STATISTICAL ANALYSIS OF A LINEAR ALGEBRA ASYMMETRIC WATERMARKING SCHEME

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ABSTRACT

We introduce a novel asymmetric watermarking scheme, involving a private key for embedding and a public key for detection, and we detail its statistical analysis, relying on Neyman-Pearson criterion. The proposed scheme solves part of the problems connected to previous watermarking approaches based on linear algebra. In particular, special attention is paid at reducing the side information required at the detector, as well as at achieving higher robustness by enphasizing the contribution of the watermark in the detection phase.

Index Terms— asymmetric watermarking, statistical analysis, linear algebra

1. INTRODUCTION

Digital watermarking and cryptography represent two different approaches to achieve security in telecommunications. Recently, the scientific community has started to investigate their connections, becoming increasingly aware of the potential value and effectiveness of hybrid solutions. In particular, the analogy with public key cryptography suggests to consider asymmetric watermarking, involving a private key for embedding and a public key for detection (see [1], [2] and [3] for a detailed survey and a critical discussion).

Starting from [4], we are exploiting linear algebra tools for designing asymmetric watermarking schemes. We stress that our approach substantially improves previous ones. Indeed, the eigenvector watermarking scheme introduced in [5] has been defeated by an effective attack (see [6], Section 4.4) and the weakness of the method presented in [7] has been demonstrated in [8]. On the other hand, in the scheme proposed in [9] the watermark cannot be chosen arbitrarily, but it turns out to be heavily dependent on the host image (see in particular statement c) of the Theorem on p. 787, which shows that the watermark is forced to be a suitable multiple of a sequence deterministically extracted from the original image). As a consequence, the method of [9] is appropriate just for copyright protection, where only one key is assigned to each image, but definitely not for fingerprinting, where every recipient is identified by its own key. On the contrary, our approach is suitable also for fingerprinting, allowing the insertion into any image of different watermarking sequences (even sequentially into the same image, see [10]).

In this paper, a substantial step forward is introduced, consisting of two main improvements. First, we reduce the side information required by the detector, and second we enforce the robustness of the method by introducing a new parameter β which emphasizes the contribution of the watermark in the detection phase. These two aspects are fundamental for the viability of the proposed method in practical applications. A detailed statistical analysis of the watermarking scheme is provided under the standard assumption (for analytical purposes, see [11]) that the cover image is i.i.d. zero mean Gaussian distributed.

Correspondingly, Section 1 provides a formal description of the method, while Section 2 is devoted to its statistical analysis. Finally, Section 3 collects some numerical results and concluding remarks.

2. WATERMARKING SCHEME

We are going to construct an asymmetric watermarking procedure suitable for digital fingerprinting. Let V be a feature space of dimension d (for instance, the space $\mathbb{R}^d$ corresponding to the entries in the top left corner of the DCT of a digital image), fix an original $\phi \in V$ and let $\{u_1, \dots, u_n\}$ be an ordered set of users. For each $i = 1, \dots, n$, the user u_i is associated with a secret signature $s_i \in V$ such that $\{\phi, s_1, \dots, s_n\}$ is an orthogonal set of vectors.

2.1. Watermark embedding

For each user u_i the following algorithm is implemented:

1) *Watermark setting:* set $\psi_i := \alpha s_i$ with $0 < \alpha << 1$ in order to meet the usual imperceptibility requirement

2) *Watermark insertion:* watermark the copy of ϕ assigned to user u_i by setting $\phi_i = \phi + \psi_i$.

3) *Definition of auxiliary matrices:* choose a $d \times d$ orthogonal matrix M_i and let $\phi = M_i v_i$ and $\psi_i = M_i w_i$ (i.e. v_i, resp. w_i, are the coordinates of ϕ_i, resp. ψ, in the basis given by the columns of M_i). Fix a real parameter $\beta \geq 1$, which controls the impact of the watermark energy in the detection phase, and let $b_1 := \frac{v_i+\beta w_i}{\|v_i+\beta w_i\|}$ and complete it to an orthonormal basis $(b_1, b_2, \ldots, b_d)$ of $\mathbb{R}^d$ such that $< b_1, b_2 >=< v_i, w_i >$ (where $< ... >$ denotes linear span). If N_i is the matrix with b_t as the t-th column ($t = 1, \ldots, d$) and $\Pi_i = (e_i|e_2| \ldots |e_{i-1}|e_1|e_{i+1}| \ldots |e_d)$ is the matrix that permutes the i-th coordinate vector e_i and e_1, then

$$A_i = \Pi_i \Delta N_i^T \tag{1}$$

where

$$\Delta = \begin{pmatrix} 1 & 0 & \ldots & & & & \\ 0 & S & 0 & \ldots & & & \\ 0 & 0 & K & \ldots & & & \\ \vdots & & & \ddots & \vdots & & \\ 0 & \ldots & & & K & 0 & \ldots \\ 0 & \ldots & & & & 0 & \ldots \\ \vdots & & & & & \vdots & \end{pmatrix}$$

with $K >> 1$ an integer appearing k times and $0 < S << K$ a secret real parameter.

4) *Public key releasing:* release to the public the detection key $D_i = \|v_i + \beta w_i\| A_i M_i^T$. Since A_i is not invertible, the secrets ϕ and ψ_i cannot be reconstructed from D_i.

With this definition of the public key the side information needed by the detector is substantially reduced in comparison with previous approaches [4], [10].

2.2. Watermark detection

Let now ϕ_e be an extracted feature. The watermark detection is accomplished by the decision function

$$\delta_i(\phi_e) = \begin{cases} 1 & \textit{if } |\mathrm{sim}(e_i, D_i\phi_e)| \geq T \\ 0 & \textit{otherwise} \end{cases} \tag{2}$$

where $T = T(\beta)$ is a suitable threshold and

$$\mathrm{sim}(e_i, D_i\phi_e) = \frac{e_i^T D_i \phi_e}{\|D_i \phi_e\|} \tag{3}$$

Definitions (2) and (3) for the detector are motivated by the following fact:

Proposition 1. *For every $i = 1, \ldots, n$, we have*

$$\begin{aligned} \mathrm{sim}(e_i, D_i\phi_i) &= \frac{\|v_i\|^2 + \beta\|w_i\|^2}{\|D_i\phi_i\|} \\ \mathrm{sim}(e_i, D_i\phi) &= \frac{\|v_i\|^2}{\|D_i\phi\|} \end{aligned}$$

In particular, if $\beta >> 0$ then there exists a decision threshold $T = T(\beta)$ with $0 < T << 1$ such that $\delta_i(\phi_i) = 1$ and $\delta_i(\phi) = 0$.

The introduction of parameter β, which regulates the impact of the watermark energy at the detector, is therefore crucial for the aim of increasing the robustness of the method.

3. STATISTICAL ANALYSIS

For the sake of simplicity we consider the case $n = 1$, so that A is as in (1) with Π_1 the identity matrix. Let now ϕ_e be an image to be tested. We have two alternative hypotheses:

$$\begin{aligned} H_0 &: \phi_e \text{ does not contain } w \\ H_1 &: \phi_e \text{ contains } w \end{aligned}$$

The detection aims at defining a test of hypothesis H_1 versus the alternative H_0. The *Neyman-Pearson detection criterion* (see for instance [12]) maximizes the probability of watermark detection P_d subject to a constraint on the maximum false detection probability $\bar{P}_f$. The threshold T in (2) is calculated in order to satisfy $P\{|\mathrm{sim}(e_1, D\phi_e)| \geq T|H_0\} = \bar{P}_f$.

In order to apply the Neyman-Pearson criterion it is necessary to know the statistical models of the watermark, the host image and the noise introduced by the attacks.

3.1. Hypothesis H_0

We are interested in determining the probability of false alarm P_f where the similarity detector $\mathrm{sim}(e_1, D\phi_e)$ is used with input ϕ_e, a non watermarked image. For mathematical tractability, we assume that vector ϕ_e is i.i.d. zero-mean Gaussian with variance $\sigma^2_{\phi_e}$.

Note that since $D = \|v + \beta w\| AM^T$, with M an orthogonal matrix, it follows that $D\phi_e$ is Gaussian distributed with covariance matrix $\|v + \beta w\|^2 \Delta^2$. Therefore, $e_1^T D\phi_e$ is Gaussian with zero mean and variance $\sigma^2_{\phi_e} \|v + \beta w\|^2$. On the other hand, the term $\|D\phi_e\|$ has the same distribution as the random variable $\|v + \beta w\|(x_1^2 + S^2 x_2^2 + K^2 \sum_{i=3}^{k+2} x_i^2)^{\frac{1}{2}}$, where k is the number of elements equal to K in the diagonal of A and $x_i \sim \mathcal{N}(0, \sigma^2_{\phi_e})$ for every i. Since $K >> \max\{1, s\}$, we can consider instead the approximation $\|v + \beta w\| K (\sum_{i=3}^{k+2} x_i^2)^{\frac{1}{2}}$. Notice that this random variable is independent of $e_1^T D\phi_e$. This fact will be very useful in the computation of P_f.

Moreover, we can normalize by $\|v + \beta w\| \sigma_{\phi_e}$ both the numerator and the denominator of the similarity function to arrive at the following approximately equivalent problem:

$$P_f = Pr\left\{ \frac{x_1}{K\left(\sum_{i=3}^{k+2} x_i^2\right)^{\frac{1}{2}}} > T \right\}$$

where T is the threshold and $x_i \sim \mathcal{N}(0, 1)$ for all i. Equivalently, we can write

$$P_f = Pr\left\{ x_1 > TK \left(\sum_{i=3}^{k+2} x_i^2 \right)^{\frac{1}{2}} \right\}$$

The term $R = (\sum_{i=3}^{k+2} x_i^2)^{\frac{1}{2}}$ follows a generalized Rayleigh distribution with pdf $P_R(r) = \frac{r^{k-1}}{2^{\frac{k-2}{2}}\Gamma\left(\frac{k}{2}\right)} e^{-\frac{r^2}{2}}$, $r \geq 0$, $k \geq 1$ where $\Gamma()$ denotes the Gamma function. Then, in order to compute P_f we can fix a value of x_1 and determine the probability that R is smaller than $\frac{x_1}{TK}$. Finally, we must average the result with respect to the pdf of x_1. Hence

$$P_f = \int_{-\infty}^{+\infty} P_r \left\{ R < \frac{x_1}{TK} \right\} f_{x_1}(x_1) dx_1$$

The cumulative distribution function of R for the case k even, i.e. $k = 2m$, is $F_R(r) = 1 - e^{-\frac{r^2}{2}} \sum_{n=0}^{m-1} \frac{1}{n!} \frac{r^{2n}}{2^n}$ $r \geq 0$. Hence

$$\begin{aligned} P_f &= 1 - \sum_{n=0}^{m-1} \frac{1}{n! 2^n \sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{x_1^2}{2T^2K^2}} \frac{x_1^{2n} e^{-\frac{x_1^2}{2}}}{(TK)^{2n}} \mathrm{d}x_1 \\ &= 1 - \sum_{n=0}^{m-1} \frac{1}{n! 2^n (TK)^{2n}} \frac{2^{n+1}}{\sqrt{2\pi}\sqrt{2}} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\left(1+\frac{1}{(TK)^2}\right)^{n+\frac{1}{2}}} \\ &= 1 - \sum_{n=0}^{m-1} \frac{TK\Gamma\left(n+\frac{1}{2}\right)}{n! \left((TK)^2+1\right)^{n+\frac{1}{2}} \sqrt{\pi}} \\ &= 1 - \sum_{n=0}^{m-1} \frac{TK\,(2n-1)!!}{2^n n! \left((TK)^2+1\right)^{n+\frac{1}{2}}} \end{aligned}$$

where

$$n!! := \begin{cases} n \times (n-2) \ldots 5 \times 3 \times 1 & n > 0 \text{ odd} \\ n \times (n-2) \ldots 6 \times 4 \times 2 & n > 0 \text{ even} \\ 1 & n = -1, 0 \end{cases}$$

Notice that P_f depends solely on the product TK. This means that for a fixed $\bar{P}_f$ we just have to make the product TK constant (as in the experimental results reported in Fig. 1).

3.2. Hypothesis H_1

Let us consider now hypothesis H_1 and assume that $\phi_e = \phi_1 + n$, where the components of n are i.i.d. $\mathcal{N}(0, \sigma_n^2)$. We have $e_1^T D\phi_e = \|v\|^2 + \beta\|w\|^2 + n_1'$ where $n_1' \sim \mathcal{N}(0, \|v + \beta w\|^2 \sigma_n^2)$. On the other hand,

$$\begin{aligned} D\phi_e = (&\|v+\beta w\| c_1 + n_1', \|v+\beta w\| S c_2 + n_2', \\ &K n_3', \ldots, K n_{k+2}', 0, \ldots, 0)^T \end{aligned} \tag{4}$$

where $c_1^2 + c_2^2 = \|v + w\|^2$ and $n_i' \sim \mathcal{N}(0, \|v + \beta w\|^2 \sigma_n^2)$ for all i.

By assuming $K >> \max\{1, s\}$, we can write $\|D\phi_e\|^2 = K^2 \sum_{i=3}^{k+2} {n_i'}^2$ and $e_1^T D\phi_e$ and $\|D\phi_e\|^2$ can be regarded as being approximately independent. Now the probability of correct detection P_d is

$$P_d = Pr\left\{ \left(\|v\|^2 + \beta\|w\|^2 + n_1'\right)^2 > T^2 \left(K^2 \sum_{i=3}^{k+2} {n_i'}^2 \right) \right\}$$

If we assume $\|v + \beta w\|^2 >> n_1'$, then we can make the following simplification:

$$P_d = Pr\left\{ \left(\|v\|^2 + \beta\|w\|^2\right)^2 + 2\left(\|v\|^2 + \beta\|w\|^2\right) n_1' > T^2 K^2 \sum_{i=3}^{k+2} {n_i'}^2 \right\}$$

If $n_i'' = \frac{n_i'}{\|v+\beta w\|\sigma_n}$, then $n_i'' \sim \mathcal{N}(0, 1)$, for all i and we can write

$$P_d = Pr\left\{ \mu_z + \sigma_z n_1'' > \sum_{i=3}^{k+2} {n_i''}^2 \right\}$$

where $\mu_z = \frac{\left(\|v\|^2+\beta\|w\|^2\right)^2}{\|v+\beta w\|^2 \sigma_n^2 T^2 K^2}$ and $\sigma_z = \frac{2\left(\|v\|^2+\beta\|w\|^2\right)}{\|v+\beta w\|\sigma_n T^2 K^2}$. The term $s = \sum_{i=3}^{k+2} {n_i''}^2$ is chi-squared distributed with k degrees of freedom and pdf $f_S(s) = \frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)} s^{\frac{k}{2}-1} e^{-\frac{s}{2}}$ $s \geq 0$. If $k = 2m$, the cumulative distribution function becomes $F_S(s) = 1 - e^{-\frac{s}{2}} \sum_{i=0}^{m-1} \frac{1}{i!} \left(\frac{s}{2}\right)^i$.

Finally,

$$P_d = \frac{1}{\sqrt{2\pi}\sigma_z} \int_0^{\infty} F_S(s) e^{-\frac{(s-\mu_z)^2}{2\sigma_z^2}} ds$$

We will numerically solve this integral in the following experimental results.

Notice that we can rewrite both μ_z and σ_z in a way that makes more explicit the dependence with the Document-to-Watermark Ratio $DWR = \frac{\|v\|^2}{\|w\|^2}$ and the Watermark-to-Noise Ratio $WNR = \frac{\|w\|^2}{d\sigma_n^2}$ (this also shows the dependence on d). Namely, $\mu_z = \frac{(DWR+\beta)^2 WNR\, d}{(DWR+\beta^2)T^2K^2}$ and $\sigma_z = \frac{2\sqrt{\mu_z}}{KT}$.

3.3. Hypothesis H_1 with the original as input

Let us calculate now the probability of deciding that H_1 holds when the detector is given the original. The derivations are almost identical to the previous case with the difference that now $e_1^T D\phi_e = \|v\|^2 + n_1'$ and $c_1^2 + c_2^2 = \|v\|^2$ in (4). The probability of false positive when the detector is given the noisy original P_f' is

$$P_f' = Pr\left\{ \left(\|v\|^2 + n_1'\right)^2 > T^2 \left(K^2 \sum_{i=3}^{k+2} {n_i'}^2 \right) \right\}$$

and assuming $\|v\|^2 >> n_1'$ we have

$$P_f' = Pr\left\{ \|v\|^4 + 2\|v\|^2 n_1' > T^2 K^2 \sum_{i=3}^{k+2} {n_i'}^2 \right\}$$

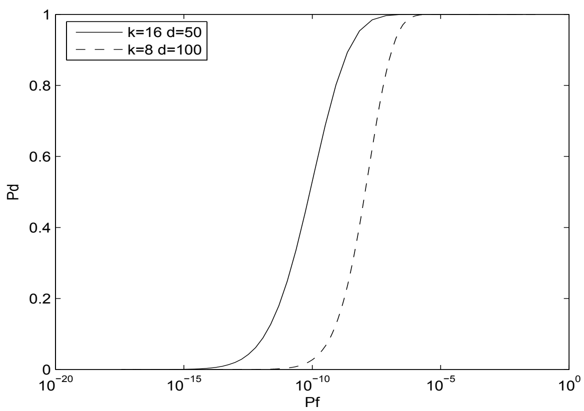
If $n_i'' = \frac{n_i'}{\|v+\beta w\|\sigma_n}$, then $n_i'' \sim \mathcal{N}(0, 1)$ for all i and we can write

$$P_f' = Pr\left\{ \mu_z + \sigma_z n_1'' > \sum_{i=3}^{k+2} {n_i''}^2 \right\}$$

where $\mu_z = \frac{\|v\|^4}{\|v+\beta w\|^2 \sigma_n^2 T^2 K^2}$ and $\sigma_z = \frac{2\|v\|^2}{\|v+\beta w\| \sigma_n T^2 K^2}$.

Exactly as above, we finally obtain

$$\begin{aligned} \mu_z &= \frac{(DWR)^2 \; WNR \, d}{(DWR + \beta^2) T^2 K^2}, \; \sigma_z = \frac{2\sqrt{\mu_z}}{KT} \\ P'_f &= \frac{1}{\sqrt{2\pi}\sigma_z} \int_0^\infty F_S(s) e^{-\frac{(s-\mu_z)^2}{2\sigma_z^2}} ds \end{aligned}$$



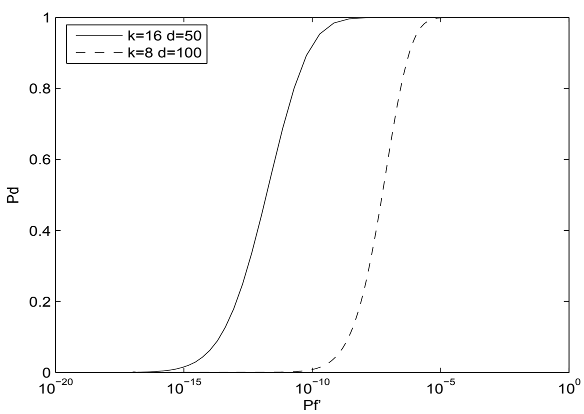


Fig. 1. ROC curves for $DNR = 30dB$

4. CONCLUSIONS

Performance of the Newman-Pearson criterion can be evaluated through the ROC (Receiver Operating Characteristic) curves, which plots P_d against P_f and against P'_f (see Fig. 1), and depend on the Document-to-Noise Ratio (DNR) $\sigma^2_{\phi_e}/\sigma^2_n$. In both cases $\beta = 5000$ and the product kd, which represents the amount of information at the detector side, is kept constant. Hence we see that we can reach an arbitrarily high detection probability and simultaneously a false positive probability bounded under a fixed threshold, thus demonstrating the effectiveness of our method.

Future work will be devoted to its statistical evaluation on more realistic model of the host data.

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