CORNER-GUIDED IMAGE REGISTRATION BY USING EDGES

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***Abstract*— This paper proposes an image registration method. Edges are detected from images and partitioned into segments as matching primitives. Then, corners on the edges are detected to guide registration. A similarity metric is proposed based on the number of pairs of matching segments. Corner mappings are sequentially tried along a segment, from which a transformation is obtained. The corner mappings are evaluated by the similarity metric under their resulting transformation. By this means, corner mappings are established by utilizing whole images. Since the sensitivity of transformation parameters to the accuracy of corner mappings, as many corner mappings as possible are used. Experimental results show that the proposed method is robust, especially when there is no integral corresponding edges between two images.**

Index Terms: Image Registration, Corner-Guided, Edges

I. Introduction

Image registration is an important task in computer vision. It comprises of four elements [1]: (1) a feature space, (2) a search space, (3) a search strategy, and (4) a similarity metric. In the past, a number of registration algorithms have been presented. Those algorithms can be classified into two categories, intensity-based and feature-based. Viola *et al.* [2] and Collignon *et al.* [3] proposed mutual information as a similarity metric to align two images. Mutual information is calculated based on only intensity, and it is an effective similarity metric especially for multimodal images. However, information-theoretic methods do not incorporate spatial or higher-level-feature information, which are also useful.

The commonly used features include edges, contours, textures, etc. Bay *et al.* [4] extracted edges and matched straight line segments between two wide-baseline images. Xia *et al.* [5] proposed matching curves (edges) by "super-curves". Bartoli *et al.* [6] introduced curves into registration and obtained a global solution by minimizing the registration error over all points and curves.

This paper proposes an image registration method based on edges and corners. The goal is to register images based on non-ideal edges, as edge detection is influenced by noise and other factors. Edges are detected and one-pixel-wide curves are extracted from edge images. Then corners along the curves are detected, with which edges are partitioned into segments as matching primitives. At this point, there are two ways to register images using edges and corners. One is to optimize a registration function of a transformation, which is defined to be the similarity metric under the transformation. But unfortunately, a registration function is usually neither a smooth function [7] nor a linear function and hence any optimization algorithm may fail to converge.

The other way is to look for correspondences of salient feature points. Assume the misalignment can be accounted for by an affine transformation. Since three pairs of non-collinear points completely determine an affine matrix, it suffices to establish three pairs of correspondences. However, in general there is no way to fulfill this task unless two images are already correctly registered. This paper proposes establishing corner correspondences by utilizing the information of whole images. For a set of point correspondences, an affine matrix is determined. The set of correspondences are preserved if the affine matrix results in a high similarity metric. Thus, the information about registration quality is implicitly introduced into the step of building corner correspondences.

The paper is organized as follows. Section II describes the similarity metric. Section III discusses how to determine the transformation parameters. Section IV shows the experimental results and conclusion is presented in Section V.

II. Similarity Metric Between Edge Images

This section discusses the proposed similarity metric. Let $I_r(x,y)$ represent a reference image, $I_f(x,y)$ a float image, T a transformation, and $S(I_r, I_f)$ a similarity metric between the two images. Registration is to search the maximizer $\hat{T}$ of a registration function. Formally,

$$\hat{T} = \arg\max_{T} S(I_r(x,y), I_f(T(x,y))). \tag{1}$$

To measure the similarity metric between $I_r(x,y)$ and $I_f(x,y)$, three steps are taken. (1) Detect edges and junctions. (2) Partition the edges into segments with the detected junctions. (3) Compute the distances between segments.

A. Detect Edges and Junctions

A Canny operator [8] is used to detect edges from images. It is applied because of its ability of providing "continuous" edge segments. For each edge, one-pixel-wide curves are extracted. 'Curve' here is refered to a sequence of edge points. Then, junctions are detected on those curves with methods such as the one in [9]. 'Junction' is defined to be the points of maximal curvature along a curve, and used as 'corner'. Nevertheless, edge detection is not essential to registration and the proposed method aims to align images based on non-ideal edges. And, no constraint is placed on corner detection techniques except that junctions are limited to lie on the curves.

B. Partition edges into segments

With those detected corners, each curve is partitioned into segments. This step is essential, because for the reasons such as noise, not all edges/curves of the reference image would have their correspondences in the float image. As a compromise, it is expected that parts of some edges in one image can be matched to parts of edges in the other.

Since a curve is a sequence of points, the task of partitioning can be sequentially tackled by dividing the curve with corners. Two factors considered are: (1) the lengths of segments L_s, and (2) the number of corners lying on a segment. Longer segments are more likely caused by physical objects than by noise, so only those segments of lengths greater than a threshold are used. Affine transformations preserve straight lines and any two lines can be aligned up by an affine transformation. A corner is where the direction of a curve changes fastest, and hence the two segments connected with the same corner usually (but not necessarily) have different directions. So segments are required to contain at least N_{jct} corners, in order to avoid too many mappings between straight lines. In this paper $N_{jct} = 1$.

Fig. 1 shows an original image and its segments. 7 longest edges are retained from the edge detection result, where $L_s = 40$ and $N_{jct} = 1$. Segments are portrayed in different widths on each curve, and they are used as matching primitives in Section II-C.

(a) Original image (b) Segments

Fig. 1. An image and its segments.

C. Assessing Segment Similarity

Let $SE_r, r = 1, 2, \cdots, m$, denote a segment in the reference image and $SE_f, f = 1, 2, \cdots, n$, in the float. A natural similarity metric is to compute the distance between the two segments as closed sets (also refer to [6]). However, this similarity metric admits only the geometric distance of SE_r to SE_f, but no information about the shapes. Another way is to use the average of the distances of points $\mathbf{p}$ on SE_r to SE_f. Formally,

$$d(r, f) = d(SE_r, SE_f) = \frac{1}{L_r} \sum_{\mathbf{p} \in SE_r} d(\mathbf{p}, SE_f). \quad (2)$$

Wherein, L_r is length of SE_r, and $d(\mathbf{p}, SE_f)$ is the distance of $\mathbf{p}$ to SE_f.

When $d(r, f)$ is less than a threshold T_d, SE_r and SE_f are called a pair of matched segments. T_d is typically chosen as $1 \sim 2$. For a particular T, three similarity metrics are defined as follows.

$$S_1(I_r(x, y), I_f(T(x, y))) = |\{(r, f) : d(r, f) \le T_d\}|. \quad (3)$$

$$S_2(I_r(x, y), I_f(T(x, y))) = |\{f : \exists r, d(r, f) \le T_d\}|. \quad (4)$$

$$S_3(I_r(x, y), I_f(T(x, y))) = |\{r : \exists f, d(r, f) \le T_d\}|. \quad (5)$$

S_1 is the number of pairs of matched segments, while S_2 (S_3) is the number of segments in $I_f(T(x, y))$ ($I_r(x, y)$) that can be matched to reference (float) segments.

III. DETERMINE TRANSFORMATION

This section deals with how to determine the transformation matrix. As discussed above, a registration function is usually neither a smooth function nor a linear function, and any algorithm may fail in optimizing the registration function. So we wish to make the proposed algorithm not rely much on optimization techniques. The idea is to establish corner correspondences and then determine the affine matrix T without involving optimization.

In order to correspond corners robustly, global information is incorporated. Specifically, for any set of corner correspondences, an affine matrix T is obtained. Then the similarity metrics S_1, S_2 and S_3 in (3), (4) and (5) are employed in determining if T is close to the true matrix or not. Assume there exist M corners in $I_r(x, y)$, and N in $I_f(x, y)$. Then the total number of sets of 3 pairs of correspondences is $\binom{N}{3} \cdot M^3$ (match float corners to reference ones). For the reason of computational cost, it is necessary to reduce the large number of corner correspondences. The rest of this section presents how to kick out corner correspondences step by step.

A. Constrain Transformations

An affine matrix T is represented as $T = (A, \boldsymbol{t})$, where

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \boldsymbol{t} = \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix}.$$

A can be decomposed as the product of rotation, scaling, and shearing [1] [5]. Under T a point $\boldsymbol{p}$ is transformed to $\boldsymbol{p'}$ via

$$\begin{aligned} \boldsymbol{p'} &= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \boldsymbol{p} + \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \cdot \boldsymbol{p} + \boldsymbol{t} \end{aligned} \quad (6)$$

For the purpose of reducing the corresponding candidates for $\boldsymbol{p}$, let us analyze the range of the distance from $\boldsymbol{p}$ to $\boldsymbol{p'}$.

$$\begin{aligned} \|\boldsymbol{p'} - \boldsymbol{p}\|_2 &= \|(A - I) \cdot \boldsymbol{p} + \boldsymbol{t}\|_2 \\ &\le \|(A - I) \cdot \boldsymbol{p}\|_2 + \|\boldsymbol{t}\|_2 \\ &\le \|(A - I)\|_2 \cdot \|\boldsymbol{p}\|_2 + \|\boldsymbol{t}\|_2 \end{aligned} \quad (7)$$

Simple calculation gives

$$A = \begin{pmatrix} \cos\theta \cdot s_x & k\cos\theta \cdot s_x + \sin\theta \cdot s_y \\ -\sin\theta \cdot s_x & -k\sin\theta \cdot s_x + \cos\theta \cdot s_y \end{pmatrix}.$$

$\|A-I\|_2 = \sigma_1(A-I)$, where $\sigma_1(A-I)$ is the largest singular value of $A-I$, i.e. the square root of the largest eigenvalue of $(A-I)^T(A-I)$ [10] (pp. 375-383).

It is assumed that we are able to obtain a rough estimation of the correct affine matrix (also refer to [3]), which means $\|(A-I)\|_2$ and $\|\boldsymbol{t}\|_2$ are small enough. In particular, it is assumed that $|\theta|, |\Delta x| = |s_x - 1|, |\Delta y| = |s_y - 1|$ and $|k|$ are sufficiently small and accordingly first-order approximation gives

$$A - I = \begin{pmatrix} \Delta x & k+\theta \\ -\theta & \Delta y \end{pmatrix}. \quad (8)$$

However, it is still not easy to get an analytic form for singular values of $A-I$ shown in (8). An estimation [10] (pp. 202-208) is

$$\|A-I\|_2 \leq \sqrt{2}\|A-I\|_\infty,$$

$$\|A-I\|_\infty \leq \max\{|\Delta x| + |k+\theta|, |-\theta| + |\Delta y|\} \quad (9)$$

When $|\theta| = 0.1$, $|\Delta x| = |\Delta y| = |k| = 0.05$, $\|A-I\|_2 \leq 0.2\sqrt{2}$.

B. Sequentially Choose Corners

As discussed in Section II-A, extracted curves are sequences of edge points, which hence implies that the corners on each curve are also ordered. Let $J^f_{k,i}$ denote the i_{th} corner on the k_{th} curve of the float image. Three corners are to be chosen on the k_{th} curve with $J^f_{k,i}$ being the first one J^f_1. The second corner J^f_2 is chosen as $J^f_{k,j}$ such that $i < j \leq i + N_{run}$, and the third $J^f_3 = J^f_{k,l}$ such that $j < l \leq i + N_{run}$. N_{run} controls the number of combinations (J^f_1, J^f_2, J^f_3) with $J^f_1 = J^f_{k,i}$. For example, $N_{run} = 2$ admits only one combination, i.e. $(J^f_{k,i}, J^f_{k,i+1}, J^f_{k,i+2})$. In this paper, $N_{run} = 4$.

The above process is performed from the first corner ($i = 1$) on each curve. Set $J^f_1 = J^f_{k,i+1}$ when all cases of (J^f_1, J^f_2, J^f_3) are exhausted with $J^f_1 = J^f_{k,i}$. This step in conjunction with the above forms a sequential process of choosing corners.

Once J^f_1, J^f_2, J^f_3 are chosen, the corresponding corner candidates J^r_1, J^r_2, J^r_3 in $I_r(x,y)$ are picked up by the technique in Section III-A. J^r_1, J^r_2, J^r_3 are required to be different corners. A further constraint can be placed on the candidates that they lie on the same curve. But it turned out to be not necessary, since in practice, there are not a lot of corresponding candidates for a corner. In fact, the cases can be easily eliminated (see Section III-C) in which J^r_1, J^r_2, J^r_3 do not lie on the same curve.

C. Decide on Corner Correspondences

Given J^f_1, J^f_2, J^f_3 corresponding to J^r_1, J^r_2, J^r_3 respectively, an affine matrix T is obtained by solving the following equation:

$$\underbrace{\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}}_{\mathbf{X}} \cdot \underbrace{\begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{pmatrix}}_{\mathbf{T'}} = \underbrace{\begin{pmatrix} u_1 & v_1 \\ u_2 & v_2 \\ u_3 & v_3 \end{pmatrix}}_{\mathbf{U}}, \quad (10)$$

where $(x_i, y_i), i = 1,2,3$, are the coordinates of J^f_i, (u_i, v_i) are of J^r_i. If $\mathbf{X}$ tends to be singular, which means J^f_1, J^f_2, J^f_3 are collinear, then this set of correspondences are discarded.

By assumption that $\|(A-I)\|_2$ and $\|\boldsymbol{t}\|_2$ are small, the set of correspondences from J^f_1, J^f_2, J^f_3 to J^r_1, J^r_2, J^r_3 can be roughly evaluated by the elments of T. If T does not satisfies the assumption, then discard this set of correspondences. Else, the similarity metrics S_1, S_2, and S_3 in Section II-C are computed under T. This step is important, as the global information are incorporated by this means. S_1, S_2, and S_3 measures the similarity between the entire edge images rather than local regions.

For each $J^f_{k,i}$, let $J^{r,1}$ and $J^{r,2}$ be two correspondence candidates, which result in two affine matrixes T_1 and T_2. T_2 is said to be better than T_1 if the following conditions hold.

1) $S_1(T_2) > \max(S_1(T_1)/2, N_1)$,
2) $S_2(T_2) > \max(S_2(T_1)/2, N_2)$,
3) $S_3(T_2) > \max(S_3(T_1)/2, N_3)$, and
4) $S_1(T_2) + S_2(T_2) + S_3(T_2) > S_1(T_1) + S_2(T_1) + S_3(T_1)$.

N_1, N_2, N_3 are used to further narrow down acceptable affine transformations. In general, T_2 is better, if the average of the three similarity metrics increase (condition 4), while the individuals do not decrease too much (condition 1-3).

The above process concludes with the best correspondence corner for each $J^f_{k,i}$. Nevertheless, the best may not imply the right for $J^f_{k,i}$ because of the deficiency of all steps involved in computing similarity metrics. N_c best pairs of correspondences are picked up based on the average metric $\frac{1}{3}(S_1(T) + S_2(T) + S_3(T))$. Since there is sensitivity of T to the accuracy of corner mappings, N_c ought to be at least a litter greater than 3. Then (10) becomes an over-constrained equation of more equations than unknowns, and is solved by a minimum-least-square technique. $N_c = 10$ in this paper.

IV. Experimental Result

This section shows experimental results. $I_f(x,y)$ is generated by rotating $I_r(x,y)$ by 10°. Note that the rotation center is the upper-left corner and downwards is the positive direction of vertical coordinate. The true transformation from $I_f(x,y)$ to $I_r(x,y)$ is

$$T = \begin{pmatrix} 0.9848 & 0.1736 & 0 \\ -0.1736 & 0.9848 & 0 \end{pmatrix}.$$

Fig. 2 shows an example of wrong correspondences. It is difficult to reject such correspondences if local information is utilized only. The correspondences result in $S_1 = S_2 = S_3 = 1$, and hence are not considered to be correct. Fig. 3 shows an example of correct correspondences, with $I_r(x,y)$ and $I_f(x,y)$ same as above, resulting in $S_1 = 7$, $S_2 = 7$, $S_3 = 4$, and

$$T = \begin{pmatrix} 0.9652 & 0.1620 & 1.8532 \\ -0.1786 & 0.9846 & -0.6798 \end{pmatrix}.$$

Fig. 4 illustrates the sensitivity of the transformation matrix to the accuracy of corner correspondences. Although those corners are pretty well mapped in human vision, they end up with $S_1 = 2$, $S_2 = 2$, $S_3 = 1$, and

$$T = \begin{pmatrix} 0.7821 & 0.1996 & 29.1062 \\ -0.1795 & 0.9963 & -0.9377 \end{pmatrix}.$$

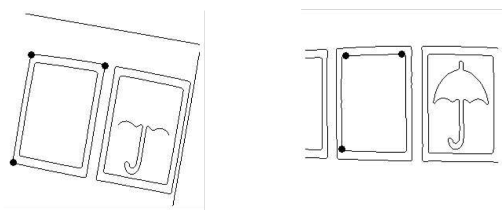
This motivates using over 3 pairs of corner correspondences. And Fig. 5 shows 10 pairs of corresponding corners, where the numbers next to corners indicate the mappings. A minimum-least-square solution is obtained for T:

$$T = \begin{pmatrix} 0.9875 & 0.1675 & 1.0488 \\ -0.1670 & 0.9920 & -1.9775 \end{pmatrix},$$

which is more close to the true matrix than others above.

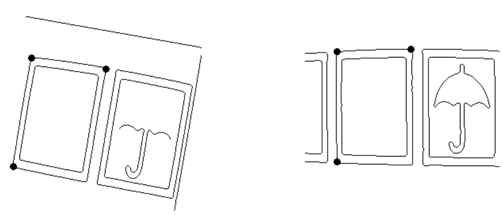
Fig. 6 shows two edge images in which some edges do not have their integral correspondences, but segments do. The float image is generated by translating the reference by (15, 15). The experiment shows that the proposed method can robustly map corners and determine the transformation matrix T:

$$T = \begin{pmatrix} 0.9987 & 0.0009 & -15.0321 \\ 0.0002 & 0.9995 & -14.9563 \end{pmatrix}.$$



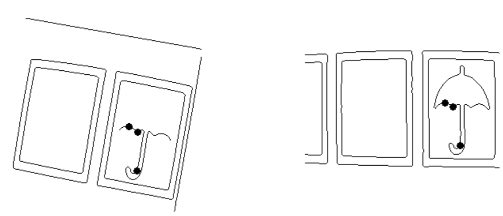
(a) Float corners (b) Reference corners

Fig. 2. Wrongly mapped corners



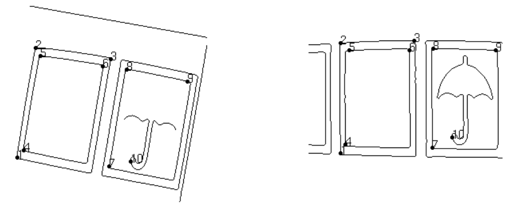
(a) Float corners (b) Reference corners

Fig. 3. Corretly mapped corners



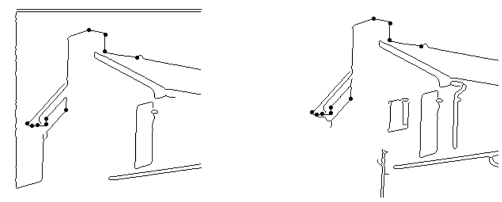
(a) Float corners (b) Reference corners

Fig. 4. Sensitivity of T to the accuracy of correspondences



(a) Float corners (b) Reference corners

Fig. 5. Over 3 corner correspondences are used



(a) Float corners (b) Reference corners

Fig. 6. Not all edges have correspondences in the other image

V. Conclusion

This paper proposes a corner-guided image registration method using edges. Edges are detected and partitioned into segments as matching segments. Corner correspondences are established utilizing global information to determine the transformation parameters. Experimental results show that the proposed method can work robustly on non-ideal edges, and hence will be useful to multimodal image registration.

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