# A GLOBAL-TO-LOCAL 2D SHAPE REGISTRATION IN IMPLICIT SPACES USING LEVEL SETS

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**ABSTRACT**

This work deals with the problem of shape registration. Broadly speaking, the problem is that of establishing point-wise correspondences in two different shapes of arbitrary dimension and topology. This problem is a fundamental component in numerous image and vision applications. We propose a variational framework for a dense global-to-local 2D shape registration. Affine transformations are accounted for using Vector Distance Functions. Based on this representation, a dissimilarity measure between the two shapes is minimized to recover the global matching parameters. The local coordinate transformation between the two shapes is explicitly estimated by solving a regularized non-linear *PDE*-based motion model. Various experimental results are presented and discussed to show the potential of the proposed framework with a finite element (*FE*)-based validation of its performance.

***Index Terms—*** Shape Registration, Active Contours, Implicit Representations, Regularization.

## 1. INTRODUCTION

Shape registration is a fundamental problem in computer vision, and an important component in various medical applications. Recognition, tracking and retrieval are some other examples of applications that may benefit from shape registration. Shape registration approaches can be categorized based on three main aspects: the selected model to represent the shapes, the transformation model, and the mathematical framework chosen to recover the registration parameters. The selection of shape representation model affects greatly the performance of any existing shape registration algorithm. In [1], the authors choose to represent the shapes to be registered as the zero level sets of distance functions in a higher dimensional space. This implicit representation is known to be invariant to translations and rotations, and performs well in the case of homogeneous scales. To account for anisotropic scales, the authors proposed to maximize an information based criterion between the two shapes, namely the mutual information (*MI*). Hong et al. [2] proposed a new shape representation method and showed its potential in image matching and segmentation. This new algorithm is based on integral kernels and represents a shape as the area of intersection between the kernel and the inside and outside of the shape. Other methods have been considered to represent shapes in different application, including cloud points [3], medial axis [4] and Fourier descriptors [5].

Transformation models can be divided into two classes: global and local. The global transformation models are usually defined by a small set of parameters. These models include, among others, the rigid transformation (translations and rotations), and the affine transformations, which in addition to translations and rotations, account for anisotropic scaling and/or shearing. Such transformations can be used alone to efficiently align two shapes. However, in the case of local deformations, more complex transformations are required in order to establish dense correspondences between the two given shapes. Different techniques were developed to solve this problem, including the optical flow [6], the incremental free form deformation [1], the thin plate spline [7].
Given the transformation model and the selected shape representation model, most existing shape registration techniques are based on the optimization of a similarity/dissimilarity criterion between the two shape. For instance, the Sum of Squared Differences (*SSD*) between the data values is one popular criterion that is mostly appropriate when the two data sets have comparable values (e.g., monomodal images), while the *MI* is a stochastic measure that is more appropriate to register images with different modalities or matching shapes under scale variations. The estimation of a dense displacement field by optimization of any of these criteria is an ill-posed problem in the sense of Hadamard. To cope with this issue, other constraints are to be introduced to guarantee the well posedeness of the problem. These constraints are usually added to the cost function as a regularization term using physically based functionals relying on elasticity theory [8, 9].

A different approach for solving the non-rigid registration problem is based on active contours formulation. This approach was independently adopted by Vemuri et al. [10] for image registration, and by Bertamio et al. [11] for image segmentation and tracking. It is this variational framework that we consider, in part, in this paper in order to locally match shapes. The shapes are first globally aligned by minimizing a *SSD* criterion measuring the disparity between the two implicit representations of the source and target shapes using vector distance functions. This shape representation leads to better results in the presence of scale variations. We test this criterion to globally register various 2D shapes, and we compare the results to those obtained when using a rigid registration criterion based on Signed Distance *(SD)* representation as used in [12]. Secondly, the aligned shapes are implicitly represented using their *SD* maps and then we let the level sets of the *SD* of the warped source shape evolve towards those of the target shape under some constraints borrowed from elasticity theory. This allows to establish a dense one-to-one displacement field between the shapes. Various experimental results are presented to show the potential of the proposed framework. Correlation coefficients are computed as a measure of the alignment accuracy. Finally, we propose a *FE*-based approach to quantitatively validate the non-rigid alignment model.

## 2. VECTOR DISTANCE FUNCTION AND GLOBAL ALIGNMENT

Vector Distance Functions (*VDF*) were proposed in [13] as an efficient way of representing shapes, and they have been popular in

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various image analysis applications. Given a manifold $\mathcal{M}$ in $\mathbb{R}^n$, $(n = 2, 3)$, let $\delta(\mathbf{x}) := dist(\mathbf{x}, \mathcal{M})$ denote the Euclidean distance from a point $\mathbf{x} \in \mathbb{R}^n$ to $\mathcal{M}$, The vector distance function $V(\mathbf{x})$ is given as the derivative of the squared distance function $\eta(\mathbf{x}) := \frac{1}{2}\delta^2(\mathbf{x})$. That is, $V(\mathbf{x}) := \nabla\eta(\mathbf{x}) = \delta(\mathbf{x})\nabla\delta(\mathbf{x})$. The VDF $V(\mathbf{x})$ is an implicit representation of the manifold $\mathcal{M}$, with $\mathcal{M} = V^{-1}(\mathbf{0})$, and for each $\mathbf{x} \in \mathbb{R}^n$, $V(\mathbf{x})$ is a vector of length $\delta(\mathbf{x})$. If $\mathcal{M}$ is smooth at $\mathbf{x}$, then the VDF to $\mathcal{M}$ at $\mathbf{x}$ is given by (see [2] for more details) $V(\mathbf{x}) = \mathbf{x} - \mathbf{x}_0$, with $\mathbf{x}_0$ being the closet point to $\mathbf{x}$ on $\mathcal{M}$. An example of the VDF representation is shown on Figure 1(cols. 3 & 4).

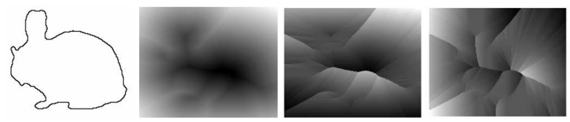

**Fig. 1**. Left to Right: Shape Contour; The signed distance representation; The x- and y-components of the *VDF* representation.

### 2.1. Global Matching

Now, let us consider a source shape, $S_1$, and a target shape, $S_2$, with corresponding VDF representations, respectively noted by $V_1$ and $V_2$. Let $A(\mathbf{x}) = SR\mathbf{x} + T$ denote a global transformation on $\mathbb{R}^n$, where $S = diag(s_i)_{1 \le i \le n}$ is a diagonal matrix whose entries are the scale factors in each direction; $R = R(\theta)$ a rotation matrix; and $T$ is a translation vector in $\mathbb{R}^n$. Given an image point $\mathbf{x}$ on the source shape, and its VDF transform, $\mathbf{x}_0 = V_1(\mathbf{x})$, and its transform by $A$, $\widehat{\mathbf{x}} = A(\mathbf{x})$, one can easily show that

$$V_2(\widehat{\mathbf{x}})) = \widehat{\mathbf{x}} - \widehat{\mathbf{x}}_0 = SR(\mathbf{x} - \mathbf{x}_0) = SRV_1(\mathbf{x}).$$

Based on this property of the *VDF* under affine transformations, we can consider the following *SSD* criterion to recover the global matching parameters that serve to align the shapes $S_1$ and $S_2$:

$$E(S, R, T) = \int_\Omega r^T.r d\mathbf{x}, \text{ with } r(\mathbf{x}) = SRV_1(\mathbf{x}) - V_2(A(\mathbf{x})), \tag{1}$$

where $\Omega$ is the image domain of the two shapes. The functional $E$ measures the dissimilarity between the VDF values of the two shapes. Minimizing this energy on the entire image domain in computationally expensive. Hence, we limit the matching space to a band around the zero level of the VDFs. This leads to minimizing

$$E(S, R, T) = \int_\Omega \delta_\alpha(V_1(\mathbf{x}), V_2(A(\mathbf{x}))).r^T(\mathbf{x}).r(\mathbf{x})d\mathbf{x}, \tag{2}$$

where $\delta_\alpha$ is an indicator function given by

$$\delta_\alpha(\mathbf{a}, \mathbf{b}) = \begin{cases} 0, & \text{if } \min(\|\mathbf{a}\|, \|\mathbf{b}\|) > \alpha, \\ 1, & \text{if } \min(\|\mathbf{a}\|, \|\mathbf{b}\|) \le \alpha. \end{cases}$$

Each parameter $p \in \{s_i's, \theta, T_i's\}$ of the transformation $A$ is recovered by solving the following evolution equation using a gradient descent scheme:

$$\frac{dp}{dt} = -2\int_\Omega \delta_\alpha r^T[\nabla_p(SR)V_1(\mathbf{x}) - \nabla V_2^T(A(\mathbf{x}))\nabla_p A(\mathbf{x})]\mathbf{dx}. \tag{3}$$

We tested this method on various 2D shapes as shown on Fig. 2, and we compared the results to those corresponding to the rigid matching, based on the signed distance representation as presented in [12]. It is clear from this figures that the *VDF* representation leads to better results since it accounts for anisotropic scales. A similar idea was previously used in [14].

## 3. LOCAL ALIGNMENT

In many applications, particularly in medical imaging, the global matching has to be completed by dense one-to-one correspondences in the presence of local deformations [15]. In this paper, we propose a new variational framework to recover dense local displacement field between the two globally aligned shapes. First, the globally aligned shapes are implicitly represented through their respective signed distance maps. Then, a dense local displacement field is explicitly recovered at each image point by solving a regularized nonlinear PDE-based motion model. The active contour model adopted in [10] for mono-modal image registration is used in this work as prototype for the proposed non-rigid matching model of shapes.

### 3.1. Implicit Shape Representation Using Distance Map

Let $S$ denote an imaged shape in $\mathbb{R}^n$ which defines a partition of the image domain $\Omega$ into the region enclosed by $S$, $\Omega_S$, and its complementary in $\Omega$, $\Omega\backslash\Omega_S$. The shape $S$ can be implicitly represented by the following distance transform $\phi_S : \Omega_S \to \mathbb{R}^+$

$$\phi_S(\mathbf{x}) = \begin{cases} 0, & \text{if } \mathbf{x} \in S, \\ +dist(\mathbf{x}, S), & \text{if } \mathbf{x} \in \Omega_S, \\ -dist(\mathbf{x}, S), & \text{if } \mathbf{x} \in \Omega\backslash\Omega_S, \end{cases} \tag{4}$$

where again, $dist(\mathbf{x}, S)$ refers to the min Euclidean distance between an image point $\mathbf{x}$ and the shape $S$. An example of such representation is shown on the $2^{nd}$ column of Fig. 1.

## 4. BASIC CURVE EVOLUTION

In this section, we present a brief overview of the planar curve evolution technique and its level set formulation. Let $\mathcal{C}(p, t) : \mathbb{R} \times [0, T) \to \mathbb{R}^2$ denote a family of closed planar curves. Assume these curves deform in time according to the following general evolution equation:

$$\frac{\partial \mathcal{C}}{\partial t} = \beta N, \text{ with } \mathcal{C}(p, t_0) = \mathcal{C}_0(p), \tag{5}$$

where, $\beta$ is the normal component of the velocity, and $N$ the inward unit normal to $\mathcal{C}(p, t)$. Now let $\mathcal{C}(p, t)$ be represented by the zero level set of a function $u : \mathbb{R}^2 \times [0, T) \to \mathbb{R}$. That is, $\mathcal{C}(p, t)$ satisfies $u(\mathcal{C}, t) = 0$. Differentiating this equation leads to the following level set representation of the flow:

$$\frac{\partial u}{\partial t} = F\|\nabla u\|, \quad F: \text{ Speed Function} \tag{6}$$

This equation offers many advantages, such as the handling of topological changes [16].

### 4.1. Proposed Local Matching Formulation

We consider the 2D case in order to facilitate the presentation. Given are two shapes, a source $S_1$ and a target $S_2$. First, the source shape is globally aligned with $S_2$ according to a global transformation model $A$ recovered as explained in section 2.1. Let $\widehat{S}_1 = A(S_1)$ denote the globally aligned source shape. To complement the global matching model, we should recover a pixel-wise displacement field $\mathbf{u} = [u_1, u_2]^T : \mathbb{R}^2 \to \mathbb{R}^2$ that establishes correspondences between the aligned source shape $\widehat{S}_1$ and $S_2$. To this end, let $\widehat{\phi}_1$ and $\phi_2$ be the implicit distance map representations of $\widehat{S}_1$ and $S_2$, respectively. We use the following evolution model to explicitly determine

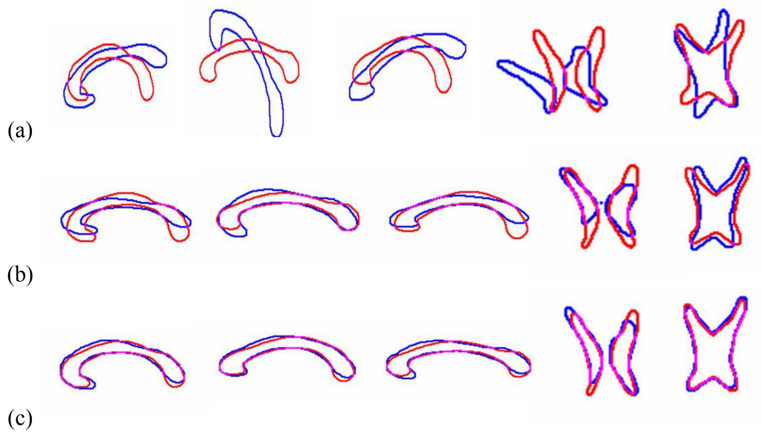


**Fig. 2**. Affine vs. rigid registration. (a) Initial Conditions. (b) Rigid registration using the *SSD* criterion in [12]. (c) Affine matching based on the *VDF* representation (Eq. (2)). Note how the use of the *VDF* representation leads to better alignments thanks to the fact of accurately taking into account anisotropic scales.

the geometric transformation between the two implicit representations, where we denote by $g = Id + \mathbf{u}$ the geometric deformation

$$\frac{\partial \mathbf{u}}{\partial t} = (\widehat{\phi}_1(\mathbf{x}) - \phi_2(g(\mathbf{x})) \frac{\nabla \phi_2(g(\mathbf{x}))}{\|\nabla \phi_2(g(\mathbf{x}))\|}, \quad \mathbf{u}(\mathbf{x}, 0) = \mathbf{0}. \quad (7)$$

In order to retain the regularity of the recovered displacement field, we add a regularization term to the evolutive model (Eq. (7)). This leads to solving the following regularized motion model in order to recover $u$:

$$\frac{\partial \mathbf{u}}{\partial t} = (\widehat{\phi}_1(\mathbf{x}) - \phi_2(g(\mathbf{x}))) \frac{\nabla \phi_2(g(\mathbf{x}))}{\| \nabla \phi_2(g(\mathbf{x})) \|} + \mathcal{R}(\mathbf{u}), \quad \mathbf{u}(\mathbf{x}, 0) = \mathbf{0}, \quad (8)$$

where $\mathcal{R}(\mathbf{u}) = \alpha \Delta \mathbf{u} + \beta \mathbf{u}$, with $\alpha$ and $\beta$ two tuning coefficients balancing the contribution of each term. The well-posedeness of this model will be addressed in future works. We used the *"minmod finite difference"* [17] to implement the evolution model whereas standard difference scheme is used to implement the regularization term. The time step is updated at each iteration according to the *CFL* condition.

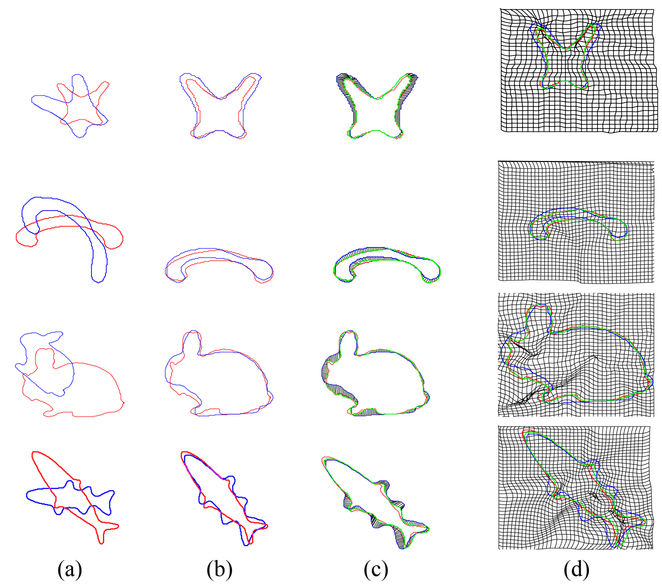


**Fig. 3**. Proposed Registration Framework. (a) Initial positions of the source shape (blue) and target (red); (b) Global alignment using the *VDF*-based *SSD* measure (Eq. (2)); (c) Established shape correspondences after local matching; (d) Space warping with globally deformed source (blue), locally deformed source (green), and target (red).

## 5. EXPERIMENTAL RESULTS

To demonstrate the potential of our global-to-local registration framework, we tested it on various 2D shapes as depicted on Fig. 3.

**Table 1**. Correlation Coefficient For our Global-To-Local Registration Framework.

| | Correlation Coefficient | | |
|---|---|---|---|
| | Input Shapes | After Global Alignment | After Local Alignment |
| Ventricle | 0.844506 | 0.952450 | 0.999214 |
| Corpus Callosum | 0.674487 | 0.946637 | 0.998453 |
| Bunny | 0.399598 | 0.946758 | 0.998817 |
| Fish | 0.530617 | 0.936664 | 0.997603 |

The source and target shapes are first globally aligned using the *VDF*-based global alignment approach (see Sec. 2.1). The local transformation coordinate between these two aligned shapes is explicitly recovered by solving the evolution equation (8). Figures 3(c) depict the overlay of the target shape and the source shape after global and local alignments. Visually, one can notice from these figures the ability of our framework to cope with very large non-linear deformations.

To quantitatively demonstrate the accuracy of our approach, we computed the correlation coefficient between the original shapes, as well as that between the target shape and the source shape after global and local alignments. To this end, we implicitly represent each shape by its distance map which we transform to a gray value image. The obtained results are summarized in table 1. These results indicate the degree of accuracy of our registration framework. In the next section we propose a $FE$-based approach to validate the non-rigid registration model.

## 6. *FE*-BASED VALIDATION APPROACH OF THE PROPOSED NON-RIGID MATCHING APPROACH

We propose a $FE$-based approach to build a ground truth pair of shapes (source and target) in order to validate our non-rigid matching model (8). Given a 2D binary image of the corpus callosum $CC$, the *Abaqus/CAE (Ver. 6.5)* [1] environment was used to generate a cubic spline fit to the points representing the outer contour of the $CC$ object and then a 2D F.E. model was built from it. A uniformly distributed pressure $P = 100N$ is applied normal to the boundary of the $CC$ which is modeled as a linear elastic material. The points on the boundary are allowed to move freely in the $x$- and $y$-directions, but are constrained to rotate around the $z$ direction. Figures 4(a) show the overlay of the non deformed mesh with the deformed one. The average displacement of the induced deformation is $2.77mm$, the min is $1.82mm$, and the max is $4.85mm$. We match

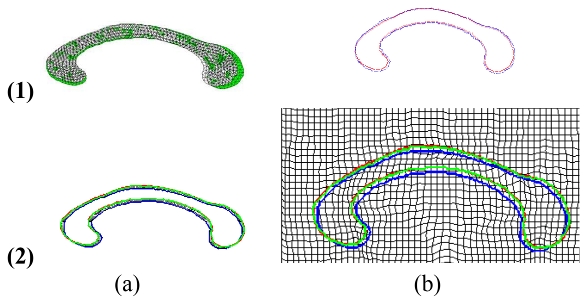


**Fig. 4**. (a) Overlayed meshes before (white) and after (green) deformation; (b) Comparison between 400 node positions on the contour: (red) Ground truth (Abaqus simulated); (blue) Corresponding positions using our local non-rigid matching algorithm. (d) Displacement vector field depicting the space warping.

the simulated deformed shape with the original one using our algorithm, and we compare the results to the $FE$-simulated ones. The average matching error is $0.9762mm$, with a $\max = 2.7458mm$, a $\min = 0.0146mm$, and a standard deviation of $0.6815$. To illustrate these results, we present on Fig. 4(b) a typical comparison between a set of $400$ randomly selected F.E. nodes positions and their corresponding recovered positions using our non-rigid matching model. Figure 4(c) depicts the matching results.

## 7. CONCLUSION

In this work, we presented a new global-to-local variational framework for 2D shape registration. The global alignment is based on minimizing a *SSD* criterion between the VDF representations of the input shapes. This vectorial representation supports both rigid and affine transformations. The local displacement field is explicitly recovered between the two aligned shapes by solving a regularized PDE-based model. Various experiments were presented to show the potential of the proposed framework. This framework is quantitatively validated using a $FE$-based approach. The extension to 3D case is straightforward and will be addressed in future works. Applications, such as shape-driven segmentation and tracking, are among the fields that may benefit from the proposed registration framework. These applications are subject of our focus.

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