

# How does the Time Delay of an ER Fluid's Response Affect Control Performance of Servo-systems?

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**Abstract**—We previously developed a particle type ER (Electro-Rheological) fluid damper in which viscosity was controlled continuously in real time for fast and precise positioning of a direct-drive motor. However, particle type ER fluids commonly have about 5-mili seconds response times. This delay needs to be taken into account when electric fields to energize ER fluids are controlled in real time. Especially, servo systems with ER fluid devices have a sampling time for the control loop that is about 1-mili second and so is shorter than the response time of ER fluids. Though the delay in this case has a great possibility of affecting the performance or stability of servo systems, there is little research in this area that has mentioned the problem. In this paper, I investigated how this time delay affected damping performance and stability of this system.

## I. INTRODUCTION

Many applications for particle type ER fluids have been suggested; such as dampers, clutches, and brakes to improve control performance or to satisfy new control demands [1], [2]. Fluids of this type commonly have a response time of about 5-mili seconds. This delay is not a serious issue when constant electric fields are impressed like in the case of damping structures [3], [4]. However, the delay needs to be taken into account when electric fields are controlled in real time. Especially, servo systems introduced with ER fluid devices have sampling times for their control loops that are about 1-mili second and so are shorter than the response times of ER fluids. Though the delay in this case has a strong possibility of having an effect on the performance or stability of these systems, there is little research in this area mentioning the problem.

In direct-drive (DD) motor systems [5] without reduction gears, disturbances directly influence motor dynamics and control accuracy because of the small amount of damping. Time delays of computers and amplifiers also destabilize the controller [6]. While a time lag cannot be compensated mechanically, damping can be done with an additional mechanical damper [7]. In an earlier study, we introduced a particle type ER fluid damper for a DD motor system for fast and precise positioning [7], [8]. Viscosity of the damper was controlled continuously in real time to improve the performance of the DD system through dynamic changing of the viscosity [8]. However, even this study of ours failed to discuss the problem of the ER fluid response time delay.

This work has been supported by grants from the Kurata Memorial Hitachi Science and Technology Foundation and the Electro-Mechanic Technology Advancing Foundation.

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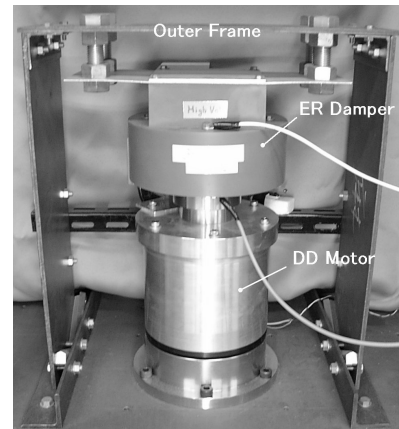


Fig. 1. ER damper and DD motor

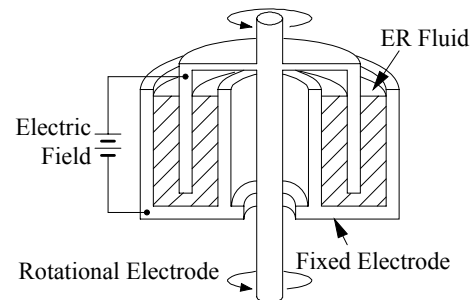


Fig. 2. Model of ER damper

There have been many studies to clarify the mechanism of the ER effect [1], but few have studied the speed of the response itself. From concerns about experimental situations, the dead time or first-order lag from mechanical and/or control factors of an experimental system could be accumulated in results of experiments; thus it may be impossible to accurately measure just the delay of the fluid. However, just factors related to the delay of an ER fluid can be extracted if the simulation results under various conditions are compared with the experimental ones. This paper investigates how time delay affects damping performance and stability by utilizing analysis, simulation, and experiment.

## II. EXPERIMENTAL SYSTEM

This study used the ER damper and DD motor system shown in Fig. 1 [7], [8].

A model of an ER damper is shown in Fig. 2. The damper uses a particle type ER fluid. It consists of a rotational electrode connected to the motor shaft and an electrode fixed to the outer part of the motor housing. The gap between the electrodes is 0.5 mm. The particle type ER fluid used in

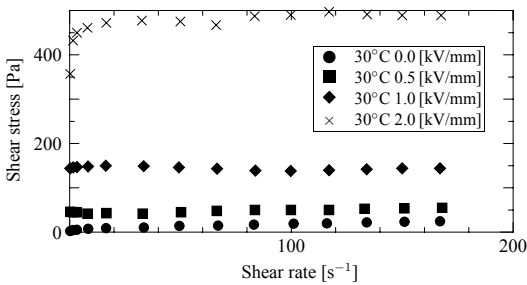


Fig. 3. Electric field strength of ARP05

this study was “ARP05” supplied by Asahi-Kasai Co. for research purposes. The characteristics of its share rate and share stress are shown in Fig. 3.

The DD motor is an outer rotor type made by NSK with a maximum torque of 30 N·m. A resolver included in the DD motor has a resolution of 491,520 ppr. The moment of inertia of the DD motor and the ER damper is  $J = 0.075$  kg·m<sup>2</sup>. The sampling time for the control is set to 0.5 ms.

### III. CONTROLLER FOR THE ER DAMPER

#### A. Construction of a Linear Viscous Damper

Since a particle type ER fluid has a high-speed response property, it can realize an arbitrary coefficient of viscosity by way of controlling the applied electric field according to the motor velocity and changing resistive force of the fluid. Thus, this section aims to construct a linear viscous damper for which the coefficient of viscosity can be changed arbitrarily, as in our earlier report [8].

First, I investigated the relationship between the applied electric field and the torque of the ER damper. While a constant electric field was applied to the damper, a speed control was implemented for the motor, and the time average command torque for the motor in a steady state was calculated. Fig. 4 shows a graph of torque and angular velocity in the case of the applied electric field: 0.0~3.0 kV/mm.

In this study, the ER effect of the damper is especially needed at a low velocity, so a low velocity level of from 0.1 to 0.5 rad/s was taken into account. According to the angular velocity, the torque at  $E = 0$  increases a little. This is considered because the particle type ER fluid behaved like a Newton fluid with some basic viscosity when  $E = 0$  as shown in Fig. 4. However, to ease the configuration, I approximated the relationship of the electric field ( $E$ )–torque ( $T_{vis}$ ) to the following quadratic form using the averages of data from 0.1 to 0.5 rad/s.

$$T_{vis} = 0.0704E^2 + 0.470E + 0.763 \quad (1)$$

Second, I attempted to make the particle type ER fluid damper behave as a linear viscous damper with controled electric field. Equation (1) is expressed as follows:

$$T_{vis} = aE^2 + bE + c \quad (2)$$

Where,  $a = 0.0704$ ,  $b = 0.470$ ,  $c = 0.763$ .

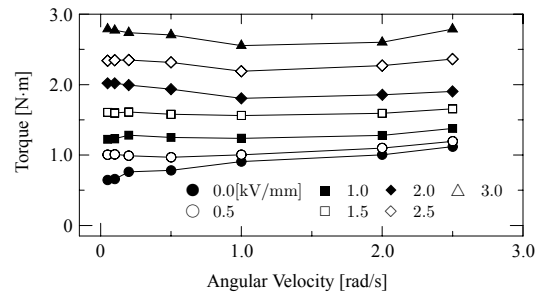


Fig. 4. Relationship of angular velocity and torque

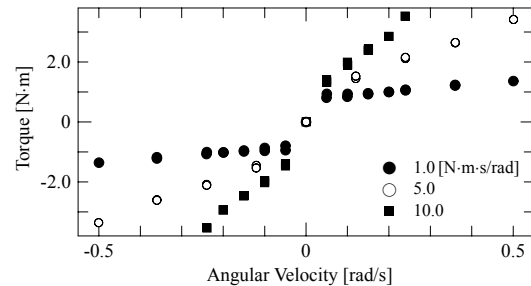


Fig. 5. Relation of angular velocity and torque (after linearization)

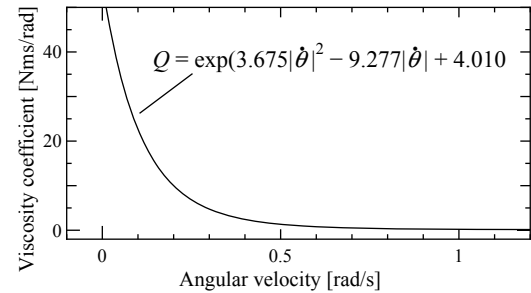


Fig. 6. Relation between  $Q$  and  $\omega$

On the other hand,  $T_{vis}$  is written using the coefficient of viscosity,  $Q$ , and angular velocity,  $\dot{\theta}$ , as follows (3):

$$T_{vis} = Q|\dot{\theta}| + c \quad (3)$$

Substituting this  $T_{vis}$  to (2) gives the following:

$$E = \frac{-b}{2a} + \sqrt{\left(\frac{b^2}{4a} + Q|\dot{\theta}|\right)}/a \quad (4)$$

Applying the electric field ( $E$ ) given by (4) to the damper brings on the viscous force set at (3). The reason that a constant value  $c$  is added in (3) is that the friction of the DD motor itself and the ER fluid causes an off-set of torque at low velocity and no electric field as shown in Fig. 4.

I applied the electric field calculated with (4) in real time to the ER damper, and then carried out experiments to derive the angular velocity–torque relation. Coefficients of viscosity,  $Q$ , were set at 1.0, 5.0, and 10.0 N·m·s/rad. The results shown in Fig. 5 indicate good linearities between velocity and torque.

#### B. Method to Energize ER Damper

the electric field applied to the ER damper was limited to under 3.5 kV/mm for safety and for the performance of the

high voltage amplifier. This would cause the saturation of the viscosity of the damper because of the not so low speed movement in transit period. As a result, the desired viscosity could not be realized and so the control performance might then worsen.

To avoid saturation, I determined the coefficient of viscosity of the damper according to the velocity of the DD motor. That is, the viscosity of the damper was kept small so as not to prevent the movement of the motor at high speed; this was realized by applying no electric field. On the other hand, the viscosity became large enough to stabilize the control of the motor at low speed; this was achieved by a large electric field as not saturated. Relation between velocity–coefficient of viscosity was expressed as (5), and as  $E = 0.0$  kV/mm when  $|\dot{\theta}| > 1.5$  rad/s, not to prevent movement of the motor at high speed.

$$Q = \exp(3.675|\dot{\theta}|^2 - 9.277|\dot{\theta}| + 4.010) \quad (5)$$

The  $Q$  corresponding to (5) is illustrated at low speed in Fig. 6.

Here, the strategy to energize the particle type ER damper was considered.

The most desirable behavior was that the coefficient of viscosity is continuously changed in the transit period, and so it is fixed at a large constant value in the positioning period. However, there is the case where there is high speed movement caused by a disturbance etc., so that the viscosity will be changed according to the speed after positioning. This might cause a large deviation since the motor could move fast just after disturbance was added.

On the other hand, it could be effective to apply a constant electric field against a disturbance. This is equal to using the ER damper as a brake. Thus, in addition to the usage as a damper, usage as a brake is combined to improve the property of resistance against disturbances. Specifically, the damper is used as a brake when the motor moves away from the reference, although the motor angle is near to the reference. Fig. 7 shows the algorithm: if used as the damper, (4) and (5) is applied. If using as a brake, a constant electric field,  $E = 3.0$  kV/mm, is applied. If at high speed, the ER damper is de-energized.

#### IV. DRAFT MODELS OF THE DELAY OF ER FLUIDS

A microscopic or transient response model of particle type ER fluids has not been clarified. It is, however, well known that an ER fluid has a 3~5 ms delay in its effect. Common mathematical models of the response delay are written in a time-lag of a first order or a dead-time style. In this paper, the response of the ER damper is captured macroscopically, and the model is then experientially considered as being approximated with a time-lag of the first order or a dead-time.

First, the situation with a dead-time is presented. Here, the basic viscosity is  $Q_0$ , the variance of the viscosity is  $Q_1$ , and the dead-time is  $L_Q$ . The transient response of the viscosity,

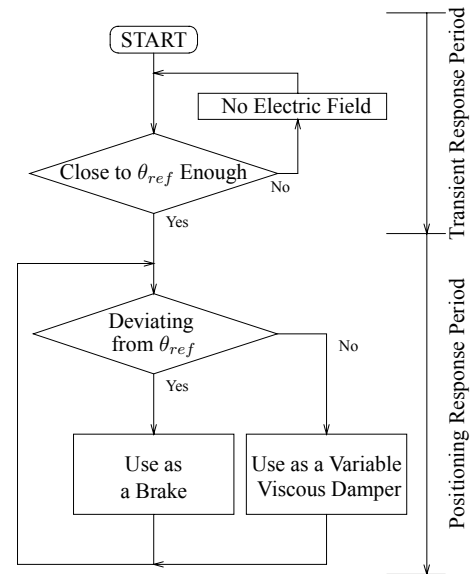


Fig. 7. Algorithm for particle-type ER damper

$Q(s)$ , can then be written as follows:

$$Q(s) = Q_1 e^{-L_Q s} + Q_0 \quad (6)$$

Second, the situation with a first-order lag is presented. Where,  $T_Q$  is the time constant,  $Q(s)$  can be written as follows:

$$Q(s) = \frac{Q_1}{T_Q s + 1} + Q_0 \quad (7)$$

#### V. THEORETICAL VERIFICATION OF STABILITY

The transfer function from the input torque,  $T$ , to the output angle,  $\theta$ , of the DD motor and ER damper system is written as follows.

$$G(s) = \frac{1}{Js^2 + Q(s)s} \cdot e^{-Ls} \quad (8)$$

The transfer function from the reference angle,  $\theta_r$ , to  $\theta$  of the model with the PD control shown in Fig. 8 is written as (9). In this chapter, the stability of the system is theoretically verified using this equation.

$$\frac{\theta}{\theta_r} = \frac{K_P e^{-Ls}}{Js^2 + \{Q(s) + K_D e^{-Ls}\}s + K_P e^{-Ls}} \quad (9)$$

##### A. The Dead Time in $Q$

In this section, the dead time lag style is considered with variations of the coefficient of viscous friction,  $Q$ . This means that the variation of  $Q$  calculated with the equation in Fig. 6 is reflected in the system after the time lag,  $L_Q$ .

The closed loop transfer function of (9) becomes as follows using (6).

$$\frac{\theta}{\theta_r} = \frac{K_P e^{-Ls}}{Js^2 + (Q_0 + Q_1 e^{-L_Q s} + K_D e^{-Ls})s + K_P e^{-Ls}} \quad (10)$$

The stability is analyzed by a parameter plane method [9] because of  $e^{-Ls}$  in (10). Substitution of  $s = j\omega$  and

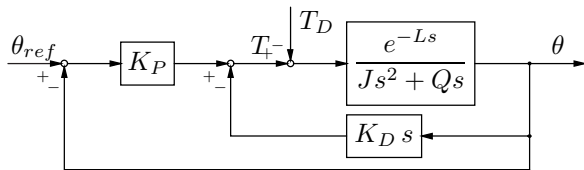


Fig. 8. Block diagram of P-D control system

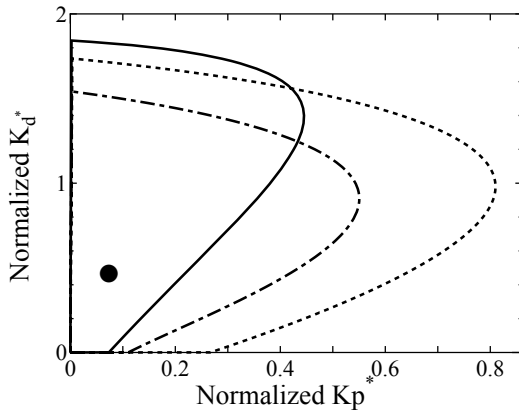


Fig. 9. Stable area of P-D control

$e^{-j\omega L} = \cos(\omega L) - j \sin(\omega L)$  to the characteristic equation gives the following stability limitation of  $K_P$  and  $K_D$ .

$$\begin{cases} K_P = (J\omega^2 - Q_1\omega \sin \omega L Q) \cos \omega L \\ \quad + \omega(Q_0 + Q_1 \cos \omega L Q) \sin \omega L \\ K_D = (J\omega - Q_1 \sin \omega L Q) \sin \omega L \\ \quad - (Q_0 + Q_1 \cos \omega L Q) \cos \omega L \end{cases} \quad (11)$$

### B. The 1st-Order Delay in Q

The closed loop function of (9) becomes the following equation using (7).

$$\frac{\theta}{\theta_r} = \frac{(T_Q s + 1)K_P e^{-Ls}}{J T_Q s^3 + \mathcal{K} s^2 + \mathcal{L} s + K_P e^{-Ls}} \quad (12)$$

Where,  $\mathcal{K} = (J + Q_0 T_Q + T_Q K_D e^{-Ls})$ ,  $\mathcal{L} = (Q_1 + Q_0) + (T_Q K_P + K_D) e^{-Ls}$ .

The same as in the previous section, the substitution of  $s = j\omega$  and  $e^{-j\omega L} = \cos(\omega L) - j \sin(\omega L)$  to the characteristic equation gives the following equations.

$$\begin{cases} K_P = \frac{\omega^2 \mathcal{M} \cos \omega L + \omega \mathcal{N} \sin \omega L}{1 + T_Q^2 \omega^2} \\ K_D = \frac{\omega \mathcal{M} \sin \omega L - \mathcal{N} \cos \omega L}{1 + T_Q^2 \omega^2} \end{cases} \quad (13)$$

Where,  $\mathcal{M} = J - Q_1 T_Q + J T_Q^2 \omega^2$ ,  $\mathcal{N} = Q_1 + Q_0 + Q_0 T_Q^2 \omega^2$ .

### C. Comparison of Stable Regions

Changing the parameter  $\omega$  in (11) and (13) gives stable regions for the models according to  $K_P$  and  $K_D$ . Fig. 9 shows stable areas that are inside of curves; dotted-line: no delay, solid line: the dead time model, and dot-dashed line: the 1st-order delay model. Normalization is applied as  $K_P = K_p^* J / L^2$  and  $K_D = K_d^* J / L$ .

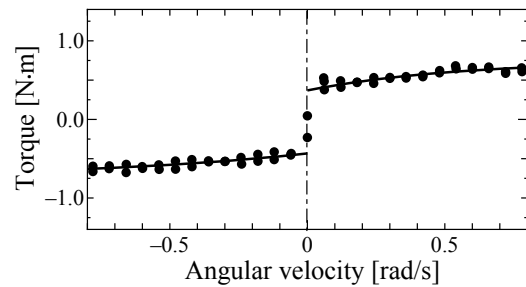


Fig. 10. Friction model

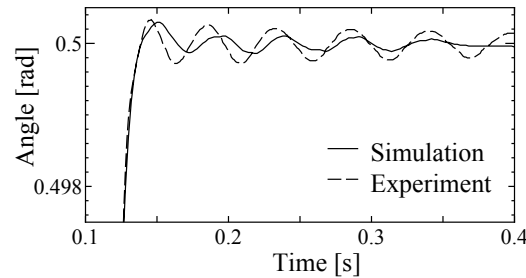


Fig. 11. Verification of friction model

For stability, transfer functions require that all coefficients of their denominator are plus. Therefore, stable regions are limited in the first quadrant of  $K_p^* - K_d^*$  plane as Fig. 9. Moreover, the areas inside curves drawn by the parameter  $\omega$  are the regions. As shown in the figure, if  $Q$  is variable and has a time delay, the stable regions become smaller.

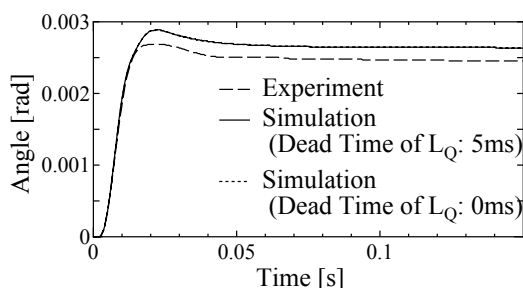
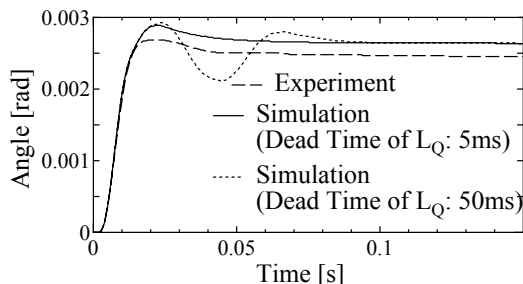
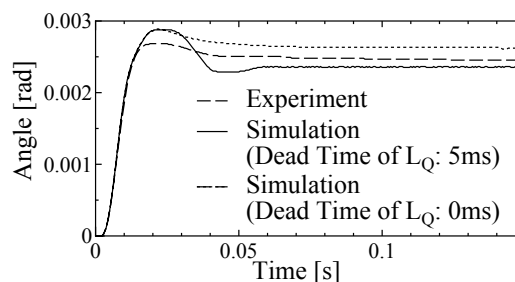
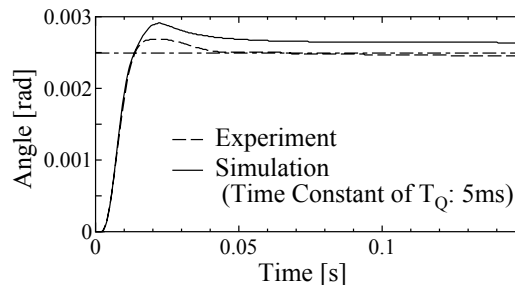
However, in this study, controlling gains for experiments and simulations in consideration of stability and performance are set at  $(K_p^*, K_d^*) = (0.0800, 0.464)$ . The point of  $(K_p^*, K_d^*)$  is shown in Fig. 9 as a dot, which is somewhat inside the stable regions. That is to say, the delay of  $Q(s)$  does not greatly affect the stability. In the next section, how the damping performance is affected is investigated.

## VI. SIMULATION EXPERIMENTS

### A. Introduction of Friction Model

In this chapter, transient responses of systems with a time delay from an ER damper are investigated by simulating the step response. Different from real experiments, a simulation model (9) without any disturbance hardly causes any vibration or other disturbing behavior; therefore, the differences of responses with a variety of lagged models are hard to confirm. Therefore, a friction model was introduced for the simulation. A time delay for the DD motor system was also included in the simulation so that the model as closely as possible resembled a real system.

The friction model had the relationship between angular velocity and command torque derived experimentally as shown in Fig. 10. The simulation result, including that of the friction model (solid lines) and the experimental results (dashed line) are shown in Fig. 11. The situation where the ER damper was de-energized was assumed in both results. The tendencies of the responses, such as the vibration cycle, were almost the same. Therefore, this friction model was used in following simulation.

Fig. 12. Result of dead time model of  $Q$ Fig. 13. The case of large  $L_Q$ Fig. 14. Result of the dead time model of  $E$ Fig. 15. Result of the 1st-order lag model of  $Q$ 

### B. Dead Time in the Coefficient of Viscous Friction $Q$

First,  $Q$  is assumed to have dead time in its variance. Fig. 12 shows the simulation results; a solid line indicates that the lag time  $L_Q$  was 5 ms, a dotted line that  $L_Q = 0$  ms, and a dashed line is the experiment.

In this case, the behavior did not differ from the situation when there was no delay in the simulation. It was also the same as the experimental result. However, a larger time delay of  $L_Q = 50$  ms gave a different response, as in Fig. 13. A larger time delay clearly causes some deterioration of the damping property.

### C. Dead Time in an Applied Electric Field $E$

This section shows the case of the dead time inserted after the command of  $E$  was outputted to the damper. That is, the electric field that accords to the command value of the viscosity at the sampling time is reflected in the system model after the time lag. In view of the damping property, the new  $Q$  is affected by the angular velocity when a new  $E$  has been inflicted on the system. Thus  $Q$  may differ from the value of the  $Q$  desired when calculating the command for the viscosity.

In Fig. 14 the solid line shows the simulation result for 5 ms of dead time, the dotted line shows that of 0 ms dead time, and the dashed line shows the experimental result. When the model had 0 ms of dead time, the result was the same as when  $Q$  had dead time and was near to the experimental result. On the other hand, when the model had 5 ms of dead time, the response showed some difference and a somewhat vibrational behavior, as shown in the figure.

### D. Time-Lag of the First Order in $Q$

The situation where the lagged model of  $Q(s)$  consists of a time-lag of the first order is considered. The simulation results of  $T_Q = 5$  ms are shown in Fig. 15 by the solid line.

The response shows little difference from that of the dead-time model of  $Q$  shown in Fig. 12. Therefore, in respect to the variance of  $Q$ , the response of the system can be considered as being hardly affected, even if the system has a first ordered delay.

### E. First Ordered Delay in $E$

The simulation results when the time-lag of the first order is inserted after the command of  $E$  are shown in Fig. 16 by the solid line. The time constant was 5 ms.

This response is not similar to the case of the 1st-order delay in  $Q$  as in Fig. 15. The phenomenon can be explained as follows. When the motor got close to the reference angle and became slower, the desired  $Q$  increased, with the desired  $E$  then becoming bigger in the same way. However, the slow variance of  $E$  resulted in an insufficient magnitude of the electric field even in the vicinity of the reference. Therefore, the coefficient of viscosity was not large enough, and so made  $K_d^* + q^*$  smaller than that of the theoretical value; which in turn brought about an inadequate damping property and so a large overshooting. On the other hand, when the motor recovered from the overshooting, the electric field became large enough. Therefore, a large frictional torque was invoked, and so the angle was unable to be returned to the reference by PD control.

### F. Model of Both Dead Time and First Ordered Lag in $E$

To investigated further, another simulation was carried out with a delay model of  $E$  that included both dead-time and 1st-order delay;  $L_Q = 2$  ms, and  $T_Q = 3$  ms. As seen in Fig. 17, the results show a similar tendency to the experimental ones, in spite of there being a little difference from the other simulation noted above. The torque data of Fig. 17 as shown in Fig. 18 also indicate the same responses for the experiment and the simulation. Moreover, that they

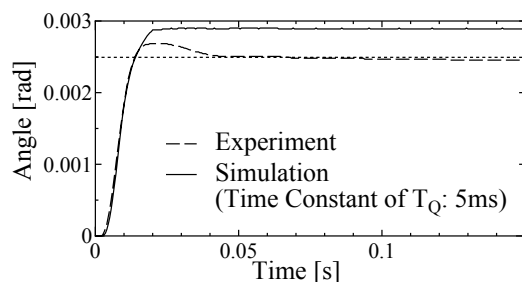
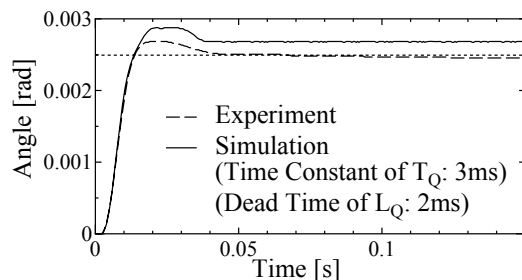
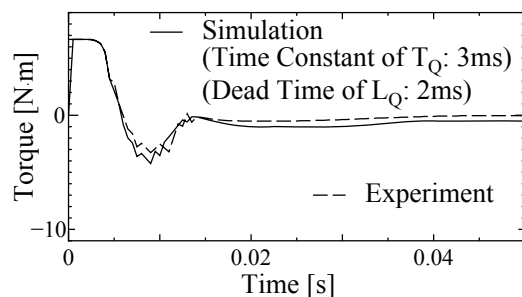
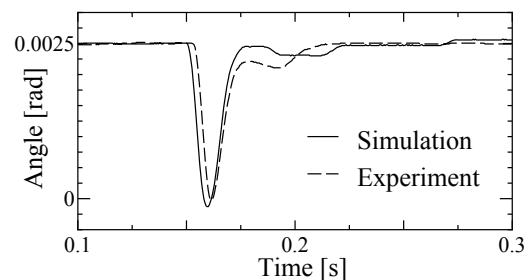
Fig. 16. Result of the 1st-order lag model of  $E$ Fig. 17. Result of  $L_Q = 2\text{ms}$ , and  $T_Q = 3\text{ms}$ 

Fig. 18. Torque data of Fig. 17

Fig. 19. Another (disturbed) response of  $L_Q = 2\text{ms}$ , and  $T_Q = 3\text{ms}$ 

show the same tendency confirmed in Fig. 19, which shows another experiment and simulation for this model.

## VII. DISCUSSIONS AND CONCLUSION

In this paper, I investigated the effects of time delays from the viscosity of a damper on control performance and stability in a high speed and precision position control system with a variable viscous damper using particle type ER fluid.

First, two basic mathematical models presenting the delay of viscous changes were introduced: a dead-time model, and a time-lag model of the first order. Then, when the coefficient of viscosity was varied and included delay in its variation, the stability of the control system was verified by

the parameter plane method. The results showed that stable regions of the parameters got smaller from the time delay. However, the parameters of the controller in this paper were inside the stable regions, and the robustness was confirmed by another calculation: it was greatly affected by variation of the system model, especially the lag time or the time constant. Nevertheless, it did not result in a destabilized state if the variations were within several dozen percent of the nominal values.

Second, responses in a simulation, which had various kinds of time delay models with viscous changing of the damper, were compared with those of experiments. The simulation model was approximated to real equipment by including a delay of the DD motor system itself and a friction model so that the response was affected only by differences in the way the viscosity manifested. Considering the mechanism of viscosity, the case where the 1st-ordered delay was inserted before applying the electric fields appeared closest to the situation of real equipment. However, the simulation results showed the largest deviation from the experimental ones. Whereas, if the delay model of the electric field had both of dead-time and 1st-ordered delay, this simulation results were approximated to the experimental response.

Most importantly, this study demonstrates the possibility that the transit response model of particle type ER fluid can be approximated by a composite function of a dead-time model and a time-lag model of the first order. For designing the controller phase, using a model that explicitly includes a delay for the ER dampers will result in stability analysis or parameter design that is more precise.

## ACKNOWLEDGMENTS

I wish to thank Prof. J. Furusho of Osaka University and Dr. A. Inoue of ER-tec Co. for useful advice.

## REFERENCES

- [1] K. Lu, R. Shen, and J. Liu, Eds., *Proceedings of the Ninth International Conference on Electrorheological Fluids and Magnetorheological Suspensions*. World Scientific, 2005.
- [2] J. Furusho and M. Sakaguchi, "New actuators using ER fluid and their applications to force display devices in virtual reality and medical treatments," *Int. J. of Modern Physics B*, vol. 13, no. 14-16, pp. 2151-2159, 1999.
- [3] D. Adams and L. Johnston, "Industrial benefits of ERF-Technology (a development report from the industry)," in *Proc. of the Eighth Int. Conf. on Electro-Rheological Fluids and Magneto-Rheological Suspensions*. World Scientific, 2001, pp. 37-42.
- [4] S. Hong, S. Choi, W. Jung, and W. Jeong, "Vibration isolation of structural systems using squeeze mode ER mounts," *Journal of Intelligent Material Systems and Structures*, vol. 13, no. 7-8, pp. 421-424, 2002.
- [5] Y. Ono, "A direct-drive motor control system and its features," *Advanced Robotics*, vol. 6, no. 2, pp. 243-254, 1992.
- [6] C. J. Kempf and S. Kobayashi, "Disturbance observer and feedforward design for a high-speed direct-drive positioning table," *IEEE Transactions on Control Systems Technology*, vol. 7, no. 5, pp. 513-526, 1999.
- [7] N. Takesue, G. Zhang, J. Furusho, and M. Sakaguchi, "Motion control of direct-drive motor by a homogeneous ER fluid," *Journal of Intelligent Material Systems and Structures*, vol. 10, no. 9, pp. 723-727, 1999.
- [8] K. Koyanagi and J. Furusho, "Direct-drive motor system with particle-type ER fluid damper," *International Journal of Modern Physics B*, vol. 19, no. 7-9, pp. 1661-1667, 2005.
- [9] D. D. Šiljak, "Analysis and synthesis of feedback control systems in the parameter plane, i—linear continuous systems," *IEEE Transactions on Applications and Industry*, vol. 83, no. A1-75, pp. 449-473, 1964.