ON THE 2D TEAGER-KAISER OPERATOR

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ABSTRACT

The Teager-Kaiser operator is a discrete version of Teager's energy operator, advanced about 16 years ago. It is a filter of the moving window type and is commonly used as an estimator of the local energy contents of a signal; it is also used as a contrast enhancer of gray level images. We state some properties of a 2D version of the operator and its responses to common images. We characterize some of its root and preconstant images, and consider the case of separable images.

Index Terms**—** Image enhancement, nonlinear filters.

1. INTRODUCTION

The Teager-Kaiser operator is a discrete nonlinear filter of the moving-window type. Nonlinear filters solve problems that are unsolvable by the use of convolution filters alone; thus, the median filter preserves sustained edges while eliminating isolated spikes, both types of signal having overlapping spectra. The one dimensional (1D) Teager-Kaiser operator [1] appeared as a discrete version of the Teager energy operator which was proposed as an estimator of the energy spent by a sinusoidal oscillator. The 1D Teager-Kaiser operator relates each input signal $x \in R^Z$ with the corresponding output signal $y \in R^Z$ via

$$y_n = x_n^2 - x_{n+1}x_{n-1} \qquad (1)$$

Constant and exponential signals are *pre-null* signals of the Teager-Kaiser operator; that is, they produce the zero signal [3]. Sinusoidal, hyperbolic and linear signals produce constant signals and we call them *preconstant signals* for the operator [5]. The signals which pass unaltered through the operator are called *root signals* for the operator.

The two dimensional (2D) version of the Teager-Kaiser operator (TK for short) has been used as a component of a contrast enhancer [2]. We study some of its properties and responses to common images; this gives some light about the behavior of the operator. We study specific cases of root, preconstant and pre-null images of the TK operator.

In Section 2, we give an initial characterization of the TK operator in terms of its responses to particular signals and briefly discuss some of its properties. In Section 3 we consider the pre-constant signals of the operator and, as a particular case, the pre-null signals. In Section 4 we consider the root images of the operator. In Section 5 we study the response of the operator to separable images. The paper is concluded in Section 6.

2. DEFINITION AND RESPONSES TO COMMON SIGNALS

Let an image be a 2D real signal, i.e. an element of $R^{Z\times Z}$. The Two-Dimensional Teager-Kaiser operator is the function TK: $R^{Z\times Z} \rightarrow R^{Z\times Z}$ that maps images into images via

$$\mathrm{TK}(x)_{m,n} = 2x_{m,n}^2 - x_{m,n-1}x_{m,n+1} - x_{m-1,n}x_{m+1,n} \qquad (2)$$

where *m* and *n* stand for rows and columns, respectively. For the visualization of images, we paint a zero with the medium gray, negative values with darker grays and positive values with lighter grays. We also write $\mathrm{TK}(x)=\{2x^2_{m,n}\}-\{x_{m,n-1}\}\{x_{m,n+1}\}-\{x_{m-1,n}\}\times\{x_{m+1,n}\}$; and, with the convention that ${}_{k,l}a_{m,n}$ means $a_{m-k,n-l}$ (two pre-sub-indexes indicating a shift in rows and columns) and the understanding that the product of images is pointwise, we write, in compact form $\mathrm{TK}(x) = 2x^2 - ({}_{0,1}x)({}_{0,-1}x) - ({}_{1,0}x)({}_{-1,0}x)$. Clearly, TK= TKR+TKC, where TKR and TKC, respectively, are the 1D Teager-Kaiser operators applied by columns and by rows, respectively, i.e.:

$$\mathrm{TKR}(x) = x^2 - ({}_{0,1}x)({}_{0,-1}x)$$
$$\mathrm{TKC}(x) = x^2 - ({}_{1,0}x)({}_{-1,0}x)$$

The TK operator does not obey neither superposition nor homogeneity; nevertheless, it obeys what we call square homogeneity. We say that an operator *O* is square homogeneous if, for each constant *c*, and each signal *s*, $O(cs)=c^2O(s)$.

2.1. Properties

Response to a sum of images: if *x* and *y* are images, then

$$\mathrm{TK}(x+y) = \mathrm{TK}(x) + \mathrm{TK}(y) + 4xy - ({}_{0,-1}x)({}_{0,1}y) - ({}_{0,1}x)({}_{0,-1}y) - ({}_{-1,0}x)({}_{1,0}y) - ({}_{1,0}x)({}_{-1,0}y)$$

In particular, if *y* is a constant image of value *d*, the output is

$$\mathrm{TK}(x+d) = \mathrm{TK}(x) + d(4xy - {}_{0,-1}x - {}_{0,1}x - {}_{-1,0}x - {}_{1,0}x) \qquad \textbf{(3)}$$

Response to a product of images: if *x* and *y* are images, then

$$\mathrm{TK}(xy) = x^2\mathrm{TK}(y) + y^2\mathrm{TK}(x) - \mathrm{TKR}(x)\mathrm{TKR}(y) - \mathrm{TKC}(x)\mathrm{TKC}(y)$$

Invariance: let $\{y_{m,n}\}=\mathrm{TK}(\{x_{m,n}\})$, then $\mathrm{TK}(\{x_{m,-n}\}) = \{y_{m,-n}\}$, $\mathrm{TK}(\{x_{-m,n}\}) = \{y_{-m,n}\}$ and $\mathrm{TK}(\{x_{-m,-n}\}) = \{y_{-m,-n}\}$.

2.2. Responses to common images

Constant image: the response is the null image.

Linear image: the response to an image $\{\alpha m+\beta n+\gamma\}$, where α, β and γ are constants, is the constant image $\{\alpha^2+\beta^2\}$.

Impulse response: The response to a 2D discrete impulse $\{\delta_{m,n}\}$ is the image 2δ (the 2D discrete impulse is given by $\delta_{m,n}=1$ if $m=n=0$ and $\delta_{m,n}=0$ otherwise.)

Sinusoidal responses: The response to an image $\{\sin(\omega_1 m+\omega_2 n+\varphi)\}$ is the constant image $\{\sin^2\omega_1+\sin^2\omega_2\}$. Let $x=\{\sin(\omega_1 m+\varphi_1)\}$ and $z=\{\sin(\omega_2 n+\varphi_2)\}$, then the response to the image xz is the image $x^2\sin^2\omega_2 + z^2\sin^2\omega_1$. Finally, the response to the linear combination $Ax+Bz$ is the image $2(2\text{-}\cos\omega_1\text{-}\cos\omega_2)\times ABxz+A^2\sin^2\omega_1+B^2\sin^2\omega_2$, so the response of the operator to a sum of sinusoidal images is their scaled product on a DC level.

Exponential responses: The images $\{e^{\alpha mn}\}$ and $\{e^{\alpha m+\beta n}\}$ are pre-null images.

*Hyperbolic response***:** The response to the images $\{\sinh(\alpha m+\beta n+\gamma)\}$ and $\{\cosh(\alpha m+\beta n+\gamma)\}$ are the constants images $0.5[\cosh(2\alpha)+\cosh(2\beta)-2]$ and $0.5[2-\cosh(2\alpha)-\cosh(2\beta)]$.

3. PRECONSTANT AND PRE-NULL IMAGES

An image that generates a constant output when it is applied to the TK operator is called a *preconstant image*, if the constant is zero we have a *pre-null image*.

In [5], the preconstant and pre-null signals of the 1D TK operator were wholly characterized. They were classified into two groups as determinate and non-determinate signals; all pre-null signals are determinate. The determinate signals are governed by rational difference equations and solve linear difference equations.

We have found that it is possible to get a preconstant image starting from a determinate preconstant signal. Let x be a 1D preconstant signal, we show below that we obtain a preconstant image by doing a linear combination of the arguments m and n of the image in the 1D argument of x.

Lemma 1. If $\{x_n\}$ is a determinate preconstant 1D signal then $\{x_{\delta n+\tau}\}$, with δ and τ integer constants, is a preconstant signal.

Proof. Since the 1D TK operator is invariant it is enough to demonstrate that $\{x_{\delta n}\}$ is a preconstant signal. Since $\{x_n\}$ is a determinate preconstant signal, for some real $B \neq 0$ it satisfies [5]

$$x_{n+1} + Bx_n + x_{n-1} = 0 \qquad \textbf{(4)}$$

The signal $\{x_{\delta n}\}$ is preconstant if for some real $\beta(\delta)$ (β is a real function of δ) the signal is a solution of the equation $x_{\delta(n+1)}+\beta(\delta)x_{\delta n} + x_{\delta(n\text{-}1)} = 0$. More generally, we will check using induction that $x_{n+\delta}+ \beta(\delta)x_n+ x_{n\text{-}\delta} = 0$. For $\delta=1$ we have $\beta(1)=B$. For $\delta=2$, we can express x_{n+1} and $x_{n\text{-}1}$ using shifted versions of (4) and we have $x_{n+2} + (2\text{-}B^2)x_n + x_{n\text{-}2} = 0$. The signal $\{x_{2n}\}$, is preconstant with $\beta(2)=2\text{-}B^2$. Now if $\{x_{(\delta\text{-}1)n}\}$ is preconstant ($\delta\geq 3$), so it obeys $x_{n+(\delta\text{-}1)} + \beta(\delta\text{-}1)x_n + x_{n\text{-}(\delta\text{-}1)} = 0$. Using (4), we rewrite the last equation as

$$-x_{n+\delta} - x_{n+(\delta-2)} + \mathrm{B}\beta(\delta-1)x_n - x_{n-(\delta-2)} - x_{n-\delta} = 0$$

and knowing that $x_{n+(\delta\text{-}2)} + x_{n\text{-}(\delta\text{-}2)} = -\beta(\delta\text{-}2)x_n$ we get $x_{n+\delta} + \beta(\delta)x_n + x_{n\text{-}\delta} = 0$ where $\beta(\delta) = -(\beta(\delta\text{-}2) + \mathrm{B}\beta(\delta\text{-}1))$. ■

Theorem 1. If $\{x_n\}$ is a determinate preconstant signal of the 1D TK operator then $\{x_{am+bn}\}$, with a and b integer constants, is a preconstant image of the TK operator.

Proof. $\mathrm{TK}(\{x_{am+bn}\})=\mathrm{TKR}(\{x_{am+bn}\})+\mathrm{TKC}(\{x_{am+bn}\})$. Since am is constant for each row and the signal $\{x_{bn}\}$ is a 1D preconstant signal, the image $\mathrm{TKR}(\{x_{am+bn}\})$ is a constant image. In a similar way, since bn is constant for each column and $\{x_{am}\}$ is a 1D preconstant signal, the image $\mathrm{TKC}(\{x_{am+bn}\})$ is constant. Thus we get that the image $\{x_{am+bn}\}$ is a preconstant image. ■

In [5], the determinate preconstant signals were classified into three types from which we can get three types of preconstant images which we show below with their respective outputs.

For $x = \{A\lambda_1^m\lambda_2^n + B\lambda_1^{-m}\lambda_2^{-n}\}$

$\mathrm{TK}(x) = \{-AB\,[(\lambda_1 - \lambda_1^{-1}) + (\lambda_2 - \lambda_2^{-2})]\}$

For $x = \{Am + Bn + C\}$

$\mathrm{TK}(x) = \{A^2 + B^2\}$

For $x = \{A\sin(\omega_1 m + \omega_2 n + \varphi)\}$

$\mathrm{TK}(x) = \{A^2(\sin^2\omega_1 + \sin^2\omega_2)\}$

Since each preconstant image $\{x_{m,n}\}$ can be obtained from a determinate preconstant signal $\{y_n\}$ by writting $\{x_{m,n}\}=\{y_{am+bn}\}$, where $a,b \in \mathrm{Z}$, for a fixed row m' of the image, $\{x_{m',n}\}=\{y_{bn+ am'}\}$ is a preconstant signal with parameters $\beta=\beta_R(b)$ and $\kappa= \kappa_R(b)$. Similarly, for a column n' of the image, $\{x_{m,n'}\}=\{y_{am+bn'}\}$ is a preconstant signal with parameters $\beta=\beta_C(a)$ and $\kappa= \kappa_C(a)$. Thus, for particular values of a and b, the rows and columns of the image $\{x_{m,n}\}$ are preconstant signals with constant parameters: β_R and κ_R for rows, and β_C and κ_C for columns. Consequently, the constant output of the image $\{x_{m,n}\}$ is given by $\kappa_R + \kappa_C$.

4. ROOT IMAGES

From (2), the starting point for the study of the root images of the TK operator is the equation

$$x_{m,n} = 2x_{m,n}^2 - x_{m,n-1}x_{m,n+1} - x_{m-1,n}x_{m+1,n} \qquad \textbf{(5)}$$

The set of solutions of this nonlinear difference equation is the set of the *root images* of the TK operator.

4.1. Determinate root images

Each determinate root signal of the 1D operator is characterized by a parameter α. If $\alpha \neq -2$, the signal is on a DC level and becomes preconstant signal when the DC level is removed [4]. Therefore every determinate root signal can be obtained from a determinate preconstant signal. On the other hand, a preconstant signal, with parameters $\beta \neq -2$ and κ, becomes a root signal when a $1/(2+\beta)$ DC level is added.

Theorem 2. Let $\{x_{m,n}\}$ be a preconstant image such that $\mathrm{TK}(x)=1/(4+ \beta_R+ \beta_C)$ and let $d = 1/(4+ \beta_R+ \beta_C)$ be a DC level, so the image $x+d$ is a root image of the TK operator.

Proof. The output of the TK operator to an image plus a DC level is shown in (3). The image $\{x_{m,n}\}$ becomes a root if $d = \mathrm{TK}(x)$ and $x = d(4x - {}_{0,\text{-}1}x - {}_{0,1}x - {}_{\text{-}1,0}x - {}_{1,0}x)$. Let suppose that $\{x_{m,n}\}$ is a preconstant image so it obeys $x_{m,n+1} + \beta_R\, x_{m,n} + x_{m,n\text{-}1} = 0$ and $x_{m+1,n} + \beta_C\, x_{m,n} + x_{m\text{-}1,n} = 0$. Hence we can write $\beta_R\, x = - {}_{0,\text{-}1}x - {}_{0,1}x$ and $\beta_C\, x = - {}_{\text{-}1,0}x - {}_{1,0}x$, and replacing it in (3) we get $\mathrm{TK}(x + d) = \mathrm{TK}(x) + dx(4 + \beta_R + \beta_C)$. Clearly $\mathrm{TK}(x + d) = x + d$ if $\mathrm{TK}(x)=1/(4+ \beta_R+ \beta_C)$ and $d = 1/(4+ \beta_R+ \beta_C)$. ■

(a) (b) (c)

Fig. 1. Contrast Modifying by adding a DC level d before filtering. (a) Original sinusoid image, (b) output gotten for $0 < d < d_R$ and (c) output obtained for $d > d_R$.

Therefore, from each preconstant that obeys

$$\kappa_R + \kappa_C = 1/(4 + \beta_R + \beta_C) \tag{6}$$

we can obtain a root image by adding a $1/(4+\beta_R+\beta_C)$ DC level.

The preconstant image $x=\{A\lambda_1^m\lambda_2^n + B\lambda_1^{-m}\lambda_2^{-n}\}$ has $\beta_R = -(\lambda_2 + \lambda_2^{-1})$, $\beta_C = -(\lambda_1 + \lambda_1^{-1})$, $\kappa_R = -AB(\lambda_2 - \lambda_2^{-1})^2$ and $\kappa_C = -AB(\lambda_1 - \lambda_1^{-1})^2$, then the image $x=\{A\lambda_1^m\lambda_2^n + B\lambda_1^{-m}\lambda_2^{-n} + 1/(4+\beta_R+\beta_C)\}$ is a root image for each A and B that satisfies (6).

The preconstant image $x=\{Am+Bn\}$, for $A,B \neq 0$, has $\beta_R = \beta_C = 2$, $\kappa_R = B^2$ and $\kappa_C = A^2$, then the image $x = \{Am+Bn+1/8\}$ is a root image if $A^2 + B^2 = 1/8$.

The preconstant image $x = \{A\sin(\omega_1 m+\omega_2 n+\varphi)\}$ has $\beta_R = -2\cos\omega_2$, $\beta_C = -2\cos\omega_1$, $\kappa_R = A^2\sin^2\omega_2$ and $\kappa_C = A^2\sin^2\omega_1$, then the image $x = \{A\sin(\omega_1 m+\omega_2 n+\varphi)+1/(4+\beta_R+\beta_C)\}$ is a root image for each A, ω_1 and ω_2 that satisfies (6); we further analyze this case below.

4.2. Sinusoid on a DC level

A 1D sinusoidal signal is preconstant for the TK operator [5]. We showed above that a sinusoidal image generates a constant output. Let $x=\{A\sin(\omega_1 m+\omega_2 n+\varphi)+d\}$, where d is a real constant, be an image who is applied to the TK operator, then we get the output

$$\{2Ad\sin(\omega_1 m + \omega_2 n + \phi)(2 - \cos\omega_1 - \cos\omega_2) + A^2(\sin^2\omega_1 + \sin^2\omega_2)\}$$

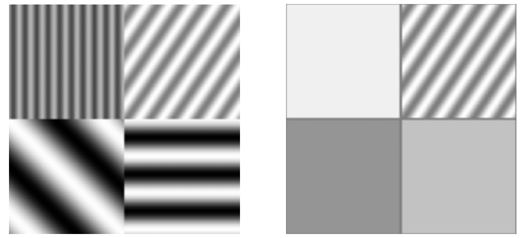
where we know that $\beta_R = -2\cos\omega_2$, $\beta_C = -2\cos\omega_1$. If the amplitude obeys $A^2=d/(\sin^2\omega_1+\sin^2\omega_2)$, then we can rewrite the output as

$$\{Ad\sin(\omega_1 m + \omega_2 n + \varphi)(4 + \beta_R + \beta_C) + d\}$$

Finally if $d=1/(4+\beta_R+\beta_C)$ the condition (6) is satisfied and we get a root image. The value of d controls the output of the operator when the image x is applied. For $d = 0$ the image generates a constant output, while for an appropriate value of d, which we call d_R, the image becomes a root.

For $d \neq 0$ and $d \neq d_R$ the output corresponds to a scaled version of x on a DC level. For values of d between 0 and d_R, the operator reduce the contrast of the original image, as we shown in Fig. (b). On the other hand, for values of d greater than d_R, the operator enhances the contrast of the image. Examples for two values of d are shown in Fig. 1.

In Fig. 2 we show the response of the operator to an image composed of 4 segments; 3 of them are taken from preconstant images and the other belongs to a root image.



(a) (b)

Fig. 2. Response (b) of the TK operator to an image (a) with root and preconstant image segments.

5. SEPARABLE IMAGES

We consider separable images as a important step in the study of the TK operator since they are relatively easier to analyze.

5.1. Response to separable images

An image is a *separable image* if it can be expressed as a certain product of 1D signals, i.e. x is a separable image if each pixel of x can be written as $x_{m,n}=u_m v_n$, where $u,v \in \mathrm{R}^{\mathrm{Z}}$; we call u and v *factor signals* and we write $u*v=\{u_m v_n\}$.

The response to a separable image can be written as

$$\mathrm{TK}(u*v) = u^2 * \mathrm{TK}_{1\mathrm{D}}(v) + \mathrm{TK}_{1\mathrm{D}}(u) * v^2 \tag{7}$$

where $\mathrm{TK}_{1\mathrm{D}}$ is the 1D TK operator.

5.2. Separable root images

A separable root image satisfies

$$u*v = u^2 * \mathrm{TK}_{1\mathrm{D}}(v) + \mathrm{TK}_{1\mathrm{D}}(u) * v^2 \tag{8}$$

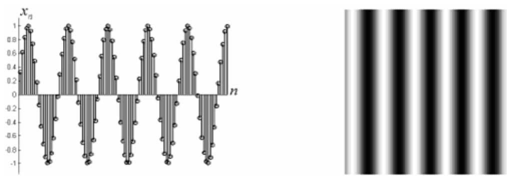
If u is a pre-null signal of the 1D TK operator, then (8) becomes $u*v=u^2*\mathrm{TK}_{1\mathrm{D}}(v)$. It can be checked that if $w=u*v$ and $z=x*y$, and if $w=z$ then $v=k_1 y$ y $u=k_2 x$, where k_1 and k_2 are numbers that obey $k_1 k_2=1$, thus $u^2=k_1 u$ and $\mathrm{TK}_{1\mathrm{D}}(v)=k_2 v$. Since u is pre-null it has to be a constant signal of value k_1. On the other hand, v has to be an *eigensignal* (i.e. the output is a scaled version of the output) with scale factor or eigenvalue $k=k_2$. Therefore a root separable image can be gotten in terms of $*$ from a k-eigensignal and a constant signal of value $1/k$.

Now, assume that u and v are not pre-null signals. If $w=u*v$ and $z=x*y$ are separable images, and if $w+z$ is separable then exist $k \in \mathrm{R}$ such that $v=ky$ or $u=kx$. Therefore in (8) is required that $\mathrm{TK}_{1\mathrm{D}}(u)=ku^2$ or $TK_{1\mathrm{D}}(v)=kv^2$. Assume $\mathrm{TK}_{1\mathrm{D}}(u)=ku^2$ then $u*v=u^2*[\mathrm{TK}_{1\mathrm{D}}(v)+kv^2]$ and $ku=u^2$, thus u has to be a constant which contradicts our choose of u. In a similar way if $\mathrm{TK}_{1\mathrm{D}}(v)=kv^2$ so v is pre-null. We conclude that the only root separable images are root signals, copied by rows or columns.

5.3. Preconstant separable images

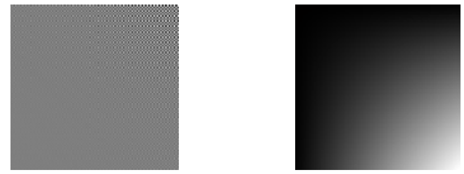
A preconstant separable image with constant κ satisfies

$$\mathrm{K} = u^2 * \mathrm{TK}_{1\mathrm{D}}(v) + \mathrm{TK}_{1\mathrm{D}}(u) * v^2 \tag{9}$$



(a) (b)

Fig. 5. A preconstant separable image. (a) A preconstant signal and (b) the image generated copying the signal by rows.



(a) (b)

Fig. 6. Pre-null images. (a) A separable image and (b) an exponential image.

where K is a constant image of value $\kappa \neq 0$. If u is a pre-null signal, the for each row m (9) becomes $\{\kappa\}=u^2_m \mathrm{TK}_{1D}(v)$. Therefore u has to be a constant signal of value k and v has to be a preconstant signal with constant $1/k$.

Now, assume that u and v are not pre-null signals. For each row we have a constant signal of value κ, so subtracting a pair of equations gotten from (7) for two rows i and j we get

$$v^2[\mathrm{TK}_{1D}(u)_i - \mathrm{TK}_{1D}(u)_j] = \mathrm{TK}_{1D}(v)(u_i^2 - u_j^2) \qquad (10)$$

If $\mathrm{TK}_{1D}(u)$ and u are constant signals, (10) is reached but u contradicts its definition. If only $\mathrm{TK}_{1D}(u)$ is constant (10) is not reached because v can not be pre-null. Finally if $\mathrm{TK}_{1D}(u)$ and u are not constant signals then exist $k \in$ R such that $\mathrm{TK}_{1D}(v)=kv^2$ and for each row m of the image we have $\{\kappa\}=v^2[ku^2_m+\mathrm{TK}_{1D}(u)_m]$. Thus v^2 has to be pre-null which contradicts the definition of v as not pre-null. Therefore preconstant separable images are preconstant signals copied by row or columns, as it is shown in Fig. 5.

5.4. Pre-null separable images

A pre-null images satisfies

$$\Theta = u^2 * \mathrm{TK}_{1D}(v) + \mathrm{TK}_{1D}(u) * v^2 \qquad (11)$$

where Θ is the null image. Clearly if u or v is the null signal θ, $u * v = \Theta$; we don't consider this case. If u is a pre-null signal then for each column m we have $\theta=u^2_m \mathrm{TK}_{1D}(v)$, so v has to be pre-null signal too. Therefore each separable image with pre-null factor signals is a pre-null image.

In Fig. 6(a) an example of pre-null separable image is shown, where the factor signals are an exponential signal and a binary signal which alternates its sign.

If u and v are not pre-null signals then for each row m we have $\mathrm{TK}_{1D}(u)_m v^2=-u^2_m \mathrm{TK}_{1D}(v)$. Since u is not pre-null has to exist $k_1 \in$ R such that $\mathrm{TK}_{1D}(v)=k_1 v^2$. Likely, for each column we followed $\mathrm{TK}_{1D}(u)=k_2 u^2$ for some $k_2 \in$ R. From (11) we get $\Theta=k_1(u^2 * v^2) + k_2(u^2 * v^2)$ which is true if and only if $k_1 = -k_2$. Therefore the factor signals who satisfy $u_{m+1}=u_m(1+k)/u_{m-1}$ and $u_{m+1}=u_m(1+k)/u_{m-1}$, for some real k, generate a pre-null separable image.

6. CONCLUSIONS

We have derived the responses of the operator to common images; this gives some light about the behavior of the operator. We have found roots and preconstant images that are derived from root and preconstant signals of the 1D TK operator. Some of these preconstant images become a root when an appropriate DC level is added and they were fully characterized.

All root and preconstant images found include negative values; the operator does not work well as a contrast enhancer for images with negative values. When a sinusoidal texture on a DC level is applied to the operator, the texture frequency and DC level determine the amount of amplification/attenuation of the output texture and of the output new DC level.

The particular case of separable images has been studied and we have characterized the root and preconstant separable images; they are root and preconstant 1D signals, copied by rows or by columns. Nevertheless not all root and preconstant images of the TK operator are separable or have a relationship with the 1D TK operator, as it was found in [7], so other strategies have to be used to solve wholly this problem.

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