A ROBUST ITERATIVE SUPER-RESOLUTION RECONSTRUCTION OF IMAGE SEQUENCES USING A LORENTZIAN BAYESIAN APPROACH WITH FAST AFFINE BLOCK-BASED REGISTRATION

V. Patanavijit†, S. Tae-O-Sot†† and S. Jitapunkul†††

ABSTRACT

Although the topic of Super-Resolution Reconstruction (SRR) has recently received considerable attention within the traditional research community, the SRR estimations are based on L1 or L2 statistical norm estimation. Therefore, these SRR methods are very sensitive to their assumed data and noise models, which limit their applications. The real noise models that corrupt the measure sequence are unknown; consequently, SRR algorithm using L1 or L2 norm may degrade the image sequence rather than enhance it. The robust norm applicable to several noise and data models is desired in SRR algorithms. This paper proposes an alternate SRR approach based on the stochastic regularization technique of Bayesian MAP estimation by minimizing a cost function. The Lorentzian norm is used for measuring the difference between the projected estimate of the high-resolution image and each low resolution image, removing outliers in the data. Tikhonov regularization is used to remove artifacts from the final result and improve the rate of convergence. In order to cope with real sequences and complex motion sequences, the fast affine block-based registration is used in the registration step of SRR. The experimental results show that the proposed reconstruction can be applied on real sequences such as Suzie sequence and confirm the effectiveness of our method and demonstrate its superiority to other super-resolution methods based on L1 and L2 norm for several noise models such as AWGN, Poisson and Salt & Pepper noise.

Index Terms— SRR, Stochastic Regularization Technique, Lorentzian Norm, Affine Block-Based Registration.

1. INTRODUCTION

SRR is considered to be one of the most promising techniques that can help overcome the limitations of optics and sensor resolution. In general, the problem of super-resolution can be expressed as that of combining a set of aliased, noisy, low-resolution, blurry images to produce a higher resolution image or image sequence. The idea is to increase the information content in the final image by exploiting the additional spatio-temporal information that is available in each of the LR images.

This section presents the literature review regarding for SRR estimation because the SRR estimation is one of the most importance parts of the SRR and directly impacts the SRR performance. R. R. Schultz and R. L. Stevenson [12-13] proposed the SRR algorithm using ML estimator (L2 Norm) with HMRF Regularization in 1996. In 1997, M. Elad and A. Feuer [6] proposed the SRR algorithm using the ML estimator (L2 Norm) with nonellipsoid constraints. M. Elad and A. Feuer [8] proposed the SRR algorithm using R-SD and R-LMS (L2 Norm) in 1999. M. Elad and A. Feuer [7] proposed the fast SRR algorithm using ML estimator (L2 Norm) in 2001. The warps are pure translations, the blur is space invariant and the same for all the images, and the noise is i.i.d. Gaussian. A. J. Patti and Y. Altunbasak proposed [1] a SRR algorithm using ML (L2 Norm) estimator with POCS-based regularization in 2001 and Y. Altunbasak, A. J. Patti, and R. M. Mersereau [20] proposed a SRR algorithm using ML (L2 Norm) estimator for the MPEG sequences in 2002. D. Rajan and S. Chaudhuri [2-3] proposed SRR using ML (L2 Norm) with MRF regularization to simultaneously estimate the depth map and the focused image of a scene in 2003. S. Farsiu and D. Robinson [15-16] proposed SRR algorithm using ML estimator (L1 Norm) with BTV Regularization in 2004. Later, they propose a fast SRR for color images [17] using ML estimator (L1 Norm) with BTV and Tikhonov Regularization in 2006.

For the data fidelity cost function, all the above super-resolution restoration methods [1-20] are based on the simple estimation techniques such as L1 Norm or L2 Norm Minimization. Therefore, these SRR methods are very sensitive to their assumed data and noise models. The success of SRR algorithm is highly dependent on the model accuracy regard the imaging process. These models do not always represent the actual imaging process, as they are merely mathematically convenient formulations of some general prior information. When the data or noise model assumptions do not faithfully describe the measure data, the estimator performance degrades. Furthermore, existence of outliers defined as data points with different distributional characteristics than the assumed model will produce erroneous estimates. Most noise models used in SRR algorithm are based on AWGN model at low power therefore SRR algorithms can effectively apply only on the image sequence that is corrupted by AWGN. With this noise model, L1 norm or L2 (quadratic) norm error are effective. For normally distributed data, the L1 norm produces estimates with higher variance than the optimal L2 norm. On the other hand the L2 norm is very sensitive to outliers and noise because the influence function increases linearly and without bound. The real noise models that corrupt the measured sequence are unknown; consequently, SRR algorithm using L1 norm or L2 norm may degrade the image sequence rather than enhance it. Therefore, the robust norm which is applicable to several noise and data models is desired in SRR algorithms. From the robust statistical estimation study [10], Lorentzian Norm is more robust than L1 and L2. Lorentzian norm is also capable of outlier rejection. The norm

†V. Patanavijit is with Faculty of Engineering, Assumption University, Bangkok, Thailand; E-mail: Patanavijit@yahoo.com.

††S. Tae-O-Sot is with Faculty of Engineering, Naresuan University (Phayao Cambus), Thailand; E-mail: Sarivio@hotmail.com.

†††S. Jitapunkul is with Faculty of Engineering, Chulalongkorn University, Thailand; E-mail: Somchai.j@chula.ac.th .

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must be more forgiving on outliers; that is, the error is increased less rapidly than L2. In this paper, we propose a robust iterative super-resolution reconstruction (SRR) algorithm using Lorentzian norm for the data fidelity cost function with Tikhonov Regularization. Whereas the data fidelity using Lorentzian norm is responsible for robustness and edge preservation while Tikhonov Regularization seeks robustness with respect to blur, outliers, and other kinds of errors not explicitly modeled in the fused images. Due to complex motion in the real sequences, the fast affine block-based registration [19] is used for registration. We show that our method's performance is superior to the technique proposed earlier [4-13], [15-17].

The organization of this paper is as follows. Section 2 reviews the main concepts of robust estimation technique in SRR framework using L1 and L2 error norm with Tikhonov Regularization. Section 3 introduces our proposed robust super-resolution reconstruction using Lorentzian error norm minimization with the affine block-based registration. Section 4 outlines the proposed solution and presents the comparative experimental results obtained by using the proposed Lorentzian norm method and by using the conventional L1 and L2 norm method. Finally, Section 5 provides the conclusion.

2. INTRODUCTION OF SRR

Assume that low-resolution frames of images $\{\mathbf{Y}(t)\}$ are our measured data and each frame contains $N_1 \times N_2$ pixels. A high-resolution frame $\mathbf{X}(t)$ is to be estimated from the low-resolution sequences and each frame contains $qN_1 \times qN_2$ pixels, where q is an integer-valued interpolation factor in both the horizontal and vertical directions. To reduce the computational complexity, each frame is separated into overlapping blocks. For convenience of notation, all overlapping blocked in a frames will be presented a column, lexicographically ordered. Namely, the overlapping blocked LR frame is $\underline{Y}_k \in \mathbb{R}^{M^2}$ $(M^2 \times 1)$ and the overlapping blocked HR frame is $\underline{X} \in \mathbb{R}^{q^2M^2}$ ($L^2 \times 1$ or $q^2M^2 \times 1$). We assume that the two images are related via the following equation:

$$\underline{Y}_k = D_k H_k F_k \underline{X} + \underline{V}_k \quad ; k = 1, 2, \dots, N \tag{1}$$

The matrix F_k stands for the geometric warp between the images $\underline{X}$ and $\underline{Y}_k$, H_k is the blur matrix which is space and time invariant, D_k is the decimation matrix and $\underline{V}_k$ is the system noise. Super resolution is an ill-posed problem [4–7]. For the under-determined cases, there exist an infinite number of solutions which satisfy (1). The solution for square and over-determined cases is not stable that means small amounts of noise in measurements will result in large perturbations in the final solution. Therefore, considering regularization in super-resolution algorithm as a means for picking a stable solution is required. The regularization can help the algorithm to remove artifacts from the final result and improve the rate of convergence.

2.1. SRR using L1 Norm with Regularized Function

A popular family of estimators in SRR is the L1 Norm estimators [4-7]. A regularization term compensates the missing information with some general prior information about the desirable HR solution, and is usually implemented as a penalty factor in the generalized minimization cost function. This estimator is defined in the SRR as follows:

$$\underline{X} = \underset{\underline{X}}{\mathrm{ArgMin}} \left\{ \sum_{k=1}^{N} \left\| D_k H_k F_k \underline{X} - \underline{Y}_k \right\|_1^1 + \lambda \cdot \left\| \Gamma \underline{X} \right\|_2^2 \right\} \tag{2}$$

The classical and simplest regularization cost functions is the Laplacian regularization [16] where the Laplacian kernel is defined as

$$\Gamma = \tfrac{1}{8}\left[1 \quad 1 \quad 1 \quad ; \quad 1 \quad -8 \quad 1 \quad ; \quad 1 \quad 1 \quad 1\right] \tag{3}$$

By the steepest descent method, the solution of equation (2) is defined as follows.

$$\hat{\underline{X}}_{n+1} = \hat{\underline{X}}_n + \beta \cdot \left\{ \begin{array}{l} \left(\sum_{k=-N}^{N} F_k^T H_k^T D_k^T \,\mathrm{sign}\left(\underline{Y}_k - D_k H_k F_k \hat{\underline{X}}_n \right) \right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma \right) \hat{\underline{X}}_n \right) \end{array} \right\} \tag{4}$$

where β is a scalar defining the step size in the direction of the gradient.

2.2. SRR using L2 Norm with Regularized Function

Another popular family of estimators in SRR is the L2 Norm estimators [12-13]. This estimator is defined in the super resolution context with the combination of the Laplacian regularization as follows:

$$\underline{X} = \underset{\underline{X}}{\mathrm{ArgMin}} \left\{ \sum_{k=1}^{N} \left\| D_k H_k F_k \underline{X} - \underline{Y}_k \right\|_2^2 + \lambda \cdot \left\| \Gamma \underline{X} \right\|_2^2 \right\} \tag{5}$$

By the steepest descent method, the solution of equation (5) is defined as

$$\begin{aligned} \hat{\underline{X}}_{n+1} &= \hat{\underline{X}}_n \\ &+ \beta \cdot \left\{ \sum_{k=1}^{N} F_k^T H_k^T D_k^T \left(\underline{Y}_k - D_k H_k F_k \hat{\underline{X}}_n \right) - \left(\lambda \cdot \left(\Gamma^T \Gamma \right) \hat{\underline{X}}_n \right) \right\} \end{aligned} \tag{6}$$

3. THE PROPOSED SRR ALGORITHM

3.1. SRR using Lorentzian Norm with Regularized Function

This paper proposes SRR using Lorentzian norm [10] that is more robust than L1 and L2 norm. The definition of this estimator is defined in the super resolution context as the following minimization problem:

$$\underline{X} = \underset{\underline{X}}{\mathrm{ArgMin}} \left\{ \sum_{k=1}^{N} f_{LOR}\left(D_k H_k F_k \underline{X} - \underline{Y}_k \right) + \lambda \cdot \left\| \Gamma \underline{X} \right\|_2^2 \right\} \tag{7}$$

$$f_{LOR}\left(\underline{X}\right) = \log\left[1 + \frac{1}{2}\left(\frac{\underline{X}}{T} \right)^2 \right] \tag{8}$$

where T is Lorentzian constant parameter. By the steepest descent method, the solution of equation (9) is defined as

$$\hat{\underline{X}}_{n+1} = \hat{\underline{X}}_n + \beta \cdot \left\{ \begin{array}{l} \sum_{k=1}^{N} F_k^T H_k^T D_k^T \cdot \psi_{LOR}\left(\underline{Y}_k - D_k H_k F_k \hat{\underline{X}}_n \right) \\ -\left(\lambda \cdot \left(\Gamma^T \Gamma \right) \hat{\underline{X}}_n \right) \end{array} \right\} \tag{9}$$

$$\psi_{LOR}\left(\underline{X}\right) = f'_{LOR}\left(\underline{X}\right) = \frac{2\underline{X}}{2T^2 + \underline{X}^2} \tag{10}$$

3.2. The Fast Affine Block-Based Registration for SRR [19]

The traditional assumption of the only translation in the registration limits the SRR to the sequences that have simple translation motion. In order to apply the proposed systems to the general sequences, the proposed SRR uses a high accuracy registration algorithm, the fast affine block-based registration [19]. In this section, we propose a scheme for estimating affine block-based motion vectors suitable for several complex motions. The estimation can be separated into 2 stages. In the first stage of the estimation algorithm, the current and reference frames are divides into 50% overlapping blocks (16x16). This stage divides the image into small areas in order to detect and estimate the local motions. The advantage of the block processing is the reduction of the computational load and the possibility of parallel processing. In the second stage, the affine motion vector of each block between the current and reference frame is computed by the M3SS (Modified Three Step Search). The M3SS is proposed to reduce a very high computational load in affine motion vector estimation. The M3SS is designed based on the popular 3SS (Three Step Search.

For the 7x7 displacement window (translation deformation) and $\pm 20^{\circ}$ degree (rotation, extraction or expansion deformation), the proposed M3SS algorithm utilizes a search pattern with $3^6 = 729$ check points on a search window in the first step. The point having the minimum error is used as the center of the search area in the subsequent step. The search window is reduced by half in the subsequent step until the search window equals to Equation (11). (The values of the parameters in this paper give the highest PSNR in the experiments of the following 3 standard sequences: Foreman, Carphone and Stefan)

$$\left[a,b,c,d,e,f\right] = [\pm 0.01, \pm 0.01, \pm 0.125 \pm 0.01, \pm 0.01, \pm 0.125] \quad (11)$$

From [19], the total number of the M3SS check points is fixed at 3.65E+3. Compared with the classical block-based estimation method (translation block-based estimation method) at 0.25 pixel accuracy and w=9, the total number of the M3SS check points has approximately 3 times more than the classical FS (Full-Search) approach but the PSNR performance of the M3SS method is 5-6 dB higher than that of the classical translational method.

4. THE EXPERIMENTAL RESULT

This section presents results obtained by the super-resolution method using the fast affine block-based registration. The experiment was implemented in MATLAB and the block size of LR images is fixed at 8x8 (16x16 for overlapping block) and the search window (w) is 7 for affine block-based registration [19] and 5 Frames for ML estimation process. We used Susie sequence as our test sequences. The sequences are in QCIF format and has complex-edge characteristic. Then, to simulate the effect of camera PSF, the images were convolved with a symmetric Gaussian low-pass filter with the size of 3x3 and the standard deviation of one. The blurred images were subsampled by the factor of 2 in each direction (88x72) and the blurred subsampled images were corrupted by Gaussian noise. The criterion for parameter selection in this paper was to choose parameters which produce both most visually appealing results and highest PSNR. Therefore, to ensure fairness, each experiment was repeated several times with different parameters and the best result of each experiment was chosen.

The first experiment was performed on three AWGN corrupted images at SNR=20, 17.5 and 15 dB. The original HR image is shown in Fig. 1(a-1) – 1(c-1). The corrupted image (40th frame) is showed in Fig. 1(a-2) – 1(c-2). The Lorentzian estimator gave the higher PSNR than L1 and L2 estimator in all cases. The result of L1, L2 and Lorentzian norm estimator at SNR=20dB are shown in Fig. 1(a-3), Fig. 1(a-4) and Fig. 1(a-5), respectively. The result of L1, L2 and Lorentzian norm estimator at SNR=17.5dB are shown in Fig. 1(b-3), Fig. 1(b-4) and Fig. 1(b-5), respectively and the result of L1, L2 and Lorentzian norm estimator at SNR=17.5dB are shown in Fig. 1(c-3), Fig. 1(c-4) and Fig. 1(c-5), respectively

The second experiment was performed on a Poisson noise corrupted images. The original HR image is shown in Fig. 1(d-1) and the corrupted image (40th frame) is shown in Fig. 1(d-2). The Lorentzian estimator gave the higher PSNR than L1 and L2 estimator result. The result of L1, L2 and Lorentzian norm estimator for Poisson Noise are shown in Fig. 1(d-3), Fig. 1(d-4) and Fig. 1(d-5) respectively.

The last experiment was performed on a Salt&Pepper noise corrupted images at D=0.015. The original HR image is shown in Fig. 12(e-1) and the corrupted image (40th frame) is shown in Fig. 1(e-2). The Lorentzian estimator also gave the higher PSNR than L1 and L2 estimator result. The result of L1, L2 and Lorentzian norm estimator for Poisson Noise are shown in Fig. 1(e-3), Fig. 1(e-4) and Fig. 1(e-5) respectively.

5. CONCLUSION

In this paper, we propose an SRR algorithm using a novel robust estimation norm function for SRR framework and fast affine block-based registration with Tikhonov Regularization. The proposed SRR can be applied on image corrupted by the several noise models and can be applied on the real complex sequence such as Susie sequence. Experimental results clearly demonstrated that the proposed algorithm the proposed algorithm is robust against several noise models (AWGN, Poisson and Salt & Pepper noise). The proposed algorithm obviously improves the result in using both the PSNR and virtualization measurements.

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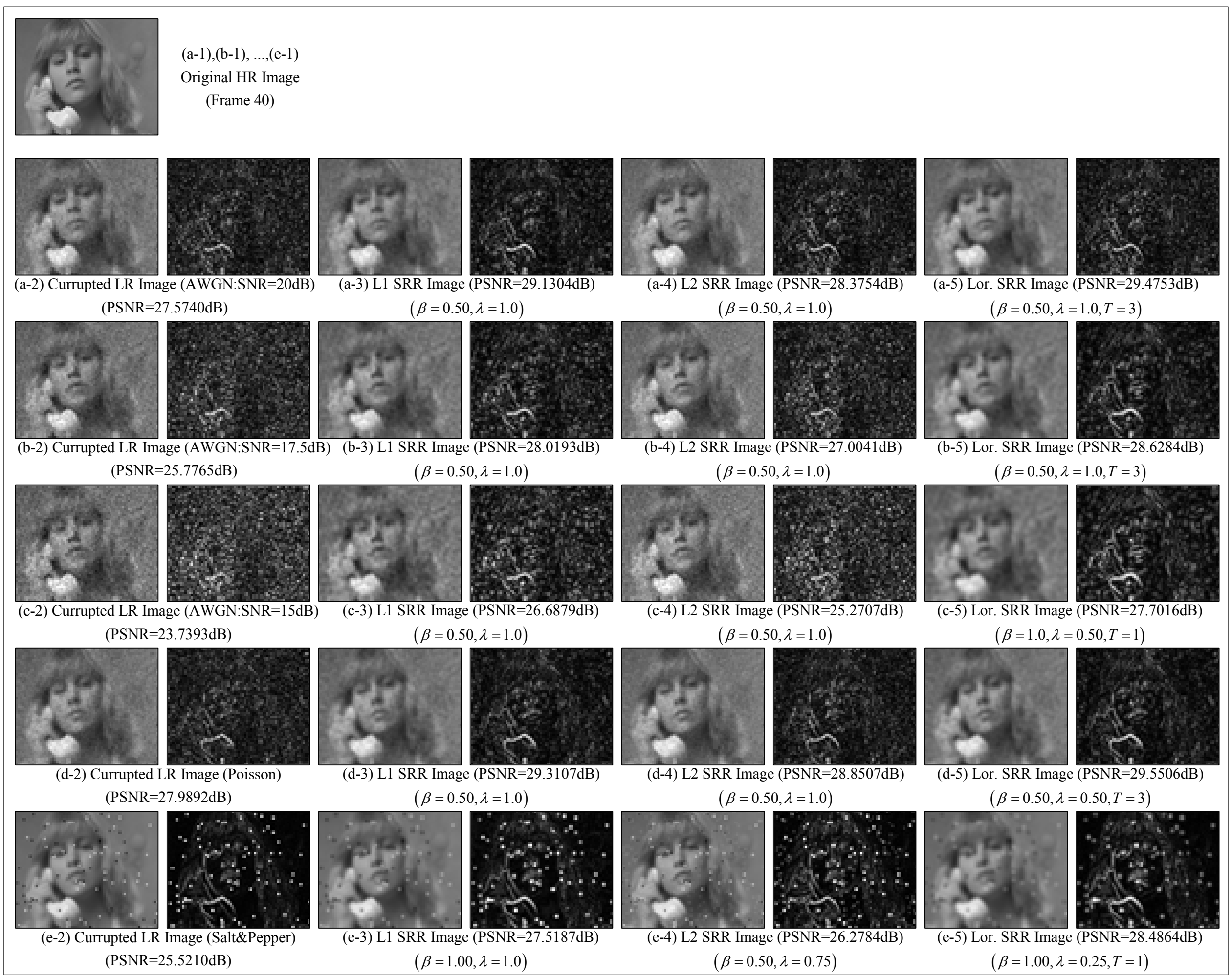

Fig. 1: The Experimental Result of Proposed Method
(The right image on our experiment result of each subfigure is the absolute difference between it’s correspond left image to the original HR image. The difference is magnified by 5.)