NONLINEAR POISSON IMAGE COMPLETION USING COLOR MANIFOLD

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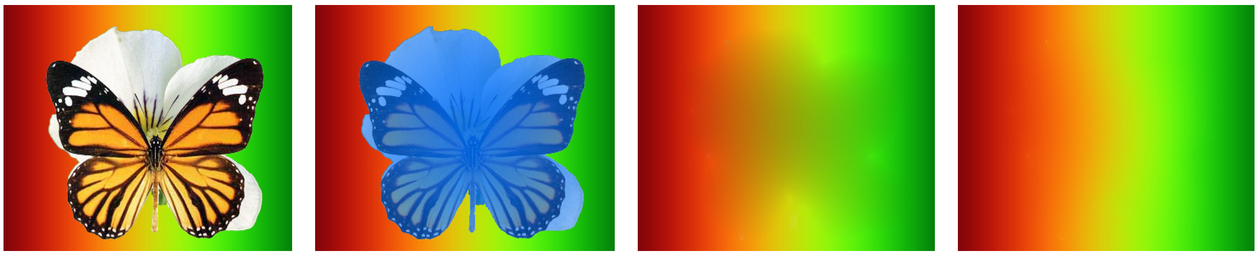

Fig. 1. From left to right: the input image, the object to be removed (unknown region), the completion result of ordinary Poisson interpolation, and the completion result of the proposed method.

ABSTRACT

While most image completion methods focus on filling regions with structures or stationary textures, few are suitable for completing large-scale missing parts on complex background with nonlinearly progressive color changes. In this paper, we propose a novel approach, termed as *nonlinear Poisson completion*, to solve this problem. The visible parts of the background serve as a training set, from which we learn the embedding nonlinear subspace of progressive colors, namely *color manifold*. A Poisson image completion procedure, which works efficiently for smoothly linear interpolation, is extended to nonlinearly recover the missing regions with iteration solution confined to the manifold. In some especially challenging cases, a simple post-processing serves to generate more natural-looking results. Experiments on both synthetic and real images verify the effectiveness of the proposed algorithm.

Index Terms— Image processing, image analysis, image restoration, interpolation

1. INTRODUCTION

Image completion/inpainting is an important tool for many image editing tasks, such as image restoration, removal of selected objects and etc.. The goals are to infer the unknown regions based on observed visual information, generating convincingly natural-looking results. However, we cannot expect one method to address all problems due to the inherent ambiguity of image completion from a single image. Designing different completion schemes according to different kinds of situations is a more reasonable and feasible choice.

One of the two major categories of previous inpainting works is based on partial differential equations (PDEs). Originated by [1], there are many derived variations [2, 3]. They are mainly applicable for small gaps and thin missing structures, e.g. edges and contours. The other category using texture synthesis [4, 5] can handle thick unknown regions with structured or stationary textures. This precondition and high running time restrict its applications.

However, the large-scale image completion tasks on backgrounds with complexly progressive colors (see Fig. 1), which are widely seen in nature and art photos, remain very challenging to the traditional methods mentioned above. Without obvious cues of structures or stationary textures, those methods suffer from the over-smoothing effects and computational inefficiency.

In this paper, we propose a novel method to solve this problem. Inspired by the points of view from machine learning and its graphic application [6], this work rests on the observation that complexly progressive colors lie in an embedding low-dimensional subspace in nature, termed as *color manifold,* which can be learned from the visible parts. With classic manifold learning algorithm, e.g. Isomap [7], we can find a Euclidean space describing the color manifold. Thus, linearly smooth interpolation in such a description space amount to completing unknown regions in a nonlinear manner in RGB space. As an effective linear interpolation tool, Poisson editing [8] is extended to this new scenario, interpolating the description values and producing smooth completion of nonlinearly progressive colors. That is why *nonlinear Poisson completion* is named. In some especially

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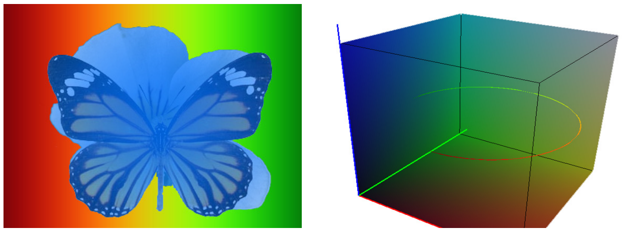


Fig. 2. Left: training samples; right: the neighborhood graph, revealing the embedding 1-dim curve of progressive colors.

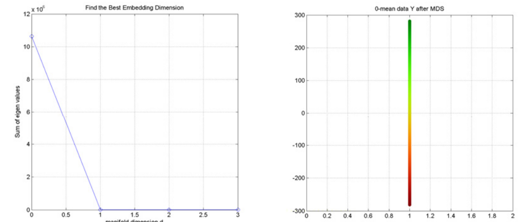


Fig. 3. Left: d-residue curve, bottoming at d = 1; right: the projective coordinates Y in the Euclidean description space.

difficult cases, perceivable discontinuous patterns exist in completion borders due to backward mapping errors. A simple and fast post-processing can assist in achieving better seamless blending.

With deeply exploiting global information, our method solves a local problem effectively and efficiently, which inspires a general intelligent image completion framework based on image analysis. The following sections will detail the algorithms above and demonstrate experimental results.

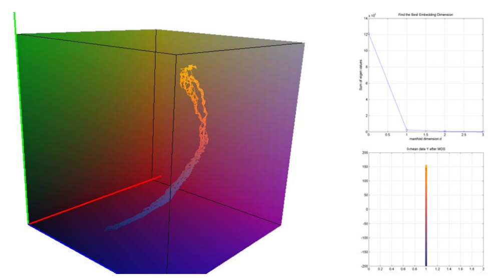
2. LEARNING COLOR MANIFOLD

With selected known pixels as a training set, we can build a color manifold by constructing a neighborhood graph for their RGB values. Like [7], each point is connected to its k-nearest neighbors and then outliers in the graph are pruned by a length threshold ε. From the structure of the neighborhood graph, we can compute the shortest path between given two points with Dijkstra algorithm. The path length is used as the interpoint geodesic distance and all geodesic distances constitute a matrix D_g, reflecting the underlying nonlinear low-dimensional geometry of the color manifold. Fig. 2 illustrates the construction of neighborhood graph.

For further computational convenience, a parametric form of the data mapped into the manifold need be obtained. Applying classical MDS to D_g will suffice, as Isomap [7]. Given observed samples $X = \{x_i\}$, MDS finds a *description space*, namely a low-dimensional Euclidean space, where the projective coordinates $Y = \{y_i\}$ best maintain the interpoint relationship of the color manifold. By converting D_g to inner product matrix $B = X^TX$, finding Y is equivalent to minimizing the function min $E(Y)$ = min$\|B - B'\|_F^2$ = min$\|B - Y^TY\|_F^2$, s.t. rank(Y) = rank(y_i) = d < 3. The global optimum is achieved when the coordinates Y is set as $Y = \Lambda_d^{1/2}U_d$, where Λ_d consists of the top d eigenvalues of B and U_d comprises corresponding normalized eigenvectors.

Note that actually we do not know the embedding low dimension d before performing MDS, computing the best d is a foremost problem. From matrix analysis theory, the residue of dimension reduction is the sum of eigenvalues excluded by Λ_d. Thus, we can plot d-residue curve and find the best d at the elbow point. Fig. 3 explains this method for the neighborhood graph shown in Fig. 2, and displays the

Left: the input image; top-right: the object to be removed; down-right: the selected training set.



Building the neighborhood graph and finding the best d.

Fig. 4. Another example of color manifold learning.

obtained projective coordinates Y in d-dim description space. Another example of color manifold learning for a nature photo is showed in Fig. 4. The discovered embedding subspace reflects intrinsic structures of the progressive colors, whose projective coordinates facilitate further operation.

3. NONLINEAR POISSON COMPLETION

After finding the Euclidean description space, linearly smooth interpolation for the projective coordinates of progressive colors is equivalent to nonlinearly smooth interpolation confined in the manifold. Consider that Poisson editing [8] is a powerful tool for linear interpolation directly in RGB channels. Poisson interpolation can be defined as an optimization problem that seeks to recover missing interior pixel values $f^*(D)$ in the smoothest manner, given unknown region D, its border ∂D and border values $f(\partial D)$. It can be formulized like:

$$\min_f \iint_D |\nabla f|^2, \quad s.t.\ f^*(\partial D) = f(\partial D) \qquad (1)$$

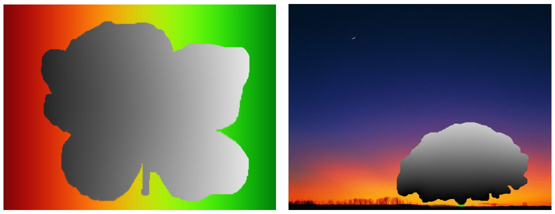


Fig. 5. Poisson interpolation results for d-dim projective coordinates over unknown regions. For visualization, black denotes normalized coordinate 0, and white 255.

The optimal solution is achieved when Euler-Lagrange equation with Dirichlet boundary condition is satisfied:

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \text{ over } D, \ s.t. \ f^*(\partial D) = f(\partial D) \qquad (2)$$

The form (2) is one kind of Poisson equation that can be discretized and solved by Gauss-Seidel iteration method with high computational efficiency.

As one linear interpolation method, direct Poisson completion in three working channels (RGB or Lab say) will result in serious blurring artifacts for large-scale loss of nonlinearly progressive colors, refer to Fig. 1. Instead, if we can map the pixels of D (including ∂D) into description space, performing the Poisson interpolation for their projective coordinates will be equivalent to nonlinearly completing D with progressive colors. Fig. 5 demonstrates two Poisson interpolation results of projective coordinates. Except a few cases, the computation is rather efficient because of the dimension reduction.

In practice, the projective coordinates are normalized to 0~255 for consistency with ordinary Poisson method. In the case that the missing region connects non-progressive parts somewhere, e.g. the root of the tree in Fig. 4, a simple manual structure propagation like [9] or a local PDE-based method can be employed first.

Post-processing When the results of projective coordinates are mapped backward to RGB space, some errors may occur and result in perceivable discontinuous patterns along the border ∂D in some challenging cases. Seamless blending can be achieved by morphologically dilating ∂D to form an expanded border, in which the Poisson completion in RGB channels is performed. Fig. 6 highlights a local region of the completion result before and after post-processing.

4. EXPERIMENTAL RESULTS

Besides the synthetic test image Fig. 1, the proposed nonlinear Poisson completion algorithm is also verified on several real nature images, as shown in Fig. 7 and Fig. 8. All the examples use images available from public sources over the Internet. During color manifold learning, we select some 1500 points in the training set and choose $k = 8$ for k-nearest connecting in neighborhood graph construction. For solving Poisson equation, the upper limit of iteration times is set to 10000 and the convergence threshold η is set to 0.01.

Fig. 6. Post-processing. From left to right: discontinuous pattern along the border after backward mapping; Dilation of the border; seamless blending result.

As seen from the results, ordinary Poisson completion in RGB space results in unpleasing blur effects and hardly converges in limited iteration times. In contrast, our proposed method produces very natural-looking results (see Fig. 1, 7 and 8) and is verified computationally efficient. Because the iterations are confined to embedding subspaces, the times to achieve convergence are greatly saved. Besides, the number of working channels are reduced to d with $d<3$. Including color manifold learning, the nonlinear Poisson completion algorithm can be completed in less than 1 minute for our test images, using non-optimized C++ code running on a CPU Pentium2.4GHz PC.

5. CONCLUSION

We proposed a novel nonlinear Poisson completion scheme to address the large-scale inpainting tasks on backgrounds with complexly progressive colors, which are challenging to traditional methods. With learning an underlying low-dimensional color manifold from training samples, nonlinearly smooth interpolation over missing regions can be achieved by solving a Poisson equation for the projective coordinates of original colors. Visually pleasing results are generated and the computation is also efficient.

The proposed scheme inspires a general framework of “learning globally, solving locally” procedures. Borrowing information from pictures at other times or other views, we can expect to solve difficult problems in immediate images. Extending the method to progressive non-stationary texture images is our future work.

6. ACKNOWLEDGMENT

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6. REFERENCES

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Fig. 7. Top-left: the input image; top-right: the object to be removed; down-left: the result by ordinary Poisson completion in RGB channels; down-right: the result by nonlinear Poisson completion; down-middle: the highlight of local comparison.

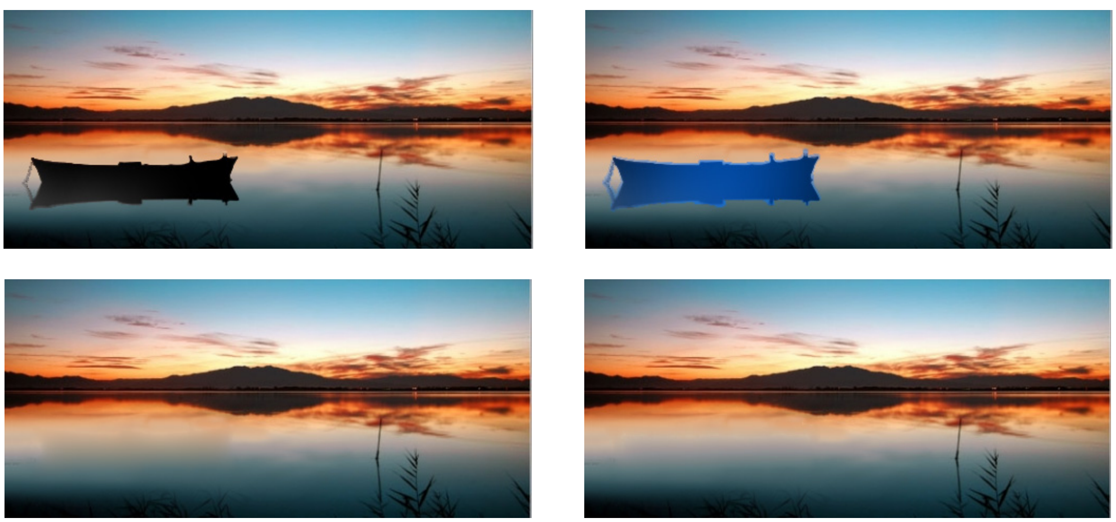

Fig. 8. Another example. Top-left: the input image; top-right: the object to be removed; down-left: the result by ordinary Poisson completion in RGB channels; down-right: the result by nonlinear Poisson completion.

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