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Model-Based Nonlinear Observers for Underwater Vehicle Navigation: Theory and Preliminary Experiments

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***Abstract*— This paper reports the analytical development and preliminary experimental evaluation of a class of exact nonlinear full state model-based observers for underwater vehicle navigation. This class of observers exploits exact knowledge of the vehicle's nonlinear dynamics, the forces and moments acting on the vehicle, and disparate position and velocity measurements. The reported observer is novel in that it estimates the full state of the vehicle and employs Lyapunov techniques to show stability. The performance of the observer is evaluated using data from single degree-of-freedom experiments with a laboratory remotely operated vehicle. High-precision measurements from a 300kHz Long Baseline (LBL) acoustic positioning systems serve as the basis for evaluating the performance of the observer. Error in the observer position estimate possesses a significantly lower standard deviation than measurements from 12kHz LBL systems alone. The performance of the observer is compared to the Extended Kalman Filter (EKF) and the error in the position estimates of these two estimators found to be comparable. These experiments are, to the best of our knowledge, the first report of the experimental implementation of exact nonlinear dynamic model-based observers and their comparison to EKFs for underwater vehicle navigation.**

***Index Terms*— Nonlinear Estimation, Observers, Underwater Vehicles, Navigation, Dynamics**

I. Introduction

This paper reports the analytical development and preliminary experimental evaluation of a new class of model-based nonlinear state observers (state estimators) for underwater vehicle navigation. Most previously reported underwater vehicle navigation observers have estimated vehicle state (position and velocity) with sensor data and kinematic plant models. The reported observer is novel because: *(i)* it exploits knowledge of the vehicle's exact nonlinear dynamics; *(ii)* the observer estimates vehicle position and velocity; *(iii)* stability of the observer is shown with Lyapunov techniques and the Kalman-Yakubovich-Popov (KYP) Lemma; and *(iv)* the performance of the observer is experimentally evaluated and compared to the performance of the Extended Kalman Filter (EKF).

The investigation of these observers is motivated by the need to provide three-dimensional navigation of underwater vehicles with a precision and update rate sufficient for tasks such as closed-loop control. Signals from the global positioning system (GPS) rapidly attenuate in water, and, traditionally, underwater vehicles have employed acoustic time-of-flight navigation systems — i.e., [19]. Recent advances in sensor technology and algorithms has enabled significant progress in underwater vehicle navigation and the reader is referred to [30] for an extensive survey. While these reported methodologies utilize navigation sensor data and kinematic models of the vehicle state dynamics, most do not employ knowledge of the vehicle's plant and actuator dynamics. The analytical development and implementation of dynamic model-based nonlinear observers promises to improve underwater navigation, and, in consequence, further increase the capabilities of underwater vehicles.

The remainder of this paper is organized as follows: Section II reviews previously reported work in nonlinear observers and the application of nonlinear observers to underwater vehicle navigation. Section III reports a full state observer for underwater vehicles with proof of asymptotic stability. Preliminary experimental evaluation of this observer is reported in Section IV.

II. Previous Results in Nonlinear Observer Theory

The origins of nonlinear observer theory lies in the development of deterministic linear observers by Luenberger [36], [37]. The literature on linear observers is extensive — the reader is referred to linear systems texts [10], [42], [25] for an introduction to the field. While Luenberger's work focused on the development of linear, deterministic observers, the work of Kalman and others focused on the development of the optimal, stochastic, linear observer. Kalman reported the first discrete-time optimal unbiased minimum variance state estimator in [26], and Kalman and Bucy extended this work to the continuous case in [28]. The Kalman Filter (KF) has been applied to nonlinear state estimation problems with great success using techniques such as the EKF and the unscented filter [24]. Numerous texts exist on Kalman filtering — [6], [11], [46] provide an entry to this literature.

A. Stability

This section reviews previously reported work in the field of nonlinear observer stability. A majority of the existing literature on the stability of nonlinear systems can be classified into four areas: (i) error linearization, (ii)

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Lyapunov techniques, (iii) frequency domain techniques, and (iv) contraction mapping.

1) Error Linearization: Early work on the development of nonlinear observers applies linear observer theory to linearized nonlinear systems. Krener and Isidori report an observer for single output nonlinear systems where the nonlinear system can be transformed to a linear system via a change of state variables and output rejection — the result is an exactly linear observer error system [32]. Banaszuk and Sluis extend this concept to nonlinear systems where an exact linearizion is not possible by employing a least-squares technique that results in an approximately linear observer error system [5].

2) Lyapunov Techniques: A common technique for showing the stability of nonlinear systems is Lyapunov's Second Method [38], and the closely related concept of passivity. Tsinias addressed the nonlinear observer where the nonlinearity is bounded and, provided the system satisfied certain sufficiency conditions, proposed an observer design that is a direct extension of the linear case [48]. Gautheir and colleagues construct an observer for systems with globally Lipschitz nonlinear functions [16]. Besançon and Hammouri report an observer that is stable for a variety of nonlinear inputs [8].

Results using passivity techniques for showing the stability of nonlinear observers have been reported by numerous authors. Berghuis and Nijmeijer propose using a passivity approach to design a nonlinear controller-observer in [7]. Shim and colleagues use output feedback passification of the error dynamics to construct a nonlinear observer [43]. Strand and Fossen use passivity methods to analyze nonlinear observers for the dynamic positioning of surface vessels [15].

3) Frequency Domain Techniques: Arcak and Kokotović proposed using the circle criterion to design observers with monotone sector nonlinearities [2] and, using Popov criteria, they develop an extended circle criterion in [3]. [1] investigates using observers designed with the circle criterion in output-feedback control and develops a modified circle criterion observer that ensures global asymptotic stability for certainty-equivalence controllers. Fan and Arcak report the design of globally convergent observers for a class of multi-variable nonlinear systems in [13]. [39] presents a new stability condition that exploits the state-dependent proprieties of observers, thus eliminating exogenous disturbances in the system.

4) Contraction Mapping: Lohmiller and Slotine introduced contraction mapping as a tool for proving the stability of nonlinear systems [34]. They reported the application of this technique to nonlinear observers in [35]. This technique has been applied to the analytical development of observers for problems such as inertial navigation [51] and dynamic positioning of surface vessels [47]. Recent work by Jouffroy provides sufficiency conditions under which the negative definite condition on the Jacobian may be relaxed such that the maximum eigenvalue of the Jacobian may be positive semi-definite [21]. Jouffroy discusses issues related to the real-time implementation of diffusion-based trajectory observers in [22].

B. Experimental Evaluation of Exact Nonlinear Observers

Numerous papers report analytical and numerical results for nonlinear observers, however papers reporting actual experimental results are rare. Fossen and Strand implemented the nonlinear observer reported in [15] to estimate the position of an ocean going vessel. The development and experimental evaluation of a nonlinear observer for estimating material damage is reported in [9], [12]. [29] reports experimental results for fault detection in electro-hydraulic positioning systems using a nonlinear observer. Görgün and colleagues report the implementation of a nonlinear observer for estimating membrane water content in fuel cells in [17]. Jouffroy reports the implementation of diffusion-based trajectory observers on data collected with a field deployed remotely operated vehicle (ROV) in [23], [22].

C. Application of State Estimators in Underwater Vehicle Navigation

To date, the development and implementation of model-based state estimators for underwater vehicle navigation has primarily focused on applying the KF or EKF to a kinematic model. Additional work has investigated using Simultaneous Localization and Mapping (SLAM), trajectory-based observers [23], or dynamic model-based Kalman Filters [20]. Ribas and colleagues report the experimental implementation of a dynamic model-based EKF in [41]. An extended discussion on the application of state estimators to underwater vehicle navigation is presented in [30].

The analytical development of a nonlinear underwater vehicle velocity observer has been previously reported by Lohmiller and Slotine in [34] using contraction mapping to show stability. In [40], Refsnes and colleagues report the analytical development of an exact dynamic model-based observer based on the underwater vehicle model presented in [14]. The observer reported herein differs from these previously reported results in that it employs an experimentally evaluated vehicle model, whose parameters can be adaptively identified, to estimate the full state of the vehicle. It is, to the best of our knowledge, the first reported experimental implementation of an exact dynamic model-based nonlinear observer for underwater vehicle navigation.

III. The Full State Observer

This section reports an exact nonlinear observer for estimating the state of a underwater vehicle. These observer incorporates (*i*) exact nonlinear models of the vehicle's dynamics; (*ii*) plant output data from disparate position and velocity sensors; and (*iii*) actuator forces and moments acting on the vehicle. We employ the experimentally validated decoupled underwater vehicle model reported in [45].

Plant: Employing the single degree-of-freedom (DOF) underwater vehicle plant reported in [45]

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix}}_{\dot{\boldsymbol{x}}(t)} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & \mu \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}}_{\boldsymbol{x}(t)} + \underbrace{\begin{bmatrix} 0 \\ \beta \end{bmatrix}}_{\boldsymbol{\beta}} \dot{x}(t)|\dot{x}(t)| + \underbrace{\begin{bmatrix} 0 \\ \alpha \end{bmatrix}}_{\boldsymbol{\alpha}} \tau(t) + \underbrace{\begin{bmatrix} 0 \\ \nu \end{bmatrix}}_{\boldsymbol{\nu}} \quad (1)$$

Where $\{\alpha, \beta, \mu, \nu\} \in \mathbb{R}^1$ are lumped parameters, $\tau(t)$ is the scalar plant input (i.e., forces and moments generated by thrusters), and $x(t)$, $\dot{x}(t)$, and $\ddot{x}(t)$ are the scalar vehicle position, velocity, and acceleration, respectively. Note that α and β are negative. Writing (1) in matrix-vector form we obtain

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{\beta}\dot{x}(t)|\dot{x}(t)| + \boldsymbol{\alpha}\tau(t) + \boldsymbol{\nu}. \qquad (2)$$

Measured Output: The measured output, $\boldsymbol{w}(t)$, for this observer consists of measurements for both position and velocity.

$$\boldsymbol{w}(t) = \boldsymbol{C}\boldsymbol{x}(t) \qquad (3)$$

where $\boldsymbol{C}$ is the output map. Note that for $\boldsymbol{C} \in \mathbb{R}^{2x1}$ or $\boldsymbol{C} \in \mathbb{R}^{2\times 2}$, the resulting output would be $\boldsymbol{w}(t) \in \mathbb{R}^1$ or $\boldsymbol{w}(t) \in \mathbb{R}^{2x1}$, respectively.

Estimated State: Define $\hat{\boldsymbol{x}}(t)$ to be the estimate of $\boldsymbol{x}(t)$. $\dot{\hat{\boldsymbol{x}}}(t)$ is the derivative with respect to time of $\hat{\boldsymbol{x}}(t)$.

Estimated Output: Define $\hat{\boldsymbol{w}}(t)$ to be the estimated output

$$\hat{\boldsymbol{w}}(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t) \qquad (4)$$

Depending on the output map, $\hat{\boldsymbol{w}}(t) \in \mathbb{R}^1$ or $\hat{\boldsymbol{w}}(t) \in \mathbb{R}^{2x1}$.

Task: Given the plant parameters $\{\alpha, \beta, \mu, \nu\}$, the input $\tau(t)$, and full state output $\boldsymbol{w}(t)$, our goal is to construct a state estimate $\hat{\boldsymbol{x}}(t)$ that converges asymptotically to $\boldsymbol{x}(t)$ as time approaches infinity, i.e., $lim_{t\to\infty}\hat{\boldsymbol{x}}(t) = \boldsymbol{x}(t)$.

State Error: The state error is defined as:

1) The error between the state estimate, $\hat{\boldsymbol{x}}(t)$, and the actual state, $\boldsymbol{x}(t)$:

$$\Delta\boldsymbol{x}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t). \qquad (5)$$

2) The derivative of $\Delta\boldsymbol{x}(t)$ with respect to time:

$$\begin{aligned} \Delta\dot{\boldsymbol{x}}(t) &= \dot{\hat{\boldsymbol{x}}}(t) - \dot{\boldsymbol{x}}(t) \\ &= \begin{bmatrix} \hat{x}(t) - x(t) \\ \dot{\hat{x}}(t) - \dot{x}(t) \end{bmatrix}. \end{aligned} \qquad (6)$$

3) The difference between the scalar estimated quadratic drag and the scalar actual quadratic drag:

$$\Delta\dot{x}_q(t) = \dot{\hat{x}}(t)|\dot{\hat{x}}(t)| - \dot{x}(t)|\dot{x}(t)|. \qquad (7)$$

Let $\Delta\dot{x}(t) = \dot{\hat{x}}(t) - \dot{x}(t)$. Notice that when $\dot{\hat{x}}(t) > \dot{x}(t)$, $\Delta\dot{x}(t) > 0$ which results in $\Delta\dot{x}(t)\Delta\dot{x}_q(t) > 0$. Similarly, if $\dot{\hat{x}}(t) < \dot{x}(t)$, $\Delta\dot{x}(t) < 0$ which results in $\Delta\dot{x}(t)\Delta\dot{x}_q(t) > 0$. Consequently, $\Delta\dot{x}(t)\Delta\dot{x}_q(t)$ is sign definite.

Output Error: We define the output error as

$$\Delta\boldsymbol{w}(t) = \hat{\boldsymbol{w}}(t) - \boldsymbol{w}(t) \qquad (8)$$

Observer: Consider the following observer:

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{\beta}\dot{\hat{x}}(t)|\dot{\hat{x}}(t)| + \boldsymbol{\alpha}\tau(t) + \boldsymbol{L}\boldsymbol{C}\Delta\boldsymbol{x}(t) + \boldsymbol{\nu} \quad (9)$$

where $\boldsymbol{L}$ is a gain matrix of appropriate dimensionality.

System: Substituting the observer and the plant into the time derivative of the state error results in the system.

$$\Delta\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \boldsymbol{L}\boldsymbol{C})\Delta\boldsymbol{x}(t) + \boldsymbol{\beta}\Delta\dot{x}_q(t) \qquad (10)$$

Defining $\tilde{\boldsymbol{A}} = \boldsymbol{A} + \boldsymbol{L}\boldsymbol{C}$ and

$$\boldsymbol{\zeta} = \begin{bmatrix} 0 & -1 \end{bmatrix} \qquad (11)$$

we obtain the error system,

$$\begin{aligned} \Delta\dot{\boldsymbol{x}}(t) &= \tilde{\boldsymbol{A}}\Delta\boldsymbol{x}(t) + \boldsymbol{\beta}\Delta\dot{x}_q(t) \\ \vartheta &= \boldsymbol{\zeta}\Delta\boldsymbol{x}(t). \end{aligned} \qquad (12)$$

The quadratic drag, $\Delta\dot{x}_q(t)$, is the input to the error system, and the velocity error, $\Delta\dot{x}(t)$ is the output.

Stability: Consider the Lyapunov function candidate

$$\phi(t) = \frac{1}{2}\Delta\boldsymbol{x}(t)^T\boldsymbol{P}\Delta\boldsymbol{x}(t) \qquad (13)$$

where $\boldsymbol{P} \in \mathbb{R}^{2\times 2}$ is a positive definite symmetric matrix.

Differentiating with respect to time yields,

$$\dot{\phi}(t) = \frac{1}{2}\left(\Delta\dot{\boldsymbol{x}}(t)^T\boldsymbol{P}\Delta\boldsymbol{x}(t) + \Delta\boldsymbol{x}(t)^T\boldsymbol{P}\Delta\dot{\boldsymbol{x}}(t)\right). \quad (14)$$

Substituting in the system (12) yields

$$\begin{aligned} \dot{\phi}(t) &= \frac{1}{2}\left(\Delta\boldsymbol{x}(t)^T\left(\tilde{\boldsymbol{A}}^T\boldsymbol{P} + \boldsymbol{P}\tilde{\boldsymbol{A}}\right)\Delta\boldsymbol{x}(t)\right) + \\ &\quad \boldsymbol{P}\boldsymbol{\beta}\Delta\boldsymbol{x}(t)\Delta\dot{x}_q(t). \end{aligned} \qquad (15)$$

If the error system satisfies the Kalman-Yakubovich-Popov (KYP) Lemma [50], [27], [33], then there exists a $\boldsymbol{P}$ such that

$$\begin{aligned} \tilde{\boldsymbol{A}}^T\boldsymbol{P} + \boldsymbol{P}\tilde{\boldsymbol{A}} &= -\tilde{\boldsymbol{Q}} \\ \boldsymbol{P}\boldsymbol{\beta} &= \boldsymbol{\zeta}^T \end{aligned} \qquad (16)$$

where $\tilde{\boldsymbol{Q}} \in \mathbb{R}^{2\times 2}$ is a symmetric, positive definite matrix. Substituting these identities into (15) results in the negative definite function

$$\begin{aligned} \dot{\phi}(t) &= -\frac{1}{2}\left(\Delta\boldsymbol{x}(t)^T\tilde{\boldsymbol{Q}}\Delta\boldsymbol{x}(t)\right) - \Delta\dot{x}(t)\Delta\dot{x}_q(t) \\ &< 0. \end{aligned} \qquad (17)$$

The proposed Lyapunov function $\phi(t)$ satisfies the following criteria: (i) $\phi(t)$ is continuously differentiable; (ii) when $\Delta\boldsymbol{x}(t) = 0, \phi(t) = 0$; (iii) $\forall\Delta\boldsymbol{x}(t) \neq 0, \phi(t) > 0$; (iv) $\phi(t)$ is radially unbounded; and (v) $\forall\Delta\boldsymbol{x}(t) \neq 0, \dot{\phi}(t) < 0$. Thus the error system is globally asymptotically stable, and the state estimate, $\hat{\boldsymbol{x}}(t)$, converges to the state, $\boldsymbol{x}(t)$, as $t \to \infty$.

IV. Experimental Evaluation

The reported observer was experimentally implemented and evaluated with data obtained with the *JHUROV*, a laboratory ROV designed for underwater vehicle research. Position was measured using a 300kHz acoustic time-of-flight positioning system possessing sub-centimeter precision [31]. A RDI Doppler sonar measured velocity at 0.03% precision and attitude was sensed with an Ixsea Phins North-seeking gyrocompass. The reader is referred to [44], [31] for an extended exposition of the *JHUROV* and its sensor suite.

Since we utilize a decoupled model for the underwater vehicle dynamics, the vehicle trajectories were commanded

TABLE I

ESTIMATED VEHICLE PARAMETERS

Experiment	α	β	μ	ν
	1/kg	1/m	1/s	m/s^2
EXPT098	$1.709e^{-3}$	$-6.118e^{-1}$	$-1.532e^{-1}$	$-2.175e^{-3}$
EXPT099	$1.371e^{-3}$	$-4.383e^{-1}$	$-1.150e^{-1}$	$-2.072e^{-3}$

along a single degree-of-freedom (DOF). The two experiments conducted employed sinusoidal trajectories in the X DOF. The first experiment, EXPT098, had an amplitude of 1 meter and a frequency of 0.25Hz. The second experiment, EXPT099, had an amplitude of 1.4 meter and a frequency of 0.15Hz. The duration of each experiment was 30 minutes. For each experiment, the vehicle model parameters were estimated using the adaptive identifier reported in [45]. Table I shows the estimated parameters used by the observer.

A. Methodology of Nonlinear Observer Evaluation

The data collected from these experiments was post-processed in Matlab. Implementing the nonlinear observer requires 300kHz Long Baseline (LBL) position measurement data, the vehicle velocity data measured by the 1200kHz Doppler sonar, the vehicle attitude data as measured by the Phins inertial measurement unit (IMU), and thrust input data. Thrust was quantified using a static thrust model (i.e., thrust is proportional to the command current). More precise thrust models are available (e.g., [4], [18]), however the necessary dynamic thruster characterization was unavailable at the time of these experiments.

The sub-centimeter precision of the 300kHz LBL position measurements is superior to the precision of 12kHz systems typically used in at-sea operations [49]. To simulate the noise characteristics of 12kHz LBL systems, random noise with a Gaussian distribution and a standard deviation of 0.25 meters was added to the 300kHz position measurements. The 0.25 meter standard deviation was chosen based on data collected with 12kHz and 300kHz LBL systems during an at-sea experiment [49]. We consider the simulated 12kHz LBL signal to be the measured value and the 300kHz position measurements to be the ground truth. The 300kHz position measurements serve as the basis for comparing the performance of the nonlinear estimators.

The position of the underwater vehicle is estimated with the following two nonlinear estimators:

1) **NLO -** The exact nonlinear observer (NLO) reported in Section III.
2) **EKF -** The Extended Kalman Filter (EKF), a version of the Kalman Filter that linearizes the plant and then uses the Kalman Filter to estimate the position [6], [11], [46]. The implementation of the EKF on these data sets is discussed in Section IV-C.

The NLO was implemented using the ODE45 numerical solver in Matlab. The logged times of the thrust and LBL data were used as the time vector used by ODE45. At every time interval, ODE45 used the most recent thrust and velocity measurements to compute the estimated velocity. Since position was sampled at a lower update rate of 1Hz, ODE45 only updated the position measurement and position estimate when it received a new position measurement — i.e. the position innovation was updated only when new position measurements were received. At present, there exist no general results for analytically selecting the optimal gains of nonlinear systems. This contrasts linear systems where techniques such as Linear Quadratic Regulation (LQR), Linear Quadratic Gaussian Regulation (LQGR), and the Kalman Filter can be employed to select the optimal gain. Consequently, a numerical simulation technique was employed for gain selection.

After estimating the position of the vehicle, we compared this estimated position and the 12kHz position measurement to the position measured by the 300kHz LBL. The result is the error in the estimated position and the 12kHz position. The error is quantified by computing the mean and the standard deviation of the error for the last half of the experiment. Figure 1 shows the histograms for Experiment 098 (EXPT098) and Experiment 099 (EXPT099).

B. Nonlinear Observer Performance

The "Nonlinear Observer" columns of Table II show the means and standard deviations (sigmas) of the NLO position estimates. Compared to the 12kHz LBL measurements, the standard deviations of the NLO position estimates are significantly lower. The means of the position error for the NLO estimates range from 0.004670 meters to 0.015371 meters — higher than the means for the 12kHz LBL measurements. This is reasonable given that the reported derivation of the NLO makes no assertion to providing an estimate resulting in zero-mean error. Histograms of the 12kHz LBL and NLO position errors for EXPT098 and EXPT099 are shown in Figure 1. Despite the increase in the mean error, the significant decrease in the standard deviations demonstrates the NLO's ability to provide position estimates with a superior precision than those of 12kHz LBL measurements alone.

C. Comparison to the Extended Kalman Filter

This section compares the performance of the NLO position estimate to the estimate computed by the Extended Kalman Filter (EKF). Since its inception in the early 1960's, the EKF has been used in a wide variety of nonlinear estimation applications with great success [6], [11], [46], including underwater vehicles [41]. Numerous other nonlinear estimators (e.g. particle filters, unscented filters, etc) have been reported, however the simplicity of the EKF makes it a logical choice for comparing the performance of the NLO.

The EKF used the underwater vehicle model (1) reported in [45] and the vehicle parameters identified in Table I. The variance of the measurement noise was 0.25 meters and the variance of the velocity measurement was 0.003 meters per second. The process noise was heuristically estimated to be 0.0005 m/s^2. The EKF estimated the velocity of the vehicle at every occurrence of either a position or a velocity measurement, and used the most recent thrust value. Given the long intervals between position measurements,

TABLE II

MEANS AND STANDARD DEVIATIONS OF THE POSITION ERRORS FOR THE 12KHZ LBL MEASUREMENTS, THE NONLINEAR OBSERVER POSITION ESTIMATE, AND THE EXTENDED KALMAN FILTER POSITION ESTIMATE.

EXPT	12kHz LBL		Nonlinear Observer		Extended Kalman Filter	
	Mean	Sigma	Mean	Sigma	Mean	Sigma
	meters	*meters*	*meters*	*meters*	*meters*	*meters*
EXPT098	0.006030	0.246136	0.015371	0.062560	0.005237	0.083186
EXPT098	0.005368	0.240783	0.004670	0.049435	0.001028	0.055849

the position estimate was updated only when a position measurement was received.

The EKF estimated the position of the vehicle for both experiments. The "Extended Kalman Filter" columns of Table II show the means and standard deviations (sigmas) of the position error for the EKF. Error histograms for the EKF are shown in Figure 1. The EKF means are lower than the NLO means — this is reasonable given that the Kalman Filter computes a state estimate with zero-mean error. The NLO provides estimates with standard deviations lower than those provided by the EKF.

V. CONCLUSIONS

This paper reports the development of an exact nonlinear dynamic model-based observer for underwater vehicle navigation. The reported observer is novel because: *(i)* it exploits knowledge of the vehicle's exact nonlinear dynamics; *(ii)* the observer estimates vehicle position and velocity; *(iii)* stability of the observer is shown with Lyapunov techniques and the KYP Lemma; and *(iv)* the performance of the observer is experimentally evaluated with data from a laboratory ROV. From the experimental data we conclude that the error in the observer position estimate is significantly lower than the error of position measurements from a 12kHz LBL acoustic positioning system alone. The performance of the observer is compared to the EKF and the experimental data demonstrates that the observer and EKF perform comparably. The reported experimental results are, to the best of our knowledge, the first known implementation of an exact NLO for underwater vehicle navigation. These experiments demonstrate the ability of exact NLOs to provide improved state estimation, and further research in multiple DOF parameter identification and state estimation techniques will enable us to employ these techniques in at-sea vehicles. The continued analytical development and implementation of dynamic model-based nonlinear observers promises to improve underwater navigation, and, in consequence, further increase the capabilities of underwater vehicles.

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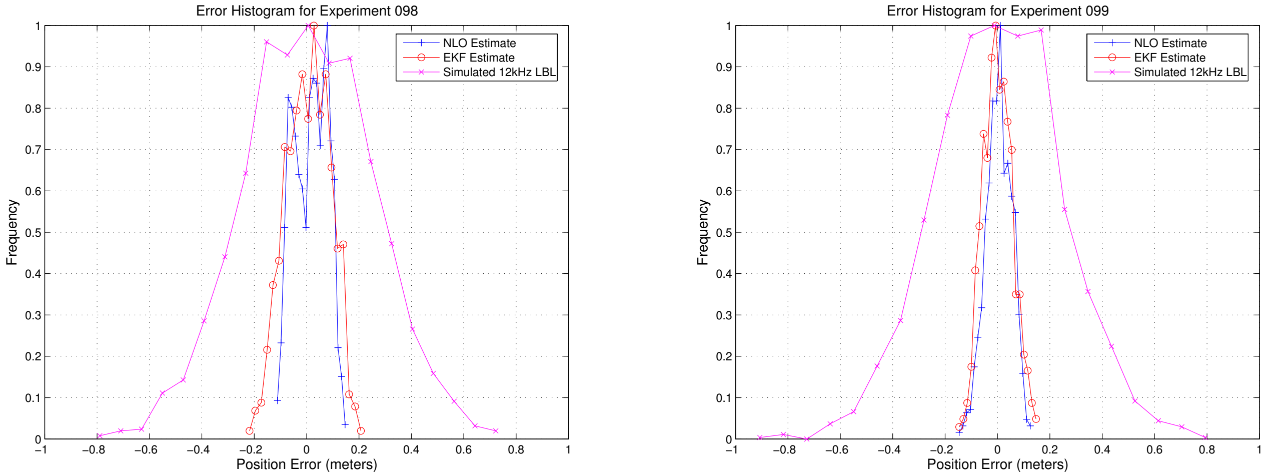

Fig. 1. Histograms of the position error for the simulated 12kHz Long Baseline (LBL) measurements, the nonlinear observer (NLO) estimate, and the Extended Kalman Filter (EKF) estimate for EXPT098, left, and EXPT099, right.

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