AN EFFICIENT METHOD FOR THE DETECTION OF PROJECTED CONCENTRIC CIRCLES

Xianghua Ying and Hongbin Zha

National Laboratory on Machine Perception
Peking University, Beijing, 100871 P.R. China
{xhying, zha}@cis.pku.edu.cn

ABSTRACT

Concentric circles are often used as calibration features since they possess good geometric properties. This paper presents an efficient method for the detection of projected concentric circles in the image plane while considering their special geometric properties. The proposed method is capable of detecting partially visible concentric circles. Experimental results demonstrate the validity of the proposed approach.

***Index Terms*—** Hough transform, concentric circles, camera calibration, ellipse detection

1. INTRODUCTION

Approaches to camera calibration using planar patterns are very popular due to their practical convenience. Features on calibration patterns can be grid [18, 12, 13] and circular [6, 2, 9, 4, 15, 16]. This paper employs concentric circles as calibration features because they have richer geometric properties than points and lines. As we know, a circle is often projected into an ellipse in the image plane, but the projection of the center of the circle is often not the center of the ellipse (see Figure 1). For two concentric circles, the centers of their image ellipses are often not coincident as shown in Figure 3. That means the projections of two concentric circles are often not two concentric ellipses. Obviously, the projection of their common center is still one image point in the image plane.

Geometric properties of concentric circles for camera calibration are discovered by many researchers [9, 4]. However, these papers usually used the general ellipse detection methods to detect the images of the concentric circles with general ellipse fitting method [3], not considering the special geometric properties of the concentric circles. Jiang and Quan [7] firstly discussed on the special detection methods for the projected concentric circles. They proposed a constructive method to detect the image of their common center. However, their method cannot deal with the occlusions cases (e.g., an image shown in Figure 4a), since it needs a recursive procedure and requires the information in the almost whole ellipses. Therefore, this paper aims at the detection of the projected concentric circles with occlusions.

As noted before, the image of the common center of concentric circles is no longer the centers of their image ellipses and cannot be detected using moment based methods [5] or Hough transform based methods [14, 17]. Heikkila [6] utilized a recursive procedure with the rough camera parameters to correct the bias between the imaged centers of circles and centers of ellipses after knowing the latter positions. That means the images of the centers of the circles on the calibration pattern are very useful from the viewpoint of calibration. In the detection methods special for the projected concentric circles as emphasized by Jiang and Quan [7], the positions of the imaged common center can be directly located without considering the camera parameters. Though our purpose is to determine the image of the common center of the two concentric circles, the main idea in this paper arises from observations on detection methods of ellipse centers based on Hough transform [14, 17].

Standard Hough transform based ellipse detection methods need a 5-dimensional parameter space that consists of the semi-axes, the center point, and the orientation. However, the 5-dimensional parameter space has a huge computational burden. To avoid such computational cost and memory requirements, Yuen et al. [17] used the properties of the ellipse to detect the center in the first step, and other parameters are recovered later. The basic idea in [17] is illustrated in Figure 1. Each pair of points is taken in turn. The midpoint of a pair is found at M. The tangent lines of the pair intersect at T. Then the line TM should pass through the ellipse center. All such lines are accumulated in the parameter space, and the peak in the parameter space is found, which is the ellipse center.

In this paper we extend the basic idea on the detection of the ellipse center in [17] to detect the image of the common center of two concentric circles. We may find lines passing through the image of the common center as shown in Figure 3, not find lines passing through the ellipse center as proposed in [17]. The details of the proposed method will be given in the main text. It should be noted that there are two methods [1, 11] for detection of the concentric circles in the images, which are very different from the method proposed in this paper, since the images of the concentric circles may be no longer concentric circles.

2. MAIN IDEA OF THE METHOD

Definition *We say that the two pairs of points* (P_1, P_2) *and* (Q_1, Q_2) *are harmonic if the cross ratio of the four points* $\{P_1, P_2; Q_1, Q_2\} = -1$ *[10].*

There is an important special case of the harmonic relation that, if Q_1 is the midpoint of (P_1, P_2), then Q_2 must be the point at infinity on the line P_1P_2. Obviously, the cross ratio is projective invariant, and then the harmonic relation of these four points is projective invariant, too. That means if four points satisfy $\{P_1, P_2; Q_1, Q_2\} = -1$, then their image points also satisfy $\{P_1', P_2'; Q_1', Q_2'\} = -1$.

For a pair of concentric circles in the calibration pattern plane, a line intersects the outer circle at two points P_1, P_2, and intersects the inner circle at two points P_3, P_4 as shown in Figure 2. Obviously, the midpoints of segments of P_1P_2 and P_3P_4 are identical, denoted as Q_1, and the point at infinity on the line is denoted as Q_2. From the definition of the harmonic relation, we have [7]:

$$\{P_1, P_2; Q_1, Q_2\} = \{P_3, P_4; Q_1, Q_2\} = -1 . \qquad (1)$$

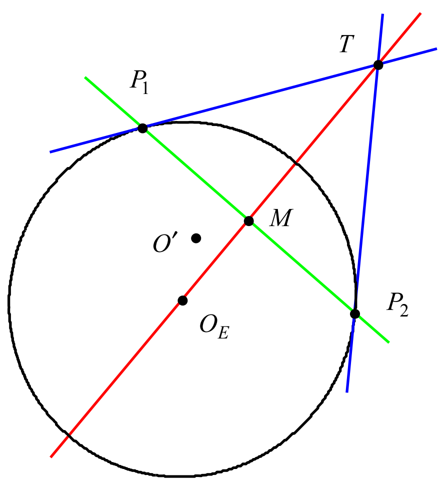


Figure 1: The geometry for ellipse detection based on Hough transform. O_E is the center of the ellipse. O' is the image of the center of the circle projected into the ellipse. In general, O_E and O' are not identical. P_1 and P_2 are two points on the ellipse. M is the midpoint of P_1 and P_2. The tangent lines of P_1 and P_2 intersect at T. Then the straight line TM may pass through the ellipse center O_E.

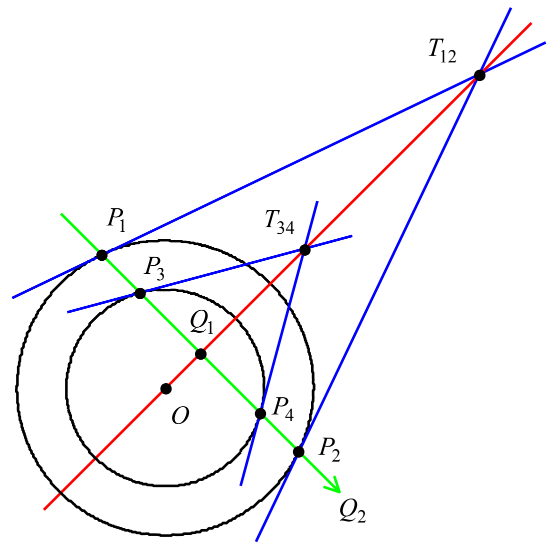


Figure 2: The geometry of two concentric circles. A line cuts the concentric circles in P_1, P_2, P_3, P_4. Q_1 is the midpoint of P_1P_2 and P_3P_4, Q_2 is the point at infinity of the line. The tangent lines of P_1 and P_2 intersect at T_{12}, and the tangent lines of P_3 and P_4 intersect at T_{34}. Obviously, the common center O lies on the line determined by T_{12}, T_{34} and Q_1.

An example of the projections of the concentric circles in the image plane is shown in Figure 3. Since the harmonic relation is projective invariant, we have:

$$\{P_1', P_2'; Q_1', Q_2'\} = \{P_3', P_4'; Q_1', Q_2'\} = -1 , \qquad (2)$$

where P_1', P_2', P_3', P_4' are the image points of P_1, P_2, P_3, P_4, respectively. From the two equations in (2), Q_1', Q_2' can be recovered. In general, the image of the midpoint Q_1' should lie inside the segment $P_1'P_2'$ and $P_3'P_4'$, then we can distinguish Q_1' and Q_2' [7].

After determine the image of the midpoint Q_1', we can also determine the intersection points T_{12}', T_{34}' of their tangent lines as shown in Figure 3. Then the line passing through the image of the common center of the two concentric circles is found, which is the line determined by T_{12}', T_{34}' and Q_1'. The proof of the correctness of the method is trivial: Symmetry properties ensure that the method works for concentric circles in the calibration pattern plane. Projective properties then ensure that it also works for the images of the concentric circles. Under perspective projection, straight lines project into straight lines, midpoint into harmonic, tangents into tangents, and circles into ellipses.

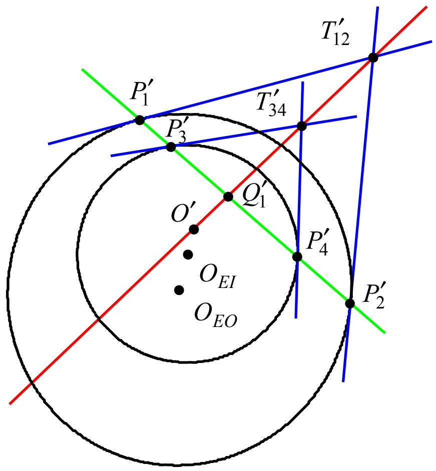


Figure 3: The geometry of projected two concentric circles. O_{EI}, O_{EO} are the centers of the ellipses which are the images of the two concentric circles, respectively. O' is the image of the common center of the concentric circles. In general, O_{EI}, O_{EO} and O' are different, but collinear [4]. A line intersects the images of the concentric circles at P'_1, P'_2, P'_3, P'_4. Q'_1 is the image of the midpoint of P_1P_2 and P_3P_4. The tangent lines of P'_1 and P'_2 intersect at T'_{12}, and the tangent lines of P'_3 and P'_4 intersect at T'_{34}. Obviously, the image of the common center O' should lie on the line determined by T'_{12}, T'_{34} and Q'_1.

All such lines are accumulated in the parameter space. Then the peak in the parameter space is found, which would be the projection of the common center of the concentric circles.

3. OUTLINE OF THE METHOD

The complete detection procedure is described below:

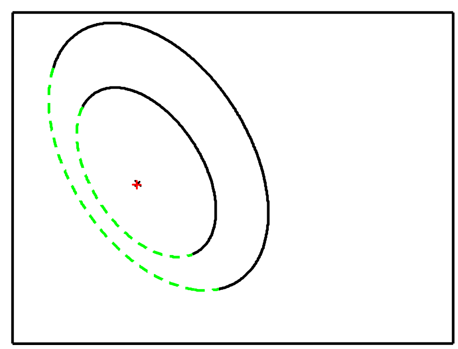
- For each four collinear edge points lying on the outer circle and inner circle respectively, find the projection of the midpoint, and determine the two corresponding intersections of the four tangent lines.
- For each set of the image of the midpoint and the two intersections, determined the line passing through the images of the common centers. All such lines are accumulated in the parameter space.
- Seek the peak in the parameter space, which is the projection of the common center of the concentric circles.
- Find better estimation on the parameters of the pair of ellipses using a homology-based optimization method [7].

4. EXPERIMENTS

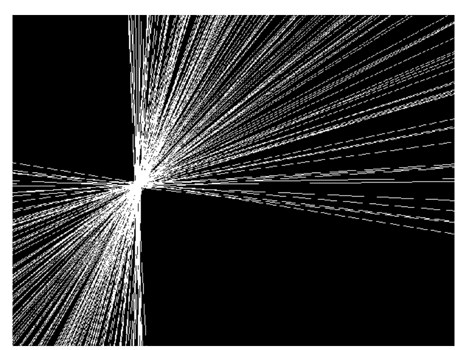
In this section, we describe the performance of the proposed detection algorithm by both simulated and real image data.

4.1. Simulated data

We use Figure 4 as a simulated example to show the detection results for the proposed method. An image containing projected concentric circles is shown in Figure 4a, where the image size is 800×600. The effects of the occlusions are simulated. The dark solid lines denote the visible parts, and the bright dashed lines denote the occluded parts. Gaussian noise with zero-mean and standard deviation 2 is added to these visible image points. Then, we use the proposed method to recover the image of the common center. The parameter space is shown in Figure 4b. After finding the peak in the parameter space, the recovered imaged common center is drawn as a '+' marker, and the ground truth of the imaged common center is marked with a cross in Figure 4a. We can see that the recovered results are satisfactory.



(a)



(b)

Figure 4: (a) A simulated image containing two projected concentric circles with occlusions. (b) The parameter space for the detection of the image of the common center.

4.2. Real data

A real image containing two projected concentric circles with the effects of occlusion is shown in Figure 5a. This image is captured using a Canon video camcorder XM2, and the image size is 720×576. From among many edge detection techniques available, we adopt the one proposed in [8] to obtain edge segments without branches. Among the resulting edge segments, we discard short ones and register the remaining ones in an edge segment list. After finding the edges in the image, the method proposed in this paper is employed, and the detected ellipses and the image of the common center are superimposed onto the original image as shown in Figure 5b.

(a)

(b)

Figure 5: (a) A real image containing two projected concentric circles with occlusions. (b) The detected ellipses are superimposed on the image.

5. CONCLUSIONS

This paper presents an efficient method for the detection of projected concentric circles in the image plane even if serious occlusions occurred in the images. This is the main contribution of this paper. The method can be employed in some concentric circle based camera calibration procedure. The validity of our proposed approach is illustrated by experiments.

6. ACKNOWLEDGEMENTS

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