Fast Computation of Zernike Moments For Rice Sorting System

Chong-Yaw Wee[1], *Raveendran Paramesran[1], Fumiaki Takeda[2]
*Corresponding author
[1]Dept. of Electrical Engineering
Faculty of Engineering
University of Malaya
50603 Kuala Lumpur, Malaysia.
Email: ravee@um.edu.my
[2]Dept. of Information
Systems Engineering
Kochi University of Technology
Kochi, Japan.
Email: takeda.fumiaki@kochi-tech.ac.jp

***Abstract*— In a rice sorting system, the separation distance of one grain from another is determined by the amount of time taken to process the image of each grain. The closer the separation distance of one grain from another will lead to higher output of grains imaged and processed. Hence, the fast computation of a reliable set of pattern features is required in a rice sorting system. Zernike moment, with its rotational invariant property makes it ideally suited to be used. Many fast computation algorithms to compute Zernike moments have been developed. In this study we propose a new application of the symmetrical property of Zernike polynomials to compute a set of Zernike moments in rice sorting system. The average overall classification time of a rice grain is reduced almost by 25.0% when the proposed technique is integrated with the q-recursive method, which currently uses the minimum time to compute a set of Zernike moments.**

Keywords – Rice Sorting Machine, Zernike moments, rotational invariant, symmetrical property, fast computation.

I. Introduction

The set of orthogonal Zernike moments has been proven to be superior to other moment functions in terms of its image representation capability and invariant property. The applications of Zernike moments in image analysis includes invariant pattern or object recognition [8], [14], image reconstruction [11], edge detection [4], image segmentation [5] and optimal corneal surfaces modeling [6]. With the orthogonal property, the set of Zernike moments can represent an image by a set of mutually independent descriptors with minimum amount of information redundancy [12]. Besides that, it also enables the separation of the individual contributions of each order moment to the reconstruction process. Hence, the image reconstruction using Zernike moments is much more easier to be performed when compared to other moment functions.

Nevertheless, the main concern of using the Zernike moments is their computational complexity which involves the factorial computation. The computation of factorial terms not only intensive but also leads to numerical instability especially for higher order moments. Many fast algorithms have been developed to bypass the computation of the factorial terms. Chong $et\ al.$ employ the recurrence relationships between successive Zernike polynomial coefficients to achieve the fast computation [2]. Another approach uses the recurrence relationships between successive Zernike polynomials such as Belkasim's [1], Prata's [7], Kintner's [10] and its simplified version (p-recursive) [13], and q-recursive [3] methods. These fast algorithms successfully reduce the computational complexity of Zernike radial polynomials. However, these methods do not use the symmetrical property of the Zernike polynomials.

In this study, the application of symmetrical property of Zernike polynomials to compute a set of Zernike moments for rice sorting system is proposed. By using the symmetrical property, only about one eighth of the entire set of Zernike radial polynomials are required to be computed instead of the entire set. This implies that the time required to compute the set of Zernike moments is significantly reduced. The technique can be easily integrated to other existing fast algorithms to further improve their computation speed.

The organization of this paper is given as follow. The brief descriptions regarding the rice sorting system is presented in section II. The basic theories of Zernike moments are briefly given in section III. The symmetrical property of Zernike polynomials and how it can be applied to reduce the computation time of Zernike moments are described in section IV. In section V, performance of the proposed technique is validated through experiments. Section VI concludes the study.

II. Rice sorting system

The rice sorting system used in this study is based on the Zernike moment feature extractor and neural network classifier. This system is constructed with a shunt, where rice grains flow, line-sensors for capturing rice grains data image and air gun for separating the rice grains. The hardware architecture of this system is shown in Figure 1.

As the rice grain falls down through a shunt to the inspection part of rice sorting system, the line sensors capture the image of it. The image is in 256-color Bitmap format (RGB). Only the blue-color component is picked up from RGB of color information to reduce the effects caused by noises during the imaging process. Then the rice grain image is converted into binary format using a threshold value. The binarized rice grain image contains pixel values in terms of '0' and '1', where '1' denotes rice and '0' denotes background.

The rice grains used in this study consist of two different classes of rice grains, namely good rice and damaged rice.

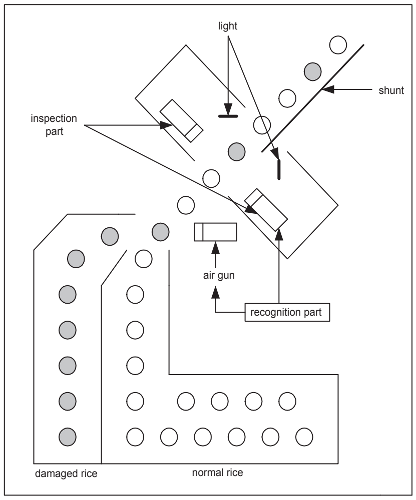

Fig. 1. Hardware architecture of the rice sorting system.

The examples of rice grains are shown in Figures 2 and 3 respectively. These rice grains are classified based on their surface and shape. The good rice has a clean surface with its shape is intact while the surface of damaged rice is not clean and broken.

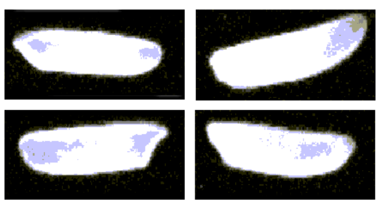

Fig. 2. Examples of good rice grain.

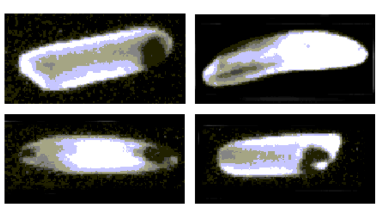

Fig. 3. Examples of damaged rice grain.

After the preprocessing stage, a set of Zernike moments up to 12th order is computed from the rice grain image. This Zernike moments set is then fed into a trained Multilayer Perceptron (MLP) classifier to separate the good and damaged rice grains. The classification result is sent to the air gun where it is activated to separate the damaged rice grains from the good one.

III. Zernike Moments

The two-dimensional Zernike moment of order p with repetition q of an image intensity function $f(x, y)$ is defined as

$$Z_{pq} = \frac{p+1}{\pi} \iint\limits_{x^2+y^2 \leq 1} V^*_{pq}(x, y) f(x, y) \mathrm{d}x\mathrm{d}y \tag{1}$$

where the Zernike polynomial, $V_{pq}(x, y)$ is defined as

$$V_{pq}(x, y) = R_{pq}(r) e^{jq\theta}, \qquad r \in [-1, 1] \tag{2}$$

with $r = \sqrt{x^2 + y^2}$ is the length of the vector from the origin to the pixel (x, y), and $\theta = \tan^{-1}(y/x)$ is the angle between the vector r and the principal x-axis. The Zernike real-valued radial polynomial is given by

$$R_{pq}(r) = \sum_{s=0}^{(p-q)/2} (-1)^s \frac{(p-s)!}{s!(\frac{p+|q|}{2} - s)!(\frac{p-|q|}{2} - s)!} r^{p-2s} \tag{3}$$

where $p - |q|$ is even, $0 \leq |q| \leq p$ and $p \geq 0$. Letting $s \rightarrow (p-k)/2$, Zernike polynomials in (2) can be rewritten as

$$V_{pq}(x, y) = \sum_{k=q}^{p} B_{pqk} r^k e^{jq\theta} \tag{4}$$

where the polynomial coefficient, B_{pqk} is defined as

$$B_{pqk} = \frac{(-1)^{\frac{p-k}{2}} (\frac{p+k}{2})!}{(\frac{p-k}{2})!(\frac{k+q}{2})!(\frac{k-q}{2})!} \tag{5}$$

Hence, Zernike moments in (1) can be represented in polar form as

$$Z_{pq} = \frac{p+1}{\pi} \int_{-1}^{1} \int_{-\pi}^{\pi} \sum_{k=q}^{p} B_{pqk} r^k e^{-jq\theta} f(r, \theta) \ r\mathrm{d}r\mathrm{d}\theta \tag{6}$$

with $\mathrm{d}x\mathrm{d}y = r\mathrm{d}r\mathrm{d}\theta$ and $-\pi \leq \theta \leq \pi$.

The set of Zernike moments possesses the inherent rotational invariant property. The invariant features are obtained by taking the magnitude values of Zernike moments since they remain the same to those image functions before and after rotation. The rotation invariant Zernike moment is extracted by considering only their magnitude values as

$$|Z^r_{pq}| = |Z_{pq} e^{-jq\alpha}| = |Z_{pq}| \tag{7}$$

where Z^r_{pq} is the Zernike moment computed from rotated image. Only the magnitude of Zernike moments with $q \geq 0$ are used since $Z_{p,-q} = Z^*_{pq}$ and $|Z_{p,q}| = |Z_{p,-q}|$ [9].

IV. Symmetrical Property of Radial Polynomials

The computational complexity of Zernike moments is mostly contributed by the computation of factorial terms in their polynomials. As shown in (3), the Zernike polynomials, $V_{pq}(r, \theta)$ can be separated into the real-valued radial polynomials, r^p and exponential angle term, exp$(jq\theta)$. The real-valued radial polynomial is the power series of the radius vector of the square image while the exponential angle term is the exponential of angle which is bounded between the radius vector and the principal x-axis.

The symmetrical characteristics of each term is explored individually and then their symmetrical characteristics are combined together to obtain the symmetrical property for Zernike polynomials. Let a $(N \times N)$ square image is mapped onto the unit disk of Zernike polynomials as shown in Figure 4.

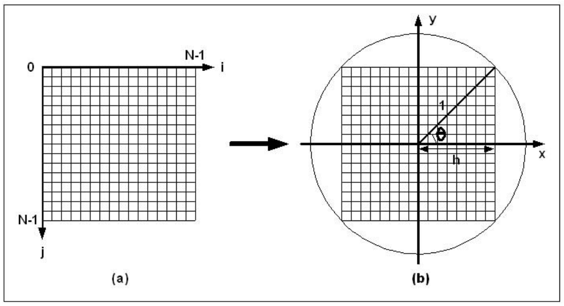

Fig. 4. (a) $(N \times N)$ square image. (b) Mapping of square image into unit disk of radial polynomials.

Let r be the vector that connects from the centroid of unit disk to each pixel of a square image. Then the radius vector r is computed as

$$r = \sqrt{x^2 + y^2} \tag{8}$$

By just considering the outermost pixels in the first quadrant of the mapped image, the r value increases as the angle between r and the principal x-axis, θ increases. The minimum value of r that touching the image edge is $1/\sqrt{2}$. Its maximum value, $r = 1$ is achieved at $\theta = 45°$. As the θ value increases again, the r value starts to decrease from its maximum value back to $1/\sqrt{2}$ at $\theta = 90°$. The increment and decrement trends of r are similar for all other inner pixels. Furthermore, the increment of r value for $0° \leq \theta \leq 45°$ is the same as the decrement of r for $45° \leq \theta \leq 90°$. Hence, within a quadrant, almost half of the computation of r can be saved.

Similar conditions are observed in the remaining three quadrants because the r value of each pixel is repeated for every $45°$. Therefore, the computation of r can be limited to just within the first $45°$. This implies that the computation time for Zernike real-valued radial polynomials is reduced to almost 12.5% of the entire set.

The percentage of reduction in computation time of Zernike real-valued radial polynomials is obtained as

$$P_d = (1 - \frac{\sum_{i=1}^{N/2} i}{N^2}) \times 100\% \tag{9}$$

The percentage of reduction P_d, in computation time increases as the resolution of square image increases.

Let θ be the angle between the radius vector r and the principal x-axis, then it can be computed as

$$\theta = \tan^{-1}(\frac{y}{x}) \tag{10}$$

The θ values in the first and fourth quadrants of a square image are the same except for their signs. All the θ values are negative in the first quadrant while positive in the fourth quadrant. Similarly, the second and third quadrants also show the same characteristics as the first and fourth quadrants where all the θ values are negative and positive in the second and third quadrants, respectively. The θ values are symmetry to the principal x-axis and hence their computation can be reduced to just within the first $180°$.

By combining the aforementioned symmetrical characteristics, the computation time of Zernike polynomials can be reduced significantly. Since the Zernike polynomials contribute most of the computation time of Zernike moments, the significant reduction in computation time for Zernike moments can be attained by using the proposed technique.

V. Experimental Study

Several numerical experiments are performed in this section to validate the performance of the proposed technique either using synthetic or actual rice grain images.

A. Synthetic Random Images

In this experiment, randomly generated images are used. The resolution of the randomly generated images, $(N \times N)$ is set equal to (64×64). Then, the Zernike moments up to different maximum moment order are computed from these synthetic images using the traditional and proposed techniques. The maximum moment order, p is varied from 5th to 40th in steps of 5, i.e. $p = 5, 10, 15, \ldots, 40$. Four existing computation methods are compared, i.e. direct, Prata's, Kintner's and q-recursive methods. The average CPU elapsed time of Zernike moments computation for 20 random images is used as the measurement of the computation speed. The average CPU elapsed time that required for the computation of entire set of Zernike moments using the traditional and proposed symmetrical techniques is shown in Table I. In the following tables, **T** is denoted as the computation methods in original form while **S** is denoted as the computation methods which are integrated with the proposed symmetrical technique.

The results from Table I show the significant reduction in average CPU elapsed time for all ZM computation methods using the proposed symmetrical technique. The reduction is most significant for direct computation method where the reduction is more than 85.0%. Prata's method also provides very significant reduction with its percentage of reduction is slightly less than direct method. Kintner's and q-recursive methods provide similar percentage of reduction which is almost 50.0%.

B. Actual Rice Grain Images

The actual rice grain images used in this study are (64×64) color images and some of the examples are shown in Figure 2 and 3. Zernike moments up to 12th order are computed from these rice grain images using the traditional and proposed symmetrical techniques. The computation process is repeated for 200 rice grain images. The average CPU elapsed time of Zernike moments up to 12th order using the traditional and proposed symmetrical techniques is shown in Table II. The

TABLE I

AVERAGE CPU ELAPSED TIME (SEC) FOR THE RANDOM IMAGES. (% = PERCENTAGE OF REDUCTION)

Order	Direct			Prata			Kintner			q-recursive		
	T	S	%	T	S	%	T	S	%	T	S	%
8	56.922	7.7233	13.57	12.625	1.8233	14.44	0.1735	0.0758	43.67	0.1705	0.0727	42.64
12	157.78	20.058	13.98	21.531	3.0768	14.29	0.2887	0.0618	45.81	0.2775	0.1352	48.72
16	226.98	32.027	14.11	32.125	4.5842	14.27	0.5334	0.2661	49.89	0.5149	0.2455	47.67
20	436.75	60.577	13.87	46.313	6.6552	14.37	0.8672	0.5100	58.81	0.8515	0.4649	46.49
24	718.37	99.351	13.83	69.751	9.8907	14.18	1.2850	0.7321	56.97	1.2166	0.5986	49.20
28	1010.6	141.69	14.02	98.875	14.347	14.51	1.7975	1.0447	58.12	1.6741	0.8589	51.31
32	1513.2	208.52	13.78	127.18	18.416	14.48	2.3743	1.3659	57.53	2.1319	1.1454	53.73

results shown in Table II are very similar to the one which obtained using the randomly generated synthetic images.

TABLE II

AVERAGE CPU ELAPSED TIME (SEC) FOR THE ACTUAL RICE GRAIN IMAGES.

Method	Techniques		Reduction Ratio (%)
	T	S	
Direct	125.78	16.955	86.52
Prata	23.876	3.4127	85.71
Kintner	0.3887	0.1949	49.86
q-recursive	0.3518	0.1745	50.40

C. Classification Results

In this experiment, only the fastest method, q-recursive method is used in the classification operation. The performances of the MLP classifier in terms of the overall processing time for single rice grain and their classification accuracy are shown in Table III. The results show that the classification accuracy of both traditional and proposed symmetrical techniques are same due to the same Zernike moment values are computed. However, the average overall classification time of a rice grain for the proposed symmetrical technique is reduced almost by 25.0% when compared to the traditional technique.

TABLE III

MLP PERFORMANCE USING ZERNIKE MOMENTS UP TO 12TH ORDER.

Hidden Nodes	Time (s)		Accuracy (%)
	T	S	
15	0.8945	0.6812	94.0
20	0.8973	0.6852	96.0
25	0.9045	0.6894	96.0
30	0.9162	0.6958	97.0
35	0.9364	0.6986	98.0
40	0.9527	0.6998	98.0
Average	**0.9169**	**0.6917**	**96.5**

VI. CONCLUSIONS

In a rice sorting system, the separation distance of one grain from another is determined by the amount of time taken to process the image of each grain. The goal to have as close a separation distance as possible is to increase the number of grains processed. Hence, the fast computation of a reliable set of pattern features is required in a rice sorting system. Zernike moment, with its rotational invariant property makes it ideally suited and many fast computation algorithms have been developed. In this study, a new technique which uses the symmetrical property of the Zernike polynomials is proposed to compute a set of Zernike moments. This technique successfully reduces the computation time of Zernike moments at least by half when it is integrated with the existing computation methods. The average overall classification time of a rice grain is reduced almost by 25.0% when the proposed technique is integrated with the currently fastest algorithm, the q-recursive method.

REFERENCES

[1] S. O. Belkasim, M. Ahmadi, and M. Shridhar, “Efficient algorithm for fast computation of zernike moments,” in *Proc. of IEEE 39th Midwest Symposium on Circuit and Systems*, vol. 3, 1997, pp. 1401 – 1404.

[2] C. W. Chong, R. Mukundan, and P. Raveendran, “An efficient algorithm for fast computation of zernike moments,” in *Proc. of Int. Conf. on Compt. Vision, Pattern Recognition and Image Processing*, Durham, North Carolina, Mac. 2002, pp. 785 – 788.

[3] C. W. Chong, P. Raveendran, and R. Mukundan, “A comparative analysis of algorithm for fast computation of zernike moments,” *Pattern Recognition*, vol. 36, no. 3, pp. 731 – 742, Mac. 2003.

[4] S. Ghosal and R. Mehrotra, “Edge detection using orthogonal moment-based operators,” in *Proc. of 11th Image, Speech and Signal Analysis (IAPR) Int. Conf. on Pattern Recognition*, vol. III, 1992, pp. 413 – 416.

[5] ——, “Segmentation of range images: An orthogonal moment-based integrated approach,” *IEEE Trans. on Robotics and Automation*, vol. 9, no. 4, pp. 385 – 399, Aug. 1993.

[6] D. R. Iskander, M. R. Morelande, M. J. Collins, and B. Davis, “Modelling of corneal surfaces with radial polynomials,” *IEEE Trans. on Biomedical Engineering*, vol. 49, no. 4, pp. 320 – 328, Apr. 2002.

[7] A. Prata. Jr. and W. V. T. Rusch, “Algorithm for computation of zernike polynomials expansion coefficients,” *Applied Optics*, vol. 28, no. 4, pp. 749 – 754, Feb. 1989.

[8] A. Khotanzad and J. H. Lu, “Classification of invariant image representations using a neural network,” *IEEE Trans. on Acoustics, Speech, Signal Processing*, vol. 38, pp. 1028 – 1038, Jun. 1990.

[9] A. Khotanzand and Y. H. Hong, “Invariant image recognition by zernike moments,” *IEEE Trans. on Patt. Anal. and Mach. Intell.*, vol. 12, no. 5, pp. 489 – 497, May 1990.

[10] E. C. Kintner, “On the mathematical properties of the zernike polynomials,” *Optica Acta*, vol. 23, no. 8, pp. 679 – 680, 1976.

[11] M. Pawlak, “On the reconstruction aspect of moment descriptors,” *IEEE Trans. on Information Theory*, vol. 38, no. 6, pp. 1698 – 1708, 1992 1992.

[12] C. H. Teh and R. T. Chin, “On image analysis by the methods of moments,” *IEEE Trans. on Patt. Anal. and Mach. Intell.*, vol. 10, no. 4, pp. 496 – 512, Jul. 1988.

[13] C. Y. Wee, P. Raveendran, and F. Takeda, “New computation methods for full and subset zernike moments,” *Information Sciences*, vol. 159, no. 3 - 4, pp. 203 – 220, Feb. 2004.

[14] C. Y. Wee, P. Raveendran, F. Takeda, T. Tsuzuki, H. Kadota, and S. Shimanouchi, “Classification of rice grain using new scale invariant zernike moments,” in *Proc. of Int. Conf. on Computer Vision, Pattern Recognition and Image Processing (CVPRIP’2002)*, Durham, South Carolina, USA, Mac. 2002, pp. 832 – 835.