TOWARD AN OPTIMAL SOLUTION FOR MULTITARGET TRACKING

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ABSTRACT

There are ever increasing number of applications of multi-target tracking and considerable research has been conducted to solve this problem. Multi-target tracking is a NP-hard problem and almost all of the present multi-target tracking algorithms are sub-optimal by finding the solution in a reduced hypothesis space. In this paper we introduce a new approach toward finding the optimal single frame solution for general multi-target tracking problem. Our proposed method finds the optimal solution using linear programming optimization method. The proposed method has been successfully applied to synthetic and real data.

***Index Terms*—** Tracking, Optimization, Association, JPDA.

1. INTRODUCTION

Multi-target tracking has several different applications and its applications has been growing by recent advances in digital imaging techniques which have made possible the automated acquisition of millions of digital images so that there are considerable demand for faster and more accurate digital image processing techniques and pattern recognition methods to analyze such huge sets of images and to address new questions. As a result designing precise fully automated tracking systems are mandatory. The field of multi-target tracking has attracted researchers with a variety of different interests which include designing faster tracking methods, innovating new mathematical tracking models and improving the previous models to overcome their shortcomings. Multi-target tracking has a broad range of applications including air traffic control, robot control, ocean surveillance, automated vehicle control, biological and cellular research [1, 2, 3, 4, 5].

The goal of multi-target tracking is associating measurements with the appropriate targets. This is the most challenging task in the multi-target tracking applications due to missing targets, new targets and false alarms. Multi-target tracking is one of the NP-hard problems and so considerable efforts have been conducted to design tractable methods by reducing the complexity. These methods include Nearest Neighbor, Joint Probabilistic Data Association and Multi-Hypotheses Data Association [6, 7, 8, 9].

The common task among all tracking methods is to reduce the hypothesis space and to find the suboptimal solution by finding the most likely hypothesis in the reduced hypothesis space. Solving the problem in a reduced hypothesis space raise some important questions such as the likelihood of the solution to be optimal regarding the reduced hypothesis space, the likelihood of finding the optimal solution in the reduced hypothesis space and the closeness of the solution to the optimal solution regarding the reduced hypothesis space.

To find an optimal solution for multi-target tracking problem, all enumerations must be evaluated. Increasing the computational power of the computers and advancements in the optimization methods, raise the question and demand for designing new methods to find the optimal solutions for NP-hard problems such as assignment problem or more specifically related to our research multi-target tracking. To do so, all possible hypotheses must be enumerated and problem must be solved on exact hypothesis space.

The focus of the proposed method in this paper is to find an optimal solution for general multi-target tracking problem.

2. PROPOSED METHOD

The association problem

$$\hat{X}_{1:K} = arg \max_{X_{1:K}} P(X_{1:K}|\, Z_{1:K}) \tag{1}$$

is a NP-hard problem, so to find the optimal solution is essentially impossible. Several different methods have been introduced to solve the problem by finding the most likely hypothesis. As a result the solution which has been found in the reduced hypothesis space is suboptimal. Virtually all association-based tracking methods made the problem tractable by searching over reduced hypothesis space

$$\{X^h_{1:K}|\, h = 1, 2, ...\} \tag{2}$$

such that we find the best member of this set

$$\hat{X}_{1:K} = X^{\hat{h}}_{1:K} \text{ where } \hat{h} = arg \max_{h} P(X^h_{1:K}|Z_{1:K}) \tag{3}$$

as the solution. The problem may be solved over reduced multi-frame hypothesis space over time $[k - n, k]$ such as

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		Meas. A	B	C
Targets	1	0	1	0
	2	1	0	0
	3	0	0	1

Fig. 1. Assignment Matrix: Three measurements A, B, C need to be assigned to three targets $1, 2, 3$. Clearly each row and column of the matrix must sum to one.

the Multi Hypothesis Tracking (MHT) algorithm [6], or over reduced single-frame hypothesis space over time $[k-1, k]$ such as Joint Probabilistic Data Association (JPDA) [10]. The original, optimal solution is found if it is included among the hypotheses, i.e., if

$$arg \max_{X_{1:K}} P(X_{1:K}|\, Z_{1:K}) \in \{X^h_{1:K}\} \tag{4}$$

A feasible approach which has been widely used is the single-frame association method. However all single-frame approaches solve the problem over a reduced hypothesis space over time $[k-1, k]$:

$$\{x^h_k|\, h = 1, 2, ...\} \tag{5}$$

and search for the best hypothesis

$$\hat{x}_k = x^{\hat{h}}_k \text{ where } \hat{h} = arg \left\{ \max_h P(x^h_k | Z_{1:k}) \right\} \tag{6}$$

as the solution. As a result if the optimal single-frame solution is included among the hypotheses of that frame, it will be found, i.e., if

$$arg \left\{ \max_{x_k} P(x_k|\, Z_k) \right\} \in \{x^h_k\} \tag{7}$$

JPDA [10] is one of the widely used single-frame algorithms to solve the multi-target tracking problem in a reduced hypothesis space.

2.1. Joint Probabilistic Data Association

Joint Probabilistic Data Association (JPDA) is a single-frame sub-optimal solution that has been widely used to solve multi-target tracking problem (1) over time $[k-1, k]$. To make the association problem tractable, JPDA employs a gating strategy to reduce the number of possible association hypotheses and to keep a subset of them. The reduced set of hypothesis contains valid association hypotheses based on the gating factor which determines the extent of the gating volume and is applied by JPDA to validate the measurements. Valid associations, a subset of association hypotheses, are generated based on validated measurements. Valid measurements fall inside the validation gate of each target while the measurements that fall outside of the target's validation gate are not considered as association candidates and are thrown away. From Bayes' rule

$$P(x_k|\, Z_{1:k}) = \lambda_k \cdot P(x_k|\, Z_{1:k-1}) \cdot P(Z_k|\, x_k) \tag{8}$$

where λ_k is a normalization constant.

The first term of (8), $P(x_k|\, Z_{1:k-1})$, is a prediction step while the second term of (8) is the likelihood of measurement Z_k given hypothesis x_k. We illustrate (8) as follows.

$$P = P(x_k|\, Z_{1:k}) = \lambda_k \cdot \left[\prod_{j \in M_k} P_D \right] \cdot \left[\prod_{j \in MD} P_{MD} \right] \cdot \left[\prod_{j \in F} P_F \right] \cdot \left[\prod_{j \in M_k} N(v_{k,j}, 0, S_{k,j}) \right] \tag{9}$$

where P_D is the probability of detection which is a constant determined based on the detection performance, P_{MD} is the probability of miss detection, P_F is the probability of false alarms, and $v_{k,j} = z^{\tau}_{k,i} - \hat{z}_{k,j}$ is an innovation term measured based on motion dynamics to associate the j^{th} measurement at time k to the i^{th} target at time $k-1$. Having the association problem specified by (9), the optimization problem is to find the best estimate among all possible hypotheses evaluated.

2.2. Optimal Probabilistic Data Association Method

Our goal in this paper is to find the optimal solution over time $[k-1, k]$ to solve (1). In our proposed Optimal Probabilistic Data Association (OPDA) method, we use Hungarian method, a linear programming optimization method, to optimize the solution.

The basis of the Hungarian method was introduced by Egervary and Konig and it has been completed later by Kuhn [11]. The Hungarian method is a class of linear programming methods for the assignment problem known as primal-dual algorithms [12]. Primal-dual algorithms are characterized by

- A primal vector and a dual feasible solution is maintained by the algorithm.
- One of the following tasks is performed by the algorithm in each iteration
  1. The primal vector is kept fixed and the dual feasible solution is changed.
  2. Dual solution is kept fixed and the primal vector is changed toward primal feasibility while satisfying the present dual solution.
- By iterating the algorithm, the primal vector progresses toward primal feasibility.

Assume the assignment problem is presented by matrix $\mathtt{X} = (\mathtt{X}_{ji})$ of order M_k and $C = (c_{ij})$ is the associated cost

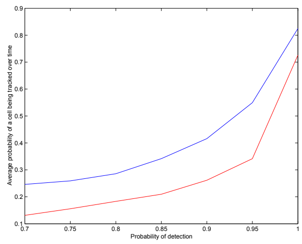


Fig. 2. Tracking Performance: OPDA (blue line) vs. Nearest Neighbour (red line) as a function of the probability of detection (P_D). For each value of probability of detection, 200 video clips each of 50 frames have been used.

matrix with the same order where M_k is the number of measurements in time k. To solve the assignment problem we wish to minimize

$$A(C, \mathrm{X}) = \sum_{i=1}^{M_k} \sum_{j=1}^{M_k} c_{ij} \mathrm{X}_{ij} \tag{10}$$

subject to

$$\begin{aligned} \sum_{j=1}^{M_k} \mathrm{X}_{ij} = 1 \quad \forall i \in [1, M_k], \\ \sum_{i=1}^{M_k} \mathrm{X}_{ij} = 1 \quad \forall j \in [1, M_k] \\ \mathrm{X}_{ij} = 0 \;\; or \;\; 1 \quad \forall i, j \in [1, M_k] \end{aligned} \tag{11}$$

Each feasible solution of (11) is an assignment problem of order M_k, i.e., each assignment is a permutation matrix. To maximize

$$B(\delta, g) = \sum_{i=1}^{M_k} \delta_i + \sum_{j=1}^{M_k} g_j \tag{12}$$

subject to

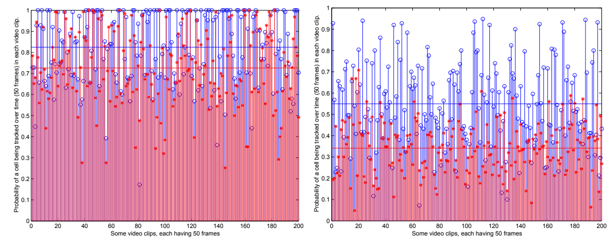
$$\delta_i + g_j \leq c_{ij}, \;\; i, j \in [1, M_k] \tag{13}$$

$\delta = (\delta_1, \delta_2, \cdots, \delta_{M_k})$ and $g = (g_1, g_2, \ldots, g_{M_k})$ must be found as the dual of (11). The constraint which is called dual feasibility condition for (δ, g) can be rewritten as

$$\bar{c_{ij}} = c_{ij} - \delta_i - g_j \geq 0, \;\; i, j \in [1, M_k] \tag{14}$$

where $\bar{C} = (\bar{c_{ij}})$ is reduced cost matrix and its elements $\bar{c_{ij}}$ are reduced cost coefficients. $B(\delta, g)$ is dual assignment function and $\sum_{i=1}^{M_k} \delta_i + \sum_{j=1}^{M_k} g_j$ is the total reduction. As a result the vectors (δ, g) are dual feasible if and only if the reduced cost matrix $\bar{C} \geq 0$ [12]. An assignment problem X and a dual feasible solution (δ, g) are optimal if complementary slackness optimality conditions for the assignment problem and its dual

$$\mathrm{X}_{ij} \times (c_{ij} - \delta_i - g_j) = \mathrm{X}_{ij} \times \bar{c_{ij}} = 0, \;\; i, j \in [1, M_k] \tag{15}$$



(a) (b)

Fig. 3. Tracking performance of OPDA(circles) vs. NN(stars) is shown by average probability that a cell has been tracked computed over each video clip (50 frames). Moreover, the average performance is computed over all 200 video clips (each contains 50 frames) and is superimposed in blue line for OPDA and in red line for NN; (a) (P_D = 100%), (b) (P_D = 95%).

is satisfied. Having these basic assumptions, Hungarian method begins with a dual feasible solution

$$\bar{c_{ij}} = c_{ij} - \delta_i - g_j = 0 \tag{16}$$

and tries to find admissible elements, i.e., an association among the cost matrix elements (C) which satisfy (16). The tracking problem can be represented in the form of an assignment problem so that a primal-dual algorithm can be applied to solve it. To represent the tracking problem in the form of an assignment matrix, the measurements Z_k of frame k are assigned to the targets x_{k-1} of frame $k-1$, giving rise to an assignment matrix (see Fig. 1) which represents the association of measurement j to target i.

To solve the assignment problem in the proposed method, we embed the probabilistic data association function as the cost function C in the Hungarian method. C is the cost function so that each element of which (c_{ij}) represents the cost of assigning measurement j in time k to target i from time $k-1$. The goal is minimizing the cost of joint association of measurements to targets. The cost matrix is computed as follows.

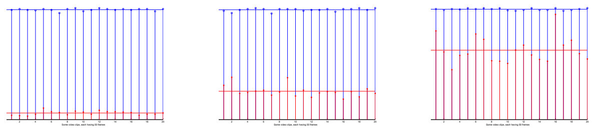
$$C = [c_{ij}] = \frac{1}{P} = \frac{1}{P_{ij}} \tag{17}$$

where each element P_{ij} is obtained by

$$P_{ij} = P_{D_{ij}} \times P_{MD_{ij}} \times P_{F_{ij}} \times N_{ij} \tag{18}$$

Finally having C as the cost function, the proposed OPDA finds the optimal assignment by satisfying (15).

As we can observe by minimizing the cost function C, at the same time we are maximizing the probabilistic data association (P) in (9). The optimal association among all possible hypotheses is found by employing Hungarian method to solve (9) and the optimal solution over frame $[k-1, k]$ is obtained.



(a) (b) (c)

Fig. 4. Performance of the proposed OPDA (circles) in comparison with standard JPDA (stars) as a function of gating factor (G_f). The average performance computed over all 20 video clips for each gating factor is superimposed in blue line for OPDA and in red line for standard JPDA; (a) $G_f = 1$, (b) $G_f = 2$, (c) $G_f = 3$.

3. RESULTS

The proposed OPDA is successfully applied to both synthetic video clips and blood stem cell image sequences. We generated 2000 video clips, each video clip composed of 50 frames and 5 objects. The performance of the algorithm is assessed based on the average percentage of frames in which a target has been correctly associated in comparison with ground truth.

The performance of the proposed method in comparison with Nearest Neighbour (NN) method is depicted in Fig. 2. The two methods are compared for different values of the probability of detection (P_D). For each value of P_D, the synthetic cell centres are tracked over time applying the OPDA and NN. As we can observe, the proposed method has outperformed NN for all values of P_D. A comparison of the two algorithms for $P_D = 100\%$, and $P_D = 95\%$ is depicted in detail, sequence by sequence, in Figs. 3(a) and (b) respectively.

Fig. 4 shows the performance of the proposed method in comparison with the standard JPDA for $P_D = 100\%$. To compare the results, the performance of JPDA is measured as a function of gate area $G_a = \pi \times G_r^2$, where $G_r = G_f \times \sigma_{sd}$ is the gate radius, set to be a multiple (G_f) of the standard deviation of the random part of the systems dynamics (sd). The results are derived for different values of $G_f \in [0.1, 5]$. For each value of G_f, 20 video clips, each one contains 50 frames are generated. As we can observe in Figs. 4(a), (b) and (c) for G_f equal to one, two and three respectively, the proposed OPDA method has outperformed standard JPDA.

4. CONCLUSIONS

This paper presents an optimal single frame solution for tracking problem. To solve the tracking as an assignment problem, our proposed method employs linear programming optimization method to find the optimal solution. Measurement to target association is accomplished based on Hungarian method in which the proposed probabilistic data association is embedded as the cost function. We can observe from the results that such an optimal tracking method produces very promising results. This is a generative algorithm and every tracking method including nearest-neighbor, PDA, JPDA, particle filtering, MHT and deformable models can be employed in this platform by designing the correct cost function.

5. REFERENCES

[1] Y. Bar Shalom, X. R. Li, and T. Kirubarajan, *Estimation with Application to Tracking and Navigation*, Wiley, 2001.

[2] V. Santibaez, R. Kelly, and M.A. Llama, "Global asymptotic stability of a tracking sectorial fuzzy controller for robot manipulators," *IEEE Tran. on SMC Part B: Cybernetics*, vol. 34, no. 1, pp. 710–718, 2004.

[3] J.H. Rife and S.M. Rock, "Design and validation of a robotic control law for observation of deep-ocean jellyfish," *IEEE Transactions on Robotics*, vol. 22, no. 2, pp. 282–291, 2006.

[4] N. Ray and S.T. Acton, "Motion gradient vector flow: An external force for tracking rolling leukocytes with shape and size constrained active contours," *IEEE Transactions on Medical Imaging*, vol. 23, no. 12, pp. 1466–1478, 2004.

[5] X. Chen, X. Zhou, and S.T.C. Wong, "Automated segmentation, classification, and tracking of cancer cell nuclei in time-lapse microscopy," *IEEE Transactions on Biomedical Engineering*, vol. 53, no. 4, pp. 762–766, 2006.

[6] D. Reid, "An algorithm for tracking multiple targets," *IEEE Transactions on Automation and Control*, vol. 24, no. 6, pp. 84–90, 1979.

[7] Y. Bar-Shalom and E. Tse, "Tracking in a cluttered environment with probabilistic data association," *Automatica*, vol. 11, pp. 451–460, 1975.

[8] A. Mansouri, A. Mitiche, and C. Langevin, "Joint space time motion based segmentation of image sequences with level set pdes," in *IEEE Workshop on Motion and Video Computing*, 2002.

[9] S. Colegrove, A. Davis, and J. Ayliffe, "Track initiation and nearest neighbours incorporated into pda," *Electrical and electronics Eng.*, vol. 6, 1986.

[10] Y. Bar Shalom and T. E. Fortmann, *Tracking and Data Association*, Academic-Press, 1988.

[11] H. W. Kuhn, "The hungarian method for the assignment problem," *Naval Research Logistics Quarterly*, vol. 2, pp. 83–97, 1955.

[12] K. G. Murthy, *Network Programming*, Prentice Hall, Englewood Cliffs, NJ., 1992.