HIGHLY ACCURATE ORIENTATION ESTIMATION USING STEERABLE FILTERS

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ABSTRACT

This paper presents a generalized theoretical framework for designing accurate steerable filters for orientation estimation. We derive the necessary properties of orientation estimation filters in their most general form. Based on our framework, we implemented a highly angular-specific filter. Numerical experiments show the enhanced accuracy of our proposed filter as compared with existing filters.

1. INTRODUCTION

Orientation estimation is an important step in many image processing algorithms. It aims to find a direction $\mathbf{r}$ of local constancy of a given signal. Typically, the classical approach is used where a particular direction $\hat{\mathbf{r}}$ is found by calculating an approximation of the first derivative in the direction $\hat{\mathbf{r}}$ on a discrete grid [1, 2, 3, 4, 5, 6], and orienting the filter responsible for calculating the derivative so that its output vanishes. This derivative condition leads to the well known *brightness constancy constraint equation* (BCCE), and the associated filtering task includes translation of the discrete signal into the continuous domain through pre-filtering and applying a directional derivative to the obtained continuous signal. The set of filters used in this process belongs to the class of nullifying filters, i.e., parametric filters whose response is zero only if the values of the parameters reach the required values. Simoncelli showed that the BCCE can be most efficiently implemented using steerable filters [7, 8]. Steerable filters are based on the idea of composing a filter from a linear combination of basis filters independent of $\hat{\mathbf{r}}$ such that any rotation of the filter can be obtained by an appropriate linear combination of the basis filters. This paper addresses the issue of creating filters using the recently proposed extended BCCE [9, 10] by formulating a general framework for design of orientation selective filters. We determine the necessary and desired features of a filter for orientation estimation, design a highly angle-specific filter, and finally show that our exemplar filter performs better than filters that are based on first order derivative approximations.

2. THEORETICAL BACKGROUND

The common goal of all differential-based orientation estimation techniques is to find a signal characteristic being conserved in a certain direction. The direction of constancy $\mathbf{r}$ can be related to the signal s through directional derivatives of s. Let the signal s be constant in direction $\mathbf{r}$ in some vicinity of point $\mathbf{x}$, $s(\mathbf{x}+u\mathbf{r}) - s(\mathbf{x}) = 0$ for some $|u| < R$. The above, clearly non-differential, condition can be expanded in a Taylor series

$$u\frac{ds}{d\mathbf{r}} + \frac{u^2}{2}\frac{d^2 s}{d\mathbf{r}^2} + \frac{u^3}{3!}\frac{d^3 s}{d\mathbf{r}^3} + \ldots = 0, \tag{1}$$

which is equivalent to the generalized BCCE proposed by Mester [9, 10] with a particular choice of coefficients. This shows that the non-derivative approaches are necessarily equivalent to the proposed generalized BCCE. Since we assume the signal to be constant within some neighborhood, each term in (1) equals zero. Thus, each linear combination of derivative terms can be used to relate the direction of constancy to the signal. However, not every combination may be equally efficient in practice. The choice of weights obtained through Taylor expansion assures that the weights are consistent with the shape of the signal s.

Let us now consider (1) in the Fourier domain. Each term has a nullifying plane in this domain, that is, the Fourier transform of each term of (1) is zero on a plane perpendicular to the direction $\mathbf{r}$. This is the fundamental property of all orientation estimation operators and follows from the fact that the entire energy of a signal, which is constant in direction $\mathbf{r}$, is concentrated on the plane. Equation (1) can be computed via a linear filter $h(\mathbf{x};\mathbf{r})$ parameterized by a unit vector $\mathbf{r}$. The response of the filter $h(\mathbf{x};\mathbf{r})$ when applied to a signal $s(\mathbf{x})$ is defined as

$$T(\mathbf{r}) = h(\mathbf{x};\mathbf{r}) * s(\mathbf{x}), \tag{2}$$

and henceforth called the response function. For the required direction of constancy the response is zero; therefore, we may write the governing equation

$$T(\mathbf{r}) = h(\mathbf{x};\mathbf{r}) * s(\mathbf{x}) = 0, \tag{3}$$

where $\mathbf{r}$ is usually parameterized by angular coordinates of the unit vector $\mathbf{r}$. In the Fourier domain, equation (2) becomes $T(\mathbf{r}) = H(\mathbf{f};\mathbf{r})S(\mathbf{f})$ with $H = \mathcal{F}\{h\}$ and $S = \mathcal{F}\{s\}$ being the Fourier transforms of the filter and signal, respectively. If the signal s is a planar wave $s(\mathbf{x}) = \sin(\mathbf{k}\cdot\mathbf{r})$, the response function can be simplified to $T(\mathbf{r}) = H(k\mathbf{r};\mathbf{r})$ where $k = |\mathbf{k}|$ is the magnitude of the wave vector $\mathbf{k}$[1]. Hence, the response function of any filter to a signal of frequency k is equivalent to the angular behavior of the Fourier transform of the filter evaluated at distance k from the origin in the frequency domain. The response function should only be zero for $\mathbf{r}$ being the direction of constancy of the signal. It is imperative therefore, to ensure that the frequency domain form of the filter H is zero only on its nullifying plane, as spurious zeroes would result in solutions unrelated to the sought for direction.

Following [11], we further restrict the possible set of filters to those separable in the frequency domain. Thus, a general filter satisfying the above constraints can be written as $H(\mathbf{f};\mathbf{r}) = F(\rho)G(\psi)$ with $F(\rho)$ being the radial part ($\rho \equiv |\mathbf{f}|$), and $G(\psi)$ the angular part (ψ is the generalized angular coordinate vector dependent on both $\mathbf{f}$ and $\mathbf{r}$). From this general form we infer two significant observations. First, the angular part $G(\psi)$ is equivalent to the response function $T(\mathbf{r})$, and it solely determines the solutions of the governing equation (3). Conversely, the radial part $F(\rho)$ describes the response of the filter to a signal of frequency ρ and allows one to incorporate previous knowledge about the signal (signal-to-noise ratio for different frequencies, typical frequencies of sought-for features of the signal, etc.) into the designed filter to further enhance its performance.

In order to ensure that our filter is steerable, we need to decompose it into a set of basis filters independent of the parameter $\mathbf{r}$. First of all, let us rewrite it in terms of angular coordinates, keeping the notation in d-dimensional form and denoting α as a generalized angle describing the rotation. Thus, we obtain $H(\mathbf{f};\alpha) = F(\rho)G(\psi-\alpha)$ and expand the angular part in an infinite series of spherical harmonics. Spherical harmonics generalize sine and cosine functions and allow for decomposition of angular functions in a fashion similar to Fourier series $G(\psi) = \sum_{i=0}^{\infty}\sum_{j=0}^{j_i} a_{ij}Y_i^j(\psi)$ where $Y_i^j(\psi)$ is a spherical harmonic of order i, a_{ij} is an expansion coefficient, and j_i is the number of spherical harmonics of order i. The constructed filter is steerable since any combination of spherical harmonics can be rotated by application of a rotation matrix $R(\alpha)$, which consists of diagonally-centered blocks $R_i(\alpha)$ rotating the i-th order spherical harmonics [12]. Therefore, a set of functions $a_{ij}(\alpha)$ dependent on the rotation angle can be obtained through $\mathbf{a}(\alpha) = R(\alpha)\mathbf{a}$. Subsequently, they describe the function $G(\psi)$ rotated by an arbitrary angle α. In summary, the spherical harmonics decomposition of the filter $H(\mathbf{f};\alpha)$ is given by

$$H(\mathbf{f};\alpha) = F(\rho)\sum_{i=0}^{\infty}\sum_{j=0}^{j_i} a_{ij}(\alpha)Y_i^j(\psi)$$

with the condition $a_{ij}(0) = a_{ij}$ for every i and j.

Filter design reduces in this case to choice of a suitable radial function $F(\rho)$ and decomposition coefficients a_{ij}. While the form of $F(\rho)$ is restricted only by the requirement that $F(0) = 0$, the set of allowed a_{ij}'s is more limited. A filter has a nullifying plane if the angular part of $H(\mathbf{f};\alpha)$ is zero for angles in that plane, which implies that the angular part $G(\psi)$ cannot be composed of spherical harmonics of even orders.

The final step consists of transforming the filter from the frequency domain form (2) to the spatial domain. Using a very elegant result developed in [13], we obtain

$$h(r,\theta;\alpha) = \sum_{i=0}^{\infty}(-\sqrt{-1})^i\mathcal{H}_i\{F(\rho)\}\sum_{j=0}^{j_i} a_{ij}(\alpha)Y_i^j(\theta), \quad (4)$$

where $\mathcal{H}_i$ stands for a d-dimensional integral transform of order i similar to the Hankel transform, and defined as

$$\mathcal{H}_i\{F(\rho)\} \equiv r^{1-d/2}\int_0^{\infty} F(\rho)J_{i+d/2-1}(r\rho)\rho^{d/2}d\rho \quad (5)$$

with $\mathbf{x}$ rewritten in terms of the spatial domain radius r and generalized angle θ. The function $J_n(r)$ is the n-th order Bessel function of the first kind and serves as the kernel of the transform. This powerful theorem gives us a semi-analytical form for every possible filter which satisfies the aforementioned restrictions. We simplify the notation by imposing the condition $a_{ij} = 0$ for even i and j, and gather terms to emphasize the fact that $h(r,\theta;\alpha)$ is indeed a steerable filter, thus obtaining

$$h(r,\theta;\alpha) = \sum_{i=0}^{\infty}\sum_{j=0}^{j_i} b_{ij}(\alpha)h_{ij}(r,\theta), \quad (6a)$$

$$b_{ij}(\alpha) = (-1)^i a_{(2i+1)j}(\alpha), \quad (6b)$$

$$h_{ij}(r,\theta) = \mathcal{H}_{2i+1}\{F(\rho)\}Y_{2i+1}^j(\theta). \quad (6c)$$

Equation (6a) can be subsequently used in the filter design process, as it defines the most general filter that possesses properties desired from an orientation estimation filter.

3. FILTER DESIGN

Having developed a firm theoretical basis for the design of filters for orientation estimation we engineer a two-dimensional filter with high angular selectivity, and show that it is a generalization of the approximated derivative filter.

As an initial step in the design process we need to define the radial and angular functions of the designed filter. To

[1] The wave vector $\mathbf{k}$ is usually defined as $\mathbf{k} = 2\pi\mathbf{f}$.

create a general-use filter, we consider filters with a Gaussian-like frequency domain radial functions given by

$$F(\rho) = \rho^m \exp(-\rho^2) \text{ with } m \in \mathbb{R}.$$

The Hankel transform (5) yields the basis radial functions

$$f_i(r) = \frac{\Gamma\left(\frac{1+m}{2}+2\right)}{(2i+2)!} \frac{\exp\left(-\frac{r^2}{8}\right)}{r^2} \times \left[(2i+1-m) M\left(\frac{m}{2}, i+1; \frac{r^2}{4}\right) + (2i+m+3) M\left(\frac{m}{2}+1, i+1; \frac{r^2}{4}\right)\right], \quad (7)$$

where $\Gamma(x)$ is the gamma function, and $M(a, b; x)$ is the Whittaker M function [14]. The angular function $G(\psi)$ was designed with the aim of increasing its slope around the roots. While the response function of the first derivative is only linear in the vicinity of the roots, we use a square wave because of its infinite sharpness. Its spherical harmonic expansion consists of only two spherical harmonics (sine and cosine functions), with the general expression of the coefficients is given by

$$a^c_{2i+1} = (-1)^i \frac{4}{(2i+1)\pi} \text{ and } a^s_{2i} = 0. \quad (8)$$

Therefore, the spatial domain formulae of the basis filters are

$$h^c_i(r, \theta) = \frac{4}{\pi} \frac{f_i(r)}{2i+1} \cos\big((2i+1)\theta\big), \quad (9)$$

$$h^s_i(r, \theta) = \frac{4}{\pi} \frac{f_i(r)}{2i+1} \sin\big((2i+1)\theta\big). \quad (10)$$

and in terms of those the response function becomes

$$T(\alpha) = \sum_{i=0}^{n} \left(h^c_i(r, \theta) \cos\alpha - h^s_i(r, \theta) \sin\alpha\right) * s(r, \theta). \quad (11)$$

The governing equation (11) can be solved analytically only for the zeroth order filter. For higher order filters only numerical solutions are available. We used the Van Wijngaarden-Dekker-Brent [15] method to solve (11), with an initial guess being the solution to the analytical zeroth order formula.

A crucial part of filter design is the transformation of a continuous filter to the discrete domain, also known as sampling. It is well-known, that performance of orientation estimation filters heavily depends on the width of the sampled filter, and can be optimized by selecting a particular value. Define the width σ to be the maximum distance r along any axis of the discrete filter mask; in this case, the optimum widths were found by filtering a set of synthetic test images made of planar waves of frequencies ranging from zero to the Nyquist frequency, and traveling at several angles between $0°$ and $45°$ to the x axis. We examined σ in the range from 0.1 to 17.5; for each value of σ, a set of 1600 images of size 30×30 with 40 different angles and frequencies was analyzed. The average error $\bar{E} = \frac{1}{n}\sqrt{\sum_{i=0}^{n}(\hat{\theta} - \theta)^2}$ was calculated and used to choose an optimal value of $\sigma = 10.1$ (valid for 11×11 filter). During the filter optimization process we found that of all tested values of the m parameter, filters with $m > 1$ gave worse results than first order derivative filters. However, we found that the case $m = 0$ yields better orientation estimates and only this value was used in the experimental section.

4. EXPERIMENTS

To compare the designed filters against the first order derivative filters, we filtered a set of test images consisting of a chirp, a parabolic and a planar wave signals, with a set of first order derivative filter and our proposed filters of tap eleven. Plots showing the errors in the estimated angle were generated, and are encoded in gray scale images on Fig. 1.

In all cases the average error of our filter was smaller than that of the first order derivative approach. The plots reveal a particular strength of our algorithm: the areas of the signal with relatively small spatial variability, such as the crest of a wave in the chirp or planar wave signals, or the bottom of the parabolic signal, show the largest improvement. This observation may be due to the inclusion of higher order derivatives, which remain non-zero even though the first order derivative might be very close to that value.

5. CONCLUSIONS AND FUTURE WORK

In this paper we have generalized the Brightness Constancy Constraint Equation to include higher order directional derivatives. We developed a theoretical basis of design of steerable filters for orientation estimation, and successfully applied it to engineer a new class of filters which perform better than first order derivative approaches, especially in the regions of low variability of the signal, giving an overall relative improvement of 10%-40%. Some features of the proposed filter, such as the angular dependency of the error, still leave space for further work and optimizations.

6. REFERENCES

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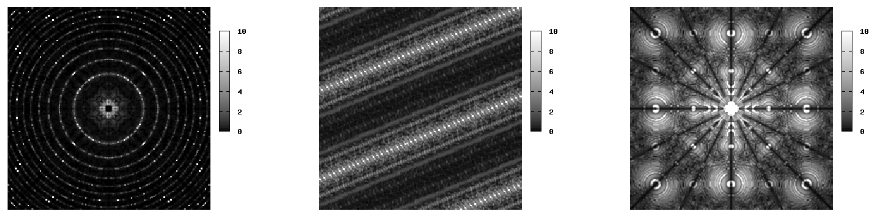
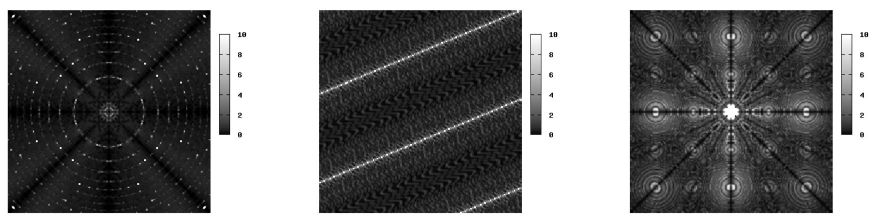


(a) Chirp signal (b) Planar wave signal. (c) Parabollic signal.



(d) 35.5% improvement. (e) 12.8% improvement. (f) 44.2% improvement.

Fig. 1. Comparison between the first order (top panel) and the proposed filter (bottom panel, $n = 7, m = 0$). Plotted are errors in degrees, in a scale from $0°$ to $10°$. As shown by quantitative error estimates, the proposed filter performs significantly better, especially in cases where signal is slowly varying.

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