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Switching Control of Mobile Robots for Autonomous Navigation in Unknown Environments

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Abstract—**This paper presents a switching controller for positioning a unicycle-like mobile robot at a desired point with final orientation avoiding obstacles in completely unknown environments. To this aim two complementary algorithms are included: the first decides whether to avoid an obstacle around its right or left side, and the second is intended to detect when an obstacle was successfully avoided. The obstacle avoidance is performed using a laser-based reactive contour-following controller. The switching controllers include the stability analysis at the switching times, using common and multiple Lyapunov functions. Finally, experimental results in a typical unicycle-like mobile robot show the performance of the proposed hybrid control system.**

I. Introduction

THE problem of programming a mobile robot to move from one place to another is of course as old as the first mobile robot. In mobile robotics almost every task to solve deals with the problem of parking [1] or with the classical behavior "move-to-goal" in behavior-based architectures [2]. The unicycle model has a nonholonomic constraint that makes it impossible to design a continuous invariant control law that guarantees to reach a final posture in Cartesian coordinates. In a seminal paper by Brockett [3] it is implied that so-called *nonholonomic* systems cannot be stabilized by a differentiable and time-invariant state feedback. Intuitively a nonholonomic constraint restricts a vehicle motion locally but not globally. For the kinematic unicycle, the nonholonomic restriction implies no sideways motion of a point on the wheel axis. Note that under this constraint, there is a feasible trajectory between any two configurations (postures). The price paid for free motion of an off-axis point is the lost of orientation control. Several works have been developed in this area; [4] uses the dynamic model of the mobile robot and achieve the objective by means of neural networks, in [5], [6], [1] a change in the coordinates of the kinematic model have been introduced. Another research objective, related to the autonomous robot navigation, is the obstacle avoidance. Regarding this problem many possibilities appear, mainly depending on the kind of obstacle and the inclusion of this behavior into the control architecture [7], [8], and [9].

This work presents a switching approach for the parking problem (Section III), i.e. the control of the robot between two arbitrary postures: the robot must reach the final posture $\begin{bmatrix} x_d & y_d & \theta_d \end{bmatrix}^T$ starting from any initial posture $\begin{bmatrix} x_i & y_i & \theta_i \end{bmatrix}^T$ as can be seen in Fig.1. This approach takes advantage of the nonholonomic constraint of the unicycle-like mobile robots by decomposing the robot movement in such a way that backward motions are avoided and the robot heading is always in the direction of the goal posture.

Next, the obstacle avoidance problem is considered in order to avoid unknown obstacles in a priori unknown positions: the chosen algorithm is a reactive contour-following (CF) controller [10], which maintains a desired robot-obstacle distance.

Finally, the robot-environment interaction is considered by proposing a switching control system including both above mentioned controllers. The combination of these strategies allows the robot to handle very real situations, including confinement or trap situations in large-scale settings. While working with this robot-environment interaction two problems arise: i) the detected obstacle must be avoided following its contour, but which side of the robot must be selected? And, ii) was the detected obstacle already avoided? In order to address these questions two algorithms are proposed in sections IV.A and IV.B. A significant part of this work is related with the stability of the system. In this context, it is important to mention that: i) the stability of the individual controllers was proved using Lyapunov theory; ii) stability at the switching times for the point-to-point controller was considered and furthermore, iii) it is proposed an stability approach for the overall control system considering multiple Lyapunov functions.

Next, in Section V experimental results in laboratory and office settings are presented. Finally in Section VI the conclusions are stated.

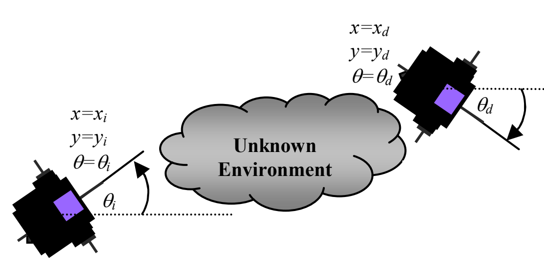

Fig.1. Problem description

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II. MOBILE ROBOT

In this paper it is considered the wheeled mobile robot of unicycle type shown in Fig.2, in which the state variables are x, y (the coordinates of the middle point of the front wheels axle) and θ (angle of the vehicle with the world X-axis [^{W}X]). A rear wheel turns freely and balances the rear end of the robot above the ground. The kinematics of the robot can be modeled by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

where v and ω are the control inputs: the forward and the angular velocity, respectively. The robot is equipped with a laser radar sensor. With reference to Fig.2, the lateral beams from 0º to 15º (and from 165º to 180º) are used to estimate the obstacle contour angle, while all of the beams are used to define a guard-zone (or safety-zone), whose purpose is to detect possible robot-obstacle collisions. This rectangular guard-zone is defined by two parameters: the desired lateral (d_{lat}) and frontal (d_{front}) distance. The minimum lateral value for a Pioneer IIIDX is about 330 millimeters.

III. PARKING PROBLEM

This section presents a switching controller approach to address the parking problem. Two continuous controllers described in subsections III.*A* and III.*B* respectively acts as subsystems. Then, in III.*C* the switching controller stability is considered.

A. Heading Control

This controller allows positioning the robot at the desired orientation angle θ_d (Fig.3). Considering a constant angular error (2), and its time derivative (3)

$$\tilde{\theta} = \theta_d - \theta \tag{2}$$

$$\dot{\tilde{\theta}} = -\omega\,. \tag{3}$$

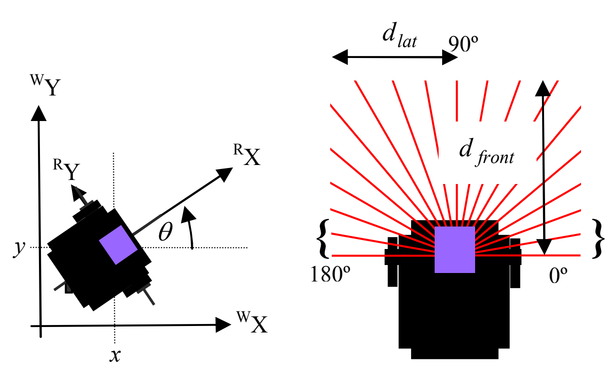

Fig.2. Unicycle-like mobile robot and laser rangefinder

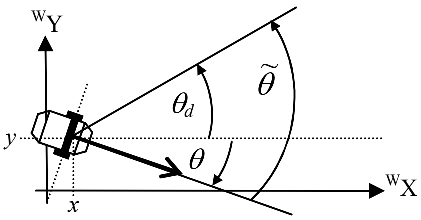

Fig.3. Controller for Angular Position

The following control actions are proposed,

$$v = 0 \tag{4.a}$$

$$\omega = K_{\tilde{\theta}} \tanh\left(k_{\theta}\,\tilde{\theta}\right), \quad K_{\tilde{\theta}} > 0 \tag{4.b}$$

The expression for the angular velocity saturates at the value of the constant $K_{\tilde{\theta}}$. The value of $k_{\tilde{\theta}} > 0$ is chosen to increase the angular velocity for small errors. Considering the following Lyapunov candidate function

$$V_{\tilde{\theta}} = \tilde{\theta}^2 / 2\,, \tag{5}$$

the asymptotic stability at the origin, that is: $\tilde{\theta}(t) \to 0$, is easily proved.

B. Positioning Control

Let us consider the controller in [11], where the robot can reach a desired destination point $\begin{bmatrix} x_d & y_d & \theta \end{bmatrix}^T$ in the work plane without specifying the final heading (Fig.4). Cartesian errors become defined as:

$$\tilde{x} = x_d - x \tag{6.a}$$

$$\tilde{y} = y_d - y \tag{6.b}$$

and the control states are calculated as

$$d = \sqrt{\tilde{x}^2 + \tilde{y}^2} \tag{7.a}$$

$$\tilde{\theta} = \theta_d - \theta = \tan^{-1}\left(\tilde{y} / \tilde{x}\right) - \theta\,. \tag{7.b}$$

The time-variation of these control states are given by

$$\dot{d} = -v\cos\left(\tilde{\theta}\right) \tag{8.a}$$

$$\dot{\tilde{\theta}} = v\sin\left(\tilde{\theta}\right) / d - \omega\,. \tag{8.b}$$

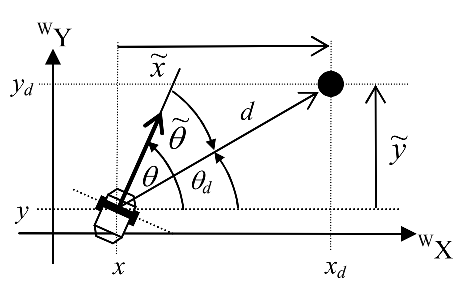

Fig.4. Controller for Target Position

Analyzing the system at the equilibrium point:

$$\chi = \begin{bmatrix} d \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 , \qquad (9)$$

the following Lyapunov candidate function is considered:

$$V_t = \tilde{\theta}^2 / 2 + d^2 / 2 \qquad (10)$$

Its time derivative along the trajectories is

$$\dot{V}_t = \tilde{\theta}\,\dot{\tilde{\theta}} + d\,\dot{d} = \tilde{\theta}\left(v \sin\left(\tilde{\theta}\right)/ d - \omega\right) - d\, v\, \cos\left(\tilde{\theta}\right). \quad (11)$$

The following control actions are defined

$$v = \frac{v_{\max}}{1+|d|}\, d \cos\left(\tilde{\theta}\right) \qquad (12.a)$$

$$\omega = \frac{v_{\max}}{1+|d|} \cos\left(\tilde{\theta}\right)\sin\left(\tilde{\theta}\right) + K_{\tilde{\theta}}\, \tanh\left(k_\theta\, \tilde{\theta}\right) \qquad (12.b)$$

finally, by replacing (12) into (11) the control system stability at the equilibrium point can be easily proved.

C. *Switching Controller for parking with final orientation*

The block diagram in Fig.5 shows this switching control system composed by the controllers described in the previous sections. The switching signal is σ_1; whereas $\sigma_1 = 1$ the controller for distance correction is active, and whereas $\sigma_1 = 0$ or $\sigma_1 = 2$ the controller for Angular Position is active. By redefining the errors according with Fig.6

$$\tilde{\theta}_1 = \theta_{d1} - \theta = \tan^{-1}\left(\tilde{y} / \tilde{x}\right) - \theta \qquad (13.a)$$

$$\tilde{\theta}_2 = \theta_d - \theta \qquad (13.b)$$

$$d = \sqrt{\tilde{x}^2 + \tilde{y}^2} \qquad (13.c)$$

and considering as individual subsystems the controllers described in sections III.*A* and III.*B*: the switching between these controllers is ruled by an automata, which logic (Fig.7) is based on three different stages, a) first the robot is oriented to the destination point by correcting the angle θ_{d1}, b) then, the robot achieves the final point without regard of its orientation and c) finally the robot corrects its orientation to the desired final heading with $\tilde{\theta}_2(t) \to 0$. Switching according this logic the robot goes straight to the target point by activating the *heading controller* before moving towards the target point and after reaching it. Regarding the stability at the switching times, it can be proved that both controllers share the closed-loop equation

$$\dot{\tilde{\theta}} = -K_{\tilde{\theta}}\, \tanh\left(k_\theta\, \tilde{\theta}\right) \qquad (14)$$

then, considering the equilibrium point $\tilde{\theta} = 0$ is easy to see that (10) is a Common Lyapunov Function for the switching system above. Therefore, the supervisor can switch among the mentioned controllers without affecting the stability of the system. In the following (see Fig.8), we show a comparison between a usual continuous parking controller [11] and the proposed switching controller.

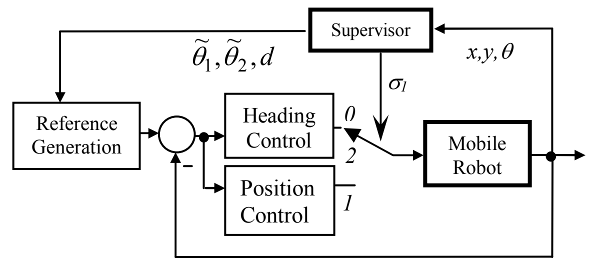

Fig.5. Block diagram of the Supervisor

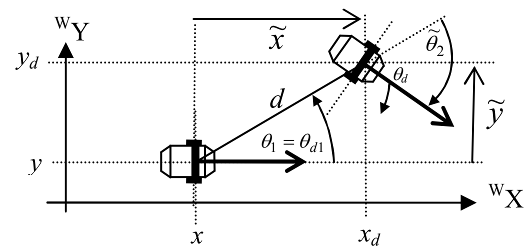

Fig.6. Angles description for the parking controller

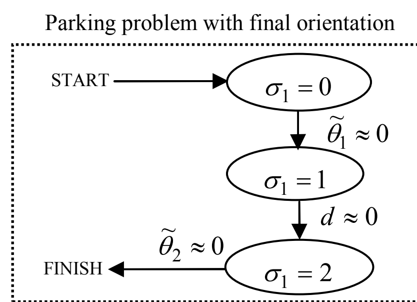

Fig.7. Parking controller: Supervisor logic

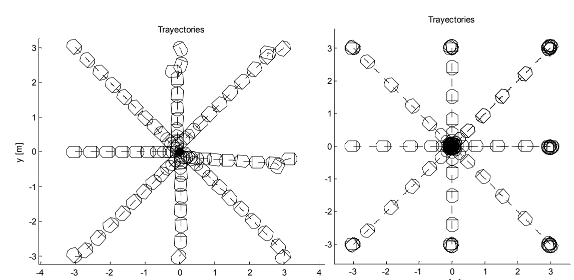

Fig.8. Left: continuous parking controller without final orientation for eight different starting points and the origin as goal. Note the path followed by the robot when moving backwards. Right: Switching parking controller with final orientation. Always reaching the final point by means of a straight path

IV. SWITCHING PARKING CONTROL

Given the robot in its initial position, it must arrive at a destination posture avoiding the obstacles between the initial and the final points (the only requirement is that there is a feasible path connecting these points). The solution proposed here is not optimal, in the sense that if two or more possible ways are available, the selection of an optimal path is not performed due to lack of global information. It is important to know the instant at which the obstacle has been avoided. To this aim, an appropriate algorithm is proposed.

A. Obstacle Avoided Detection

This algorithm requires to know the present robot position (x,y), the desired final position (x_{REF},y_{REF}) and the position (x_{000},y_{000})/(x_{180},y_{180}) of the laser beam at 0º/180º that indicates the position at the right/left side of the robot. These points can be appreciated in Fig.9. Then the problem is divided into four quadrants depending on the relation between the actual and the final points. A flag variable *OBSTpassed* is defined; the value TRUE for this variable indicates that the obstacle was actually surpassed. As an example for the case in which x_{REF}>x and y_{REF}>y, see Fig.9, the algorithm is

$$\begin{aligned} & OBSTACLE_{passed} = false \\ & if\{((x_{REF} > x) AND (y_{REF} > y) AND (x_{000} < x) AND (y_{000} < y)) \\ & OBSTACLE_{passed} = true\} \end{aligned} \tag{15}$$

B. Right/Left robot side selection algorithm

As the obstacle avoidance problem is treated by using a CF controller, it is important to detect the side of the robot that will avoid the obstacle. To this aim, the safety-zone defined by the laser range finder is employed, in such a way that, analyzing the obstacle invasion according to Fig.10, it is decided if the robot will avoid the obstacle to its left or to its right side. For brevity this algorithm is not explained in detail, but an intuitive approach can be seen in Fig.10.

C. Block diagram

The block diagram of the control system is as depicted in Fig.11, the switching signal is σ, whereas σ=0 the robot is approaching the goal point using the parking controller of Section III.*C* and will only switch to the CF controller if an obstacle is detected. The CF controller allows the robot to follow the discontinuous contour of the obstacle at a desired constant distance.

D. Stability Analysis

In order to prove asymptotic stability of this switching control system a Multiple Lyapunov Functions (MLF) [12] based approach is considered by associating a Lyapunov function to each controller (one for the parking and other for the CF) and designing a logic that guarantees that the sequence of values for these functions is decreasing.

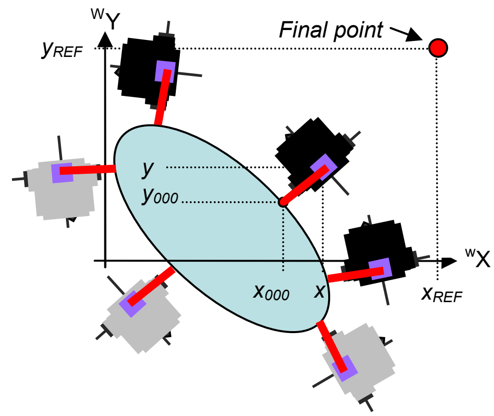

Fig.9. Obstacle avoided detection example for the III quadrant (taking xREF,yREf as the origin of the Cartesian coordinate plane and x,y of the robot as a point in space with some displacement from the origin) and for the robot following an obstacle (here with oval shape) at its right side. Black robots indicate the zone where the obstacle was avoided and the grey robots the zones where the obstacle was not yet avoided. Similar graphs can be constructed for the other quadrants and for the robot following the obstacle at its left side

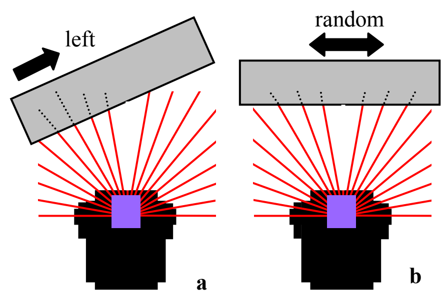

Fig.10. a) Invasion to the safety-zone is stronger at its left side and b) when the invasion is equal at both sides the selection depends on the positions of the goal-point. Finally, if the goal-point is just in front (through the obstacle) then the side to follow is randomly selected. Note that the robot heading is always in the goal direction

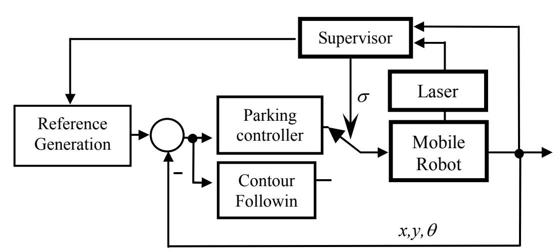

Fig.11. Control system block diagram

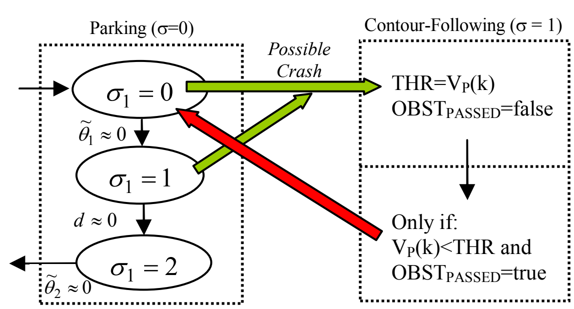

Fig.12. Supervisor logic

First, it is considered the following Lyapunov function, derived from (10) which indicates the achievement of the parking control objective

$$V_P = d^2 / 2 . \qquad (16)$$

In order to guarantee a decreasing sequence for (16) it is considered a threshold (THR) value, which takes the value of (16) each time that an obstacle is detected. Then the CF controller is activated and the control will return to the parking controller only if the obstacle was avoided and the value of (16) is less than the THR value. Note that every time the parking controller is activated, it must recalculate the control actions considering the actual point as the initial point, i.e., the robot will head at the desired final point every time that it is switched to the parking controller. Second, some assumptions must be stated: i) the CF controller is asymptotically stable and has an associated Lyapunov function which indicates the robot-obstacle error. As this function converge asymptotically to zero when the CF controller is active, it is assumed that will be always decreasing and therefore its corresponding analysis is not included here; ii) another implicit assumption is that following the obstacle contour the value of (16) will be smaller. This is a strong hypothesis heavily dependent on the IV.*A* and IV.*B* algorithms performance. However, as can be seen in the experimental results presented in this paper, this analysis approach prove asymptotic stability in typical indoor settings, iii) finally, the completely unknown environment assumption must be relaxed, since the robot navigation capability depends directly on the d_{front} and d_{lat} values (Fig.2).

V. Experimental Results

The experiments were carried out using a Pioneer IIIDX mobile robot. In the first experiment (Fig.13), it can be seen how the obstacle is detected in <1> and the value of V_d is taken as the new threshold value. Next, in <2> the obstacle is avoided but the value of V_d is greater than the threshold, so the robot keeps following the obstacle until <3> where the value of V_d is less than the threshold and the system switches to the parking controller (securing that V_d will be decreasing due to the selected logic).

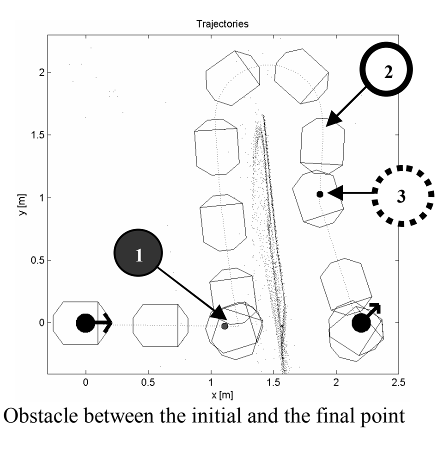

Fig.13. Obstacle between the initial and the final point

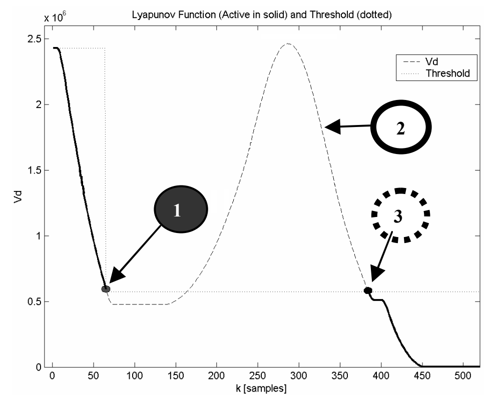

Fig.14. <1>: Obstacle detected; <2> Obstacle avoided; <3>: Switching to the parking controller

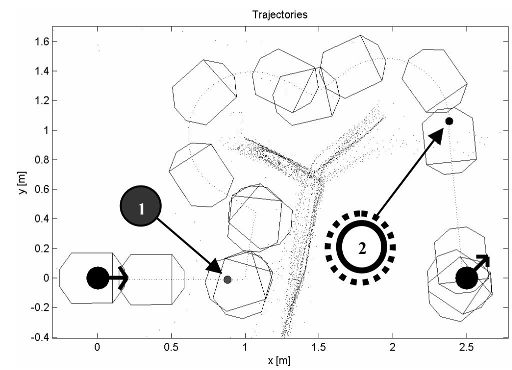

Fig.15. Avoiding a trap situation

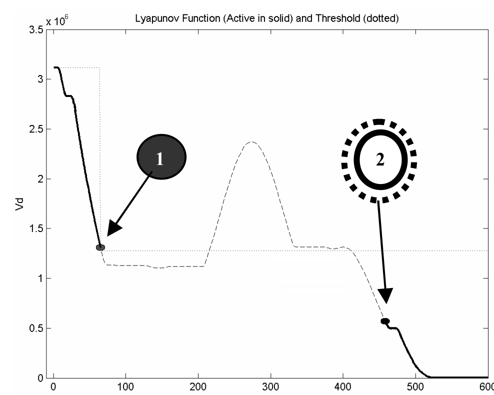

Fig.16. <1>: obstacle detected; <2>: obstacle avoided with a Vd value smaller than the threshold (direct switching case)

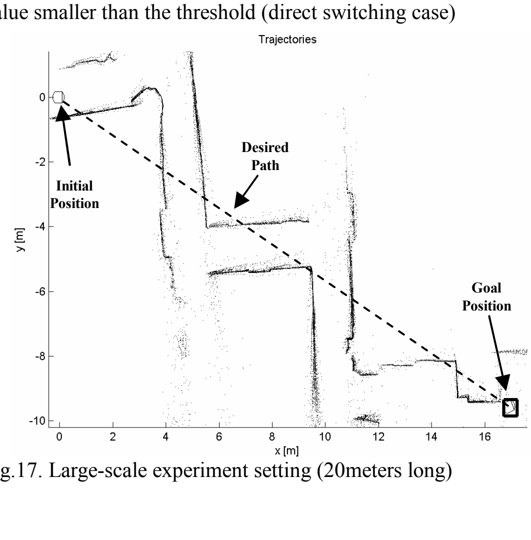

Fig.17. Large-scale experiment setting (20meters long)

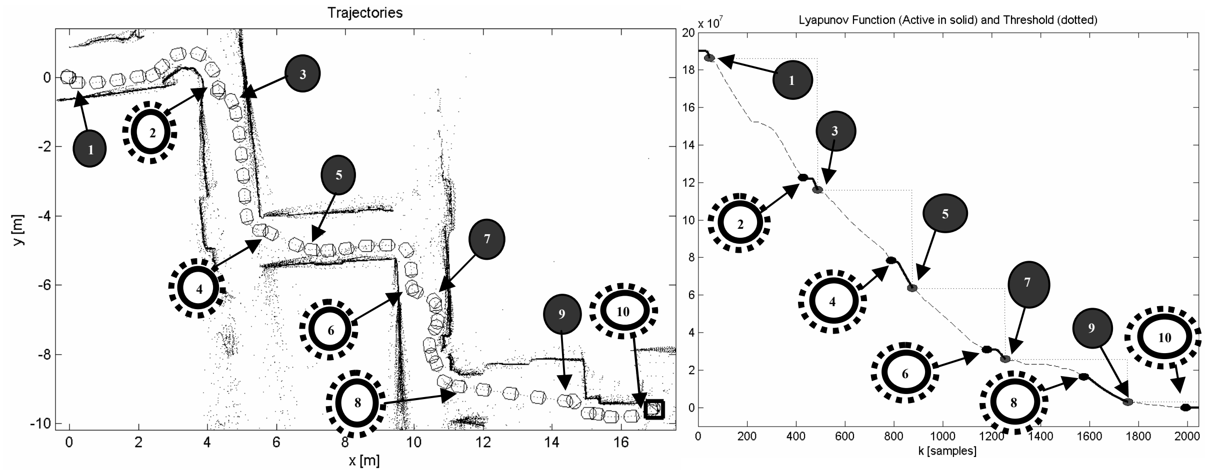

Fig.18.<(1,3,5,7,9> obstacle detected; <2,4,6,8,10>: obstacle avoided. Right Picture: Decreasing Lyapunov function.

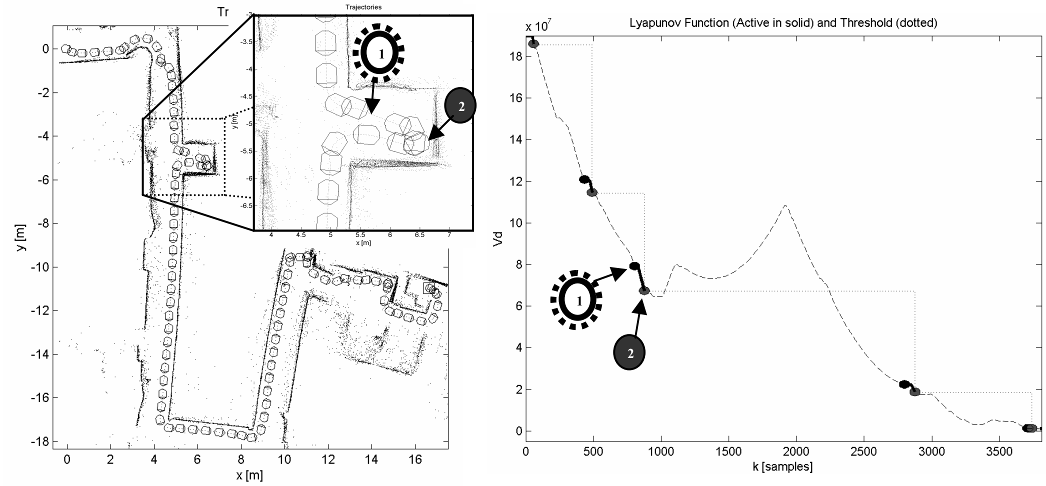

Fig.19. Same experimental setting, but blocking the corridor: <1>: obstacle avoided; <2>: obstacle detected.

In the experiment of Fig.17 the robot must arrive to the destination point crossing several doors and corridors. The result is depicted in Fig.18. Finally, the main corridor was blocked, forcing a trap situation in (Fig.19).

VI. Conclusions

In this paper, we have presented a switching controller that deals with the problem of positioning a mobile robot with final orientation by avoiding unknown obstacles. To this aim two complementary algorithms were proposed: one that allows the robot to detect when an obstacle was or not avoided; and another that selects the side to avoid the obstacle. The presented switching controllers have included the stability analysis at the switching instants. Finally, experimental results have shown the good performance of the control system in real situations.

References

[1] M. Aicardi, G. Casalino, A. Bicchi and A. Balestrino, “Closed loop steering of unicycle like vehicles via Lyapunov techniques,” IEEE Robotics and Automation Magazine, 2(1): 27-35, 1995.

[2] R.C. Arkin, *Behavior-based Robotics.* MIT Press, 1998.

[3] R. Brockett, *Differential Geometric Control Theory*. Pages 181-191. Birkhauser, 1983.

[4] R. Fierro, and F.L. Lewis, “Practical point stabilization of a Nonholonomic mobile robot using neural networks,” Proc. Conf. on Decision and Control, 1722-1727, 1996

[5] C. de Wit and O.J. Sordalen, “Exponential stabilization of Mobile Robots with non-holonomic constraints,” Trans. Automatic Control, 37, 1792-1797, 1992.

[6] A. Tayebi and A. Rachid, “A unified discontinuous state feedback controller for the path-following and the point-stabilization problems of a unicycle-like mobile robot,” IEEE Int. Conf. on Control Applications, 6, 31-35, 1997.

[7] M. Wang and J. Liu, “Autonomous robot navigation using fuzzy logic controller,” Int. Conf. on Machine Learning and Cybernetics, 26-29, 2004.

[8] R. Carelli and E. Oliveira Freire, “Corridor navigation and wall-following stable control for sonar-based mobile robots,” Robotics and Autonomous Systems, 45,235-247, 2003

[9] E. Bicho, “Detecting, representing and following walls based on low-level distance sensors,” Int. Symp. on Neural Comp., 2000.

[10] J.M. Toibero, R. Carelli and B. Kuchen, “Stable Switching Contour-Following Controller for Wheeled Mobile Robots,” IEEE ICRA, 2006.

[11] H. Secchi, R. Carelli and B. Mut, “Design of Stable Algorithms for Mobile Robots Control with Obstacle Avoidance,” 14th IFAC World Congress, 185-190, 1999.

[12] D. Liberzon, *Switching in Systems and Control.* Birkhauser, 2003.