{c})}|} \int_{\mathbf{x} \in \mathcal{U}_{\mathbf{k}(\mathbf{c})}} h_{\mathbf{k}(\mathbf{c})}(\mathbf{x}) d\mathbf{x}, \qquad (5)$$

where $\mathcal{U}_{\mathbf{k}(\mathbf{c})}$ denotes the unit cell of $\mathbf{\Lambda}_{\mathbf{k}(\mathbf{c})}$ and $|\mathcal{U}_{\mathbf{k}(\mathbf{c})}|$ the area of this unit cell.

From the Fourier transform properties it follows that

$$\beta_{\mathbf{k}(\mathbf{c})} = H_{\mathbf{k}(\mathbf{c})}(\mathbf{0}), \qquad (6)$$

where $H_{\mathbf{k}(\mathbf{c})}(\mathbf{u})$ represents the Fourier transform of $h_{\mathbf{k}(\mathbf{c})}(\mathbf{x})$ computed over the lattice $\mathbf{\Lambda}_{\mathbf{k}(\mathbf{c})}$. In other words, $\beta_{\mathbf{k}(\mathbf{c})}$ is equal to the *d.c. term* of the frequency spectrum of the overlay of the constituent colorants in $\mathbf{k}(\mathbf{c})$. The Fourier transform of (4) yields

$$H_{\mathbf{k}(\mathbf{c})}(\mathbf{u}) = H_{k_1}^{(\mathbf{d}_{k_1})}(\mathbf{u}) * \ldots * H_{k_m}^{(\mathbf{d}_{k_m})}(\mathbf{u}), \qquad (7)$$

where $H_k^{(\mathbf{d}_k)}(\mathbf{u})$ represents the Fourier transform of the halftone image $h_k^{(\mathbf{d}_k)}(\mathbf{x})$ and $\mathbf{u} = [u, v]^T$ denotes the coordinates in frequency space. Let $S_k(\mathbf{u})$ represent the Fourier transform of the halftone spot function $s_k(\mathbf{x})$. Applying the shift and convolution property of the Fourier transform on (2), $H_k^{(\mathbf{d}_k)}(\mathbf{u})$ can be written as

$$H_k^{(\mathbf{d}_k)}(\mathbf{u}) = \frac{1}{|\,\mathbf{V}_k|} S_k(\mathbf{u}) \exp\left(-2\pi j \mathbf{d}_k^T \mathbf{u}\right) \sum_{\mathbf{n}_k} \delta(\mathbf{u} - \mathbf{W}_k \mathbf{n}_k), \qquad (8)$$

where the Fourier transform of the comb function $\sum_{\mathbf{n}_k} \delta(\mathbf{x} - \mathbf{V}_k \mathbf{n}_k)$ takes non-zero values on the elements of the *reciprocal lattice* of $\mathbf{\Lambda}_k$ [11, pp. 23-24][12], which is represented by

$$\mathbf{\Lambda}_k^* = \{\mathbf{W}_k \mathbf{n}_k = (\mathbf{V}_k^{-1})^T \mathbf{n}_k \mid \mathbf{n}_k \in \mathbb{Z}^2\}. \qquad (9)$$

Using these results we can write $H_{\mathbf{k}(\mathbf{c})}(\mathbf{u})$ as in (10), which can only take non-zero values if $\mathbf{u} = \sum_{k \in \mathbf{k}(\mathbf{c})} \mathbf{W}_k \mathbf{n}_k$. Let $\mathcal{N}_{\mathbf{k}(\mathbf{c})}$ represent the set $\{\mathbf{n}_{k_1}, \ldots, \mathbf{n}_{k_m} | \sum_{k \in \mathbf{k}(\mathbf{c})} \mathbf{W}_k \mathbf{n}_k = \mathbf{0}\}$, which includes the indices of all the frequency vectors satisfying the *singularity* condition [5]. Then, (6) can be rewritten as shown in (11), where $\mathcal{N}_{\mathbf{k}(\mathbf{c})} \backslash \{\mathbf{0}\}$ denotes the elements in $\mathcal{N}_{\mathbf{k}(\mathbf{c})}$ with the exclusive of the all zero vector $\mathbf{0}$. Note that only addends indexed by variables of the summation symbol in (11) depend on the inter-separation misregistration amounts.

3. CONDITIONS FOR COLOR MISREGISTRATION SENSITIVITY

Denote by $R_{avg}^{(\mathbf{0})}$ and $R_{avg}^{(\mathbf{d})}$ the average spectrum of "prints" with inter-separation displacements $\mathbf{0}$ (perfect registration) and $\mathbf{d} = [\mathbf{d}_1, \ldots, \mathbf{d}_K]$ (misregistered), respectively. A difference in these terms represents a misregistration induced color shift[1]. In this section we consider the conditions under which these terms differ, producing sensitivity to color misregistration in the average color. In (3), there are two factors that affect the value of these terms:

- Spectral reflectances of the Neugebauer primaries,
- Fractional area coverages of the Neugebauer primaries.

[1] Strictly speaking this is a shift in average spectrum that will typically produce a corresponding color shift.

$$H_{\mathbf{k}(\mathbf{c})}(\mathbf{u}) = \frac{1}{\prod_{k \in \mathbf{k}(\mathbf{c})} \mid \mathbf{V}_k \mid} \sum_{\mathbf{n}_{k_1}} \cdots \sum_{\mathbf{n}_{k_m}} \prod_{k \in \mathbf{k}(\mathbf{c})} S_k(\mathbf{W}_k \mathbf{n}_k) \exp\left(-2\pi j \mathbf{d}_k^T \mathbf{W}_k \mathbf{n}_k\right) \delta\left(\mathbf{u} - \sum_{l \in \mathbf{k}(\mathbf{c})} \mathbf{W}_l \mathbf{n}_l\right) \tag{10}$$

$$\beta_{\mathbf{k}(\mathbf{c})} = H_{\mathbf{k}(\mathbf{c})}(\mathbf{0}) = \frac{1}{\prod_{k \in \mathbf{k}(\mathbf{c})} \mid \mathbf{V}_k \mid} \left(\prod_{k \in \mathbf{k}(\mathbf{c})} S_k(\mathbf{0}) + \sum_{(\mathbf{n}_{k_1}, \ldots, \mathbf{n}_{k_m}) \in \mathcal{N}_{\mathbf{k}(\mathbf{c})} \setminus \{\mathbf{0}\}} \prod_{k \in \mathbf{k}(\mathbf{c})} S_k(\mathbf{W}_k \mathbf{n}_k) \exp\left(-2\pi j \mathbf{d}_k^T \mathbf{W}_k \mathbf{n}_k\right) \right) \tag{11}$$

The former is affected by the spectral interactions of the colorants (inks) in their absorption bands of the spectra, the latter is a function of individual halftone separation periodicities, halftone spots and the inter-separations misregistration as shown in (11). These terms define conditions under which R_{avg} is affected by inter-separation misregistration as we show in the next sections. We also observe here that the terms $\{\beta_k\}_{k=1}^{K}$, corresponding to the total fractional area covered by the separations are independent of misregistration.

3.1. Spectral Sufficiency Condition

If the colorants are transparent with non-overlapping spectral absorption bands, it is well known that the resulting color prints have no misregistration sensitivity [2]. In our analysis this can be seen by noting that in this scenario, one can represent the reflectance of a Neugebauer primary formed by the colorants in $\mathbf{k}(\mathbf{c})$ as $R_{\mathbf{k}(\mathbf{c})}(\lambda) = R_P(\lambda) \prod_{k \in \mathbf{k}(\mathbf{c})} (1 - A_k(\lambda))$, where $R_P(\lambda)$ is the reflectance of the paper substrate and $A_k(\lambda)$ is the absorptance of the k^{th} colorant. For a given wavelength λ, then there exists a single separation k_λ for which the colorant has non-zero absorption at λ. Using this property, the average reflectance at λ can be written as

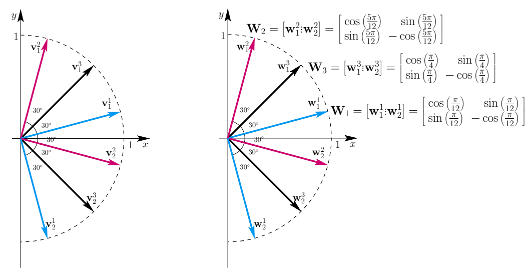
$$R(\lambda) = \left(\beta_{k_\lambda} (1 - A_{k_\lambda})^{\frac{1}{\gamma}} + (1 - \beta_{k_\lambda})\right)^\gamma R_P(\lambda), \tag{12}$$

Since the β_k terms corresponding to individual colorant separation fractional area coverages are insensitive to misregistration, $R(\lambda)$ is not affected by the inter-separation misregistration. Thus, a color halftone is insensitive to inter-separation misregistration if the aforementioned condition is satisfied. However, in most color printing systems the colorants do not obey this condition (for instance, any pair of colorants including the Black (K) colorant violates this requirement).

3.2. Periodicity Sufficiency Condition

In Sec. 2.3 we show how the fractional areas of colorant combinations can be computed by using (11). If $\mathcal{N}_{\mathbf{k}(\mathbf{c})} \setminus \{\mathbf{0}\}$ is an empty set for every possible colorant combination $\mathbf{k}(\mathbf{c})$, corresponding $\beta_{\mathbf{k}(\mathbf{c})}$ terms do not depend on the displacements $\{\mathbf{d}_k\}_{k \in \mathbf{k}(\mathbf{c})}$. Considering the equivalence between the Neugebauer primary areas and areas of colorant combinations, this defines a sufficient condition to ensure the Neugebauer primary areas $a_{c_1 \ldots c_K}$ and, thus, the average spectrum of the printed halftone $R_{avg}(\lambda)$ is invariant to inter-separation misregistration. This condition is termed the non-singularity condition in [5].

Let us visualize this case by examining the conventional 30° angular separation equi-frequency C, M, K halftones, commonly used in lithographic printing systems. Let $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$, which are formed by the basis vectors shown on the unit circle in Fig. 2(a), represent the basis matrices for the lattices $\mathbf{\Lambda}_1$, $\mathbf{\Lambda}_2$ and $\mathbf{\Lambda}_3$ for C, M and K separations, respectively. We first consider overlay of any two of these separations- for example C and M. From the frequency domain basis matrices $\mathbf{W}_1, \mathbf{W}_2$, shown in Fig. 2(b), for the halftone



(a) (b)

Fig. 2. Basis vectors in (a) spatial domain and (b) frequency domain for conventional C, M, K screens.

separations 1 and 2, it can be readily seen that $\nexists\ \mathbf{n}_1, \mathbf{n}_2 \in \mathbb{Z}^2$ that can satisfy $\mathbf{W}_1\mathbf{n}_1 + \mathbf{W}_2\mathbf{n}_2 = \mathbf{0}$. Thus, $\mathcal{N}_{12} \setminus \{\mathbf{0}\}$ is an empty set, i.e. the overlay is non-singular, and β_{12} is invariant to inter-separation misregistration. Similarly, it can be shown that $\mathcal{N}_{13} \setminus \{\mathbf{0}\}$ and $\mathcal{N}_{23} \setminus \{\mathbf{0}\}$ are also empty sets and consequently β_{13} and β_{23} are also invariant to inter-separation misregistration. Note that, in this case $\beta_{\mathbf{k}(\mathbf{c})}$ terms are only determined by the first addend in (11), which is the multiplication of the d.c. terms in each individual separation that constitutes $\mathbf{k}(\mathbf{c})$ and, therefore, the statistical randomness condition assumed by Demichel equations [13] is satisfied.

Next consider the overlay of all three separations. For this case, we can see that $\mathcal{N}_{123} \setminus \{\mathbf{0}\}$ is not an empty set and the overlay is singular. For example, $\mathbf{n}_1 = [-1, 0]^T$, $\mathbf{n}_2 = [1, 0]^T$ and $\mathbf{n}_3 = [0, 1]^T$ is a member of $\mathcal{N}_{123} \setminus \{\mathbf{0}\}$. Thus, $\mathcal{N}_{123} \setminus \{\mathbf{0}\}$ has infinite number of elements and the value β_{123} now not only depends on the halftone spots but also the inter-separation misregistrations.

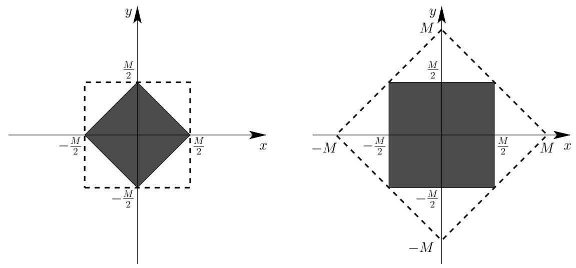
Thus, a sufficient condition that ensures $\beta_{\mathbf{k}(\mathbf{c})}$ is insensitive to inter-separation misregistration is emptiness of the set $\mathcal{N}_{\mathbf{k}(\mathbf{c})}$ which is equivalent to the non-singularity condition of [5]. In practice for typical digital printing devices, the addressable device locations are confined to a rectilinear grid, and the x and y coordinates of the vectors $\mathbf{v}_1^k, \mathbf{v}_2^k$ are constrained to take values that are multiples of the corresponding grid spacing along these two directions. Consequently, for digital printing systems the frequency domain lattices are singular and misregistration invariance cannot be assured based on halftone periodicities.

3.3. Spot Function Dependence

We observe that if the set $\mathcal{N}_{\mathbf{k}(\mathbf{c})} \setminus \{\mathbf{0}\}$ is non-empty, this alone does not ensure that $R_{avg}^{(\mathbf{d})}$ and $R_{avg}^{(\mathbf{0})}$ differ, since $\beta_{\mathbf{k}(\mathbf{c})}$ terms are functions of the constituent halftone spots and the inter-separation misregistration amounts. Depending on these, inter-separation misregistration may still have no effect on the average color of the halftone if the summation in (11) is zero.

To illustrate this, consider the overlay of two separations with lattices $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ with basis matrices $\mathbf{V}_1 = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}$ and

$\mathbf{V}_2 = \begin{bmatrix} M & M \\ M & -M \end{bmatrix}$, respectively. Define the corresponding halftone spot functions $s_1(\mathbf{x})$ and $s_2(\mathbf{x})$ as shown in Fig. (3) within the unit cells of the constituent lattices outlined by the dashed lines, respectively.



(a) $s_1(\mathbf{x})$ (b) $s_2(\mathbf{x})$

Fig. 3.

Then we see that the terms in the summation in (11), are computed at the frequency coordinates $\mathbf{W}_1\mathbf{n}_1$ and $\mathbf{W}_2\mathbf{n}_2$ for $\{\mathbf{n}_1, \mathbf{n}_2\} \in \mathcal{N}_{12}\backslash\{\mathbf{0}\}$ as

$$S_1(\mathbf{W}_1\mathbf{n}_1) = \frac{M^2}{2}\text{sinc}\left(\frac{n_{1x}-n_{1y}}{2}\right)\text{sinc}\left(\frac{n_{1x}+n_{1y}}{2}\right), \quad (13)$$

$$S_2(\mathbf{W}_2\mathbf{n}_2) = M^2\text{sinc}\left(\frac{n_{2x}+n_{2y}}{2}\right)\text{sinc}\left(\frac{n_{2x}-n_{2y}}{2}\right). \quad (14)$$

In order to compute β_{12}, we should find the indices $\mathbf{n}_1$ and $\mathbf{n}_2$ for which

$$\mathbf{W}_1\mathbf{n}_1 + \mathbf{W}_2\mathbf{n}_2 = \frac{1}{M}\begin{bmatrix} n_{1x} + \frac{n_{2x}+n_{2y}}{2} \\ n_{1y} + \frac{n_{2x}-n_{2y}}{2} \end{bmatrix} = \mathbf{0}. \quad (15)$$

The above relation implies that only indices such that both n_{2x} or n_{2y} are odd or even can contribute terms in $\mathcal{N}_{12}\backslash\{\mathbf{0}\}$. However, in these cases, the value of the sinc functions in (14) is zero. This ensures β_{12} is the multiplication of the d.c. terms in each individual separations. Therefore, the characteristics of the halftone spots can define a condition to ensure that R_{avg} is insensitive to inter-separation misregistration.

3.4. Invariant Misregistrations

It is readily seen from (11) that if the displacement of the k^{th} separation corresponds to a point on the corresponding lattice, the term $\mathbf{d}_k^T\mathbf{W}_k\mathbf{n}_k$ is integer valued and the result of the summation is identical to that for no displacement. This represents the trivial case when the inter-separation displacements are matched to the separations' lattice periodicities. Invariance is, however, also achievable for non-trivial displacements, as we illustrate next.

Consider, again, the overlay of two lattices $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$. The value of (11) depends on the multiplicative term $\exp\left(-2\pi j(\mathbf{d}_1^T\mathbf{W}_1\mathbf{n}_1 + \mathbf{d}_2^T\mathbf{W}_2\mathbf{n}_2)\right)$. If this term is equal to identity $\forall\ (\mathbf{n}_1, \mathbf{n}_2) \in \mathcal{N}_{12}\backslash\{\mathbf{0}\}$, then β_{12} is same as the one computed for the perfectly registered separations.

Suppose $\mathbf{n}_1, \mathbf{n}_2$ are integers satisfying $\mathbf{W}_1\mathbf{n}_1 + \mathbf{W}_2\mathbf{n}_2 = \mathbf{0}$, then $\mathbf{W}_2\mathbf{n}_2 = -\mathbf{W}_1\mathbf{n}_1$. If $(\mathbf{d}_1 - \mathbf{d}_2)^T\mathbf{W}_1\mathbf{n}_1 \in \mathbb{Z}$, then the value of the exponential term is always equal to 1. Now if $\mathbf{W}_2\mathbf{n}_2 \in \mathbf{\Lambda}_2^*$ and $-\mathbf{W}_1\mathbf{n}_1 \in \mathbf{\Lambda}_1^*$, hence $\mathbf{w} = \mathbf{W}_2\mathbf{n}_2 = -\mathbf{W}_1\mathbf{n}_1 \in \mathbf{\Lambda}_1^* \cap \mathbf{\Lambda}_2^*$ and as vice versa if $\mathbf{w} \in \mathbf{\Lambda}_1^* \cap \mathbf{\Lambda}_2^*$, then $\exists\ \mathbf{w} = \mathbf{W}_1\mathbf{n}_1' = -\mathbf{W}_2\mathbf{n}_2'$ for some $\mathbf{n}_1', \mathbf{n}_2' \in \mathbb{Z}$ and $\mathbf{W}_1\mathbf{n}_1' + \mathbf{W}_2\mathbf{n}_2' = \mathbf{0}$. From the definition of the reciprocal lattice it follows that if $\mathbf{d}_2 - \mathbf{d}_1$ is an element of $(\mathbf{\Lambda}_1^* \cap \mathbf{\Lambda}_2^*)^*$, i.e. the reciprocal lattice of $\mathbf{\Lambda}_1^* \cap \mathbf{\Lambda}_2^*$, then $(\mathbf{d}_1 - \mathbf{d}_2)^T\mathbf{W}_1\mathbf{n}_1$ is always an integer and thus $\exp\left(-2\pi j(\mathbf{d}_1 - \mathbf{d}_2)^T\mathbf{W}_1\mathbf{n}_1\right)$ is always equal to 1. Since the reciprocal lattice of $\mathbf{\Lambda}_1^* \cap \mathbf{\Lambda}_2^*$ is $\mathbf{\Lambda}_1 + \mathbf{\Lambda}_2$, $\mathbf{d}_1 - \mathbf{d}_2 \in \mathbf{\Lambda}_1 + \mathbf{\Lambda}_2$ defines a sufficient condition to ensure β_{12} is invariant to misregistration. In the general case with K colorants, all inter-separation displacements should be of form $\mathbf{d}_{k_i} - \mathbf{d}_{k_l} \in \mathbf{\Lambda}_{k_i} + \mathbf{\Lambda}_{k_l}$ for $\forall\ k_i, k_l \in \{1, \ldots, K\}$ to ensure the overlay is insensitive to inter-separation misregistration.

4. CONCLUSION

The mathematical analysis presented in this paper characterizes the color sensitivity of clustered-dot halftones to inter-separation misregistration. The comprehensive model presented here not only validates known scenarios under which the average color is invariant to inter-separation misregistration but also illustrates new situations in which this invariance is obtained for suitable choices of halftone spot shapes and displacements. This formulation can be applied to the selection of misregistration insensitive geometries and halftone spot functions.

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