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Noise Covariance Identification Based Adaptive UKF with Application to Mobile Robot Systems

Qi Song and Zhe Jiang
Robotics Laboratory, Shenyang Institute of Automation
Graduate School of the Chinese Academy of Sciences
Shenyang, 110016, China
Songqim & zhjiang@sia.cn

Jianda Han
Robotics Laboratory, Shenyang Institute of Automation
Chinese Academy of Sciences
Shenyang, 110016, China
jdhan@sia.cn

Abstract—**A novel adaptive Unscented Kalman Filter (UKF) based on dual estimation structure is proposed. The filter is composed of two parallel master-slave UKFs, while the master one estimates the states and the slave one estimates the diagonal elements of the noise covariance matrix for the master UKF. By estimating the noise covariance online, the proposed method is able to compensate the errors resulting from the change of the noise statistics. Such a mechanism improves the adaptive ability of the UKF and enlarges its application scope. Simulations conducted on the dynamics of an omni-directional mobile robot indicate that the performance of the adaptive UKF is superior to the standard one in terms of fast convergence and estimation accuracy.**

I. Introduction

As an extension of the traditional automatic control, autonomous control makes a mobile robot perform well under changing and uncertain environment with reduced human intervention for extended period of time. Its potential application includes satellite clusters, deep space exploration, air traffic control, and battlefield management, etc [1]. Online modeling is a key technology for autonomous controller to maintain stability and high performance in uncertain environment and in the presence of failures or damages.

Neural Networks (NN) and NN-based self learning have been proposed as one of the most effective approaches for the active modeling of unmanned vehicles in 1990s [2]. However the problems involved in NN, such as training data selection, online convergence, robustness, reliability and real-time implementation, limit its application in real systems. In recent years, the encouraging achievement in stochastic estimation makes it becoming an important direction for online modeling and model-reference control [3]. Among stochastic estimation methods, the most popular one for nonlinear system is the Extended Kalman Filter (EKF) [4]. Although widely used, EKF suffers from the deficiencies including the requirement of sufficient differentiability dynamics and its susceptibility to bias and divergence during estimation. Julier et al proposed Unscented Kalman Filter (UKF) as a derivative-free alternative to EKF [5]. Compared with EKF, UKF does not need to calculate the Jacobian or Hessians and can achieve higher accuracy than EKF at an equal computational complexity of $O(L^3)$ [6].

However, the performance of UKF closely depends on the prior knowledge about the statistics of the measure-ments and the states to be estimated. The UKF can only achieve good estimations under the assumption that such prior knowledge meets the real situation well [7], while an inaccurate assumption will lead to poor performance or even divergence of the filter. But for real implementation, it is difficult to obtain such an accurate statistics *a priori* because it is influenced by the dynamics and working environment of a mobile vehicle, both of which are time-varying and uncertain.

One of the efficient ways to overcome the above mentioned weakness is to use an adaptive algorithm. There have been many investigations in the area of adaptive filter. Hu et al [8] proposed the limiting memory of KF, which could adaptively adjust the forgetting factors according to an optimal condition. Maybeck [9] used a maximum-likelihood estimator in designing an adaptive KF that could estimate the covariance matrix of the error statistics. Lee [10] modified Maybeck's method by introducing a window scale factor, and integrating it into UKF. Loebis et al [11] used fuzzy logic techniques to update the sensor noise covariance. But in practice it is difficult to determine the increment values of the covariance matrix at each time instant.

In this paper we introduced an adaptive filter composed of two parallel UKFs. The master UKF estimates the system states while the slave one estimates the diagonal elements of the noise covariance matrix for the master UKF. By estimating the noise covariance, the proposed method is able to compensate the estimation errors resulting from the insufficient knowledge of the noise statistics. Simulations are conducted on the dynamics of an omni-directional mobile robot to verify the proposed scheme, and the results demonstrate that the performance of the adaptive UKF is superior to that of the standard one in terms of fast convergence and estimation accuracy.

II. The Adaptive Ukf Structure

As shown in Fig.1, the proposed adaptive scheme is composed of two parallel UKFs. At every time-step, the master UKF estimates the system states using the active noise covariance calculated by the slave UKF, while the slave UKF estimates the noise covariance using the innovation generated by the master UKF. The master UKF can also work independently without the slave one. Thus, the dual-UKF structure is reduced to a standard UKF with fix noise covariance. And the setting of the master UKF does not need any updates while activating/deactivating the slave one, which indicates that the slave UKF can be shut down to reduce the computational burden when the system statistics do not change a lot.

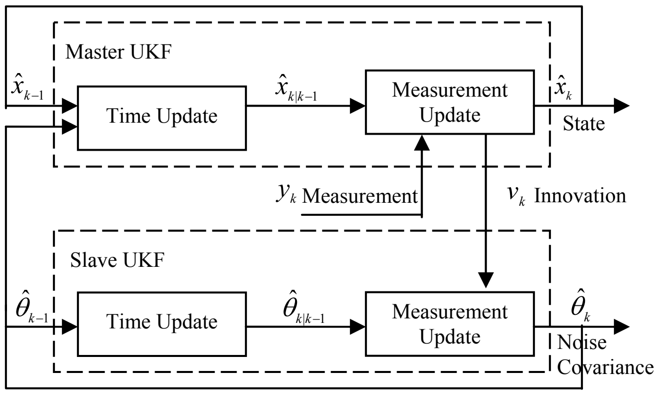


Fig. 1. The Adaptive UKF Structure

III. The Master UKF

In the proposed scheme, the setting of the master UKF is the same as that of a standard one.

Consider a general discrete-time nonlinear system:

$$\begin{cases} x_{k+1} = f(x_k, u_k) + w_k \\ y_k = h(x_k) + v_k \end{cases} \tag{1}$$

where $x_k \in R^n$ is the state vector, $u_k \in R^r$ is the input vector, $y_k \in R^m$ is the output vector at time k. w_k and v_k are, respectively, the disturbance and sensor noise vector, which are assumed to Gaussian white noise with zero mean.

The master UKF can be expressed as:

Initialization

$$\begin{cases} \hat{x}_0 = E[x_0] \\ P_{x0} = E\left[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T\right] \end{cases} \tag{2}$$

Sigma Points Calculation and Time Update

$$\begin{cases} \chi_{k-1} = \left[\hat{x}_{k-1}, \hat{x}_{k-1} + \sqrt{(n+\lambda)P_{x_{k-1}}}, \hat{x}_{k-1} - \sqrt{(n+\lambda)P_{x_{k-1}}}\right] \\ \chi^*_{k|k-1} = f(\chi_{k-1}) \\ \hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \chi^*_{i,k|k-1} \\ P_{x_{k|k-1}} = \sum_{i=0}^{2n} w_i^c \left(\chi^*_{i,k|k-1} - \hat{x}_{k|k-1}\right)\left(\chi^*_{i,k|k-1} - \hat{x}_{k|k-1}\right)^T + Q^x \\ \chi_{k|k-1} = \left[\hat{x}_{k|k-1}, \hat{x}_{k|k-1} + \sqrt{(n+\lambda)P_{x_{k|k-1}}}, \hat{x}_{k|k-1} - \sqrt{(n+\lambda)P_{x_{k|k-1}}}\right] \\ \gamma_{k|k-1} = h(\chi_{k|k-1}) \\ \hat{y}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \gamma_{i,k|k-1} \end{cases} \tag{3}$$

where

$$\begin{cases} w_0^m = \dfrac{\lambda}{n+\lambda} \\ w_0^c = \dfrac{\lambda}{n+\lambda} + (n - \alpha^2 + \beta) \\ w_i^m = w_i^c = \dfrac{\lambda}{2(n+\lambda)} \qquad i = 1, \cdots, 2n \\ \eta = \sqrt{(n+\lambda)}, \quad \lambda = n(\alpha^2 - 1) \end{cases} \tag{4}$$

Measurement Update

$$\begin{cases} P_{y_k y_k} = \sum_{i=0}^{2n} w_i^c \left(\gamma_{i,k|k-1} - \hat{y}_{k|k-1}\right)\left(\gamma_{i,k|k-1} - \hat{y}_{k|k-1}\right)^T + R^x \\ P_{x_k y_k} = \sum_{i=0}^{2n} w_i^c \left(\chi_{i,k|k-1} - \hat{x}_{k|k-1}\right)\left(\gamma_{i,k|k-1} - \hat{y}_{k|k-1}\right)^T \\ K_{x_k} = P_{x_k y_k} P_{y_k y_k}^{-1} \\ \hat{x}_k = \hat{x}_{k|k-1} + K_{x_k}\left(y_k - \hat{y}_{k|k-1}\right), \quad P_{x_k} = P_{x_{k|k-1}} - K_{x_k} P_{y_k y_k} K_{x_k}^T \end{cases} \tag{5}$$

The variables in Equation (2~5) are defined as followings: $\{w_i\}$ is a set of scalar weights, α is a constant determining the spread of the sigma points around $\hat{x}$ and is usually set as $1e-4 \le \alpha \le 1$. The constant β is used to incorporate part of the prior knowledge of the statistics of x, while β=2 is optimal for Gaussian distributions. Q^x and R^x are the disturbance and sensor noise covariance respectively. The diagonal elements of Q^x and R^x will be estimated by the slave UKF.

IV. The Slave UKF

In real application, the difference between the priori knowledge and the true state statistics is the major factor that degrades the filter's performance. Therefore, selecting appropriate covariance matrices, i.e., Q and R in Equation (4) and (5), is most important to maintain the performance and stability of the UKF. In this paper, we propose using a slave UKF to estimate the covariance online. Under the assumption that the process and measurement noises are Gaussian white, we can conclude that the relative covariance Q and R are diagonal matrices. Then the estimation of the noise covariance can be simplified as the estimation of the diagonal elements.

Supposed that the diagonal elements of the noise covariance matrix are denoted by θ and $\theta \in \Re^l$. If the dynamics of θ is known, the state equation of the slave UKF is:

$$\theta_k = f(\theta_{k-1}) + w_{\theta k} \tag{6}$$

If the dynamics of θ is unknown, it can be modelled as a non-correlated random drift vector:

$$\theta_k = \theta_{k-1} + w_{\theta k} \tag{7}$$

where $w_{\theta k}$ is the Gaussian white noise with zero mean. The innovation covariance generated by the master UKF is taken as the observation signal for the slave UKF and then according to equation (5) the observation model can be described as:

$$\hat{S}_k = g(\theta_k) = diag\left\{\sum_{i=0}^{2n} w_i^c \left(\gamma_{i,k|k-1} - \hat{y}_{k|k-1}\right)\left(\gamma_{i,k|k-1} - \hat{y}_{k|k-1}\right)^T + R^x\right\} \quad (8)$$

The measurement of $\hat{S}_k$ received by the slave UKF is

$$S_k = diag\{v_k v_k^T\} \quad (9)$$

where the v_k is innovation and can be written as:

$$v_k = y_k - \hat{y}_{k|k-1} \quad (10)$$

and y_k and $\hat{y}_{k|k-1}$ are, respectively, the real measurements in (1) and the relative estimations.

As discussed above, a recursive algorithm of the slave UKF can be formulated as:

Initialization

$$\begin{cases} \hat{\theta}_0 = E[\theta_0] \\ P_{\theta_0} = E\left[\left(\theta_0 - \hat{\theta}_0\right)\left(\theta_0 - \hat{\theta}_0\right)^T\right] \end{cases} \quad (11)$$

Sigma Points Calculation and Time Update

$$\begin{cases} \vartheta_{k-1} = \left[\hat{\theta}_{k-1}, \hat{\theta}_{k-1} + \sqrt{(l+\lambda)P_{\theta_{k-1}}}, \hat{\theta}_{k-1} - \sqrt{(l+\lambda)P_{\theta_{k-1}}}\right] \\ \vartheta^*_{k|k-1} = f_\theta\left(\vartheta_{k-1}\right) \\ \hat{\theta}_{k|k-1} = \sum_{i=0}^{2n} w_{\theta i}^m \vartheta^*_{i,k|k-1} \\ P_{\theta_{k|k-1}} = \sum_{i=0}^{2n} w_{\theta i}^c \left(\vartheta^*_{i,k|k-1} - \hat{\theta}_{k|k-1}\right)\left(\vartheta^*_{i,k|k-1} - \hat{\theta}_{k|k-1}\right)^T + Q^\theta \\ \vartheta_{k|k-1} = \left[\hat{\theta}_{k|k-1}, \hat{\theta}_{k|k-1} + \sqrt{(l+\lambda)P_{\theta_{k|k-1}}}, \hat{\theta}_{k|k-1} - \sqrt{(l+\lambda)P_{\theta_{k|k-1}}}\right] \\ \varsigma_{k|k-1} = g\left(\vartheta_{k|k-1}\right) \\ \hat{S}_{k|k-1} = \sum_{i=0}^{2n} w_{\theta i}^m \varsigma_{i,k|k-1} \end{cases} \quad (12)$$

Measurement Update

$$\begin{cases} P_{S_k S_k} = \sum_{i=0}^{2n} w_{\theta i}^c \left(\varsigma_{i,k|k-1} - \hat{S}_{k|k-1}\right)\left(\varsigma_{i,k|k-1} - \hat{S}_{k|k-1}\right)^T + R^\theta \\ P_{\theta_k S_k} = \sum_{i=0}^{2n} w_{\theta i}^c \left(\vartheta_{i,k|k-1} - \hat{\theta}_{k|k-1}\right)\left(\varsigma_{i,k|k-1} - \hat{S}_{k|k-1}\right)^T \\ K_{\theta_k} = P_{\theta_k S_k} P_{S_k S_k}^{-1} \\ \hat{\theta}_k = \hat{\theta}_{k|k-1} + K_{\theta_k}\left(S_k - \hat{S}_{k|k-1}\right),\ P_{\theta_k} = P_{\theta_{k|k-1}} - K_{\theta_k} P_{S_k S_k} K_{\theta_k}^T \end{cases} \quad (13)$$

where Q^θ and R^θ are the process and measurement covariance respectively. The values of the weights $w_{\theta i}^m, w_{\theta i}^c$ can be calculated by Equation (4).

V. SIMULATIONS

The simulations are carried out with respect to the dynamics of the omni-directional mobile robot developed in SIA. (see Fig. 2).

Fig. 2. 3-DOF omni-directional mobile robot

A. Dynamics and UKF Setting

The dynamic model of the mobile robot is [13]:

$$\begin{cases} \left(2Mr^2 + 3nI_w\right)\ddot{x}_w + 3n^2 I_w \dot{y}_w \dot{\varphi}_w + 3n^2 c\dot{x}_w \\ \quad = nr\left(\beta_1 u_1 + 2u_2 \cos\varphi_w + \beta_2 u_3\right) \\ \left(2Mr^2 + 3nI_w\right)\ddot{y}_w - 3n^2 I_w \dot{x}_w \dot{\varphi}_w + 3n^2 c\dot{y}_w \\ \quad = nr\left(\beta_3 u_1 + 2u_2 \sin\varphi_w + \beta_4 u_3\right) \\ \left(3nI_w L^2 + I_v r^2\right)\ddot{\varphi}_w + 3n^2 cL^2 \dot{\varphi}_w = nrL\left(-u_1 - u_2 - u_3\right) \end{cases} \quad (14)$$

$$\begin{cases} \beta_1 = -\sqrt{3}\sin\varphi_w - \cos\varphi_w \\ \beta_2 = \sqrt{3}\sin\varphi_w - \cos\varphi_w \\ \beta_3 = \sqrt{3}\cos\varphi_w - \sin\varphi_w \\ \beta_{41} = -\sqrt{3}\cos\varphi_w - \sin\varphi_w \end{cases} \quad (15)$$

where x_w, y_w, φ_w represent the displacement in x and y direction and rotation respectively, u_1, u_2, u_3 are actuated torques on each joint. Other parameters of (14), (15) and their initial values in the simulations are listed in Table I.

TABLE I : ROBOT PARAMETERS

Symbol	Physical Meaning	Value in Simulation
c	friction coefficient	0.0009 kgm^2/s
I_w	Inertia on motor axis	0.0036 kgm^2
M	mass	120 kg
I_v	inertia	45 kgm^2
r	wheel radius	0.06m
L	centroid – wheel distance	0.273 m
n	motor gear ratio	15

The state and measurement vector of the master UKF are selected as:

$$\begin{cases} x = [x_w, y_w, \varphi_w, \dot{x}_w, \dot{y}_w, \dot{\varphi}_w]^T \\ y = [\dot{x}_w, \dot{y}_w, \dot{\varphi}_w]^T \end{cases} \quad (16)$$

Assumed that the initial state of the system is $x_{T0} = \bar{0}$ and sampling interval is T_s=0.01s. The initial parameters of the master UKF are set as:

$$\begin{cases} \hat{x}_0 = x_{T0} \\ \hat{P}_{x0} = diag\{10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}, 10^{-8}\} \\ \alpha = 1, \quad \beta = 2 \end{cases}$$

B. Estimation with Incorrect Process Noise Covariance

The performance of the adaptive UKF with incorrect process noise covariance is tested in this section.

In this simulation, we assume that the true process noise covariance is

$$Q_T^x = diag\{10^{-10}, 10^{-10}, 10^{-10}, 10^{-6}, 10^{-6}, 10^{-6}\}$$

and the observations are corrupted by zero mean additive white noise with covariance $R_T^x = diag\{10^{-8}, 10^{-8}, 10^{-8}\}$.

In the master UKF, the prior knowledge of the process noise covariance is $Q_0^x = diag\{10^{-12}, 10^{-12}, 10^{-12}, 10^{-8}, 10^{-8}, 10^{-8}\}$ and $R^x = R_T^x$.

In the slave UKF, the states to be estimated are taken as the diagonal elements of Q^x:

$$\theta_Q = diag\{Q^x\} = [\theta_Q^{x_w}, \theta_Q^{y_w}, \theta_Q^{\varphi_w}, \theta_Q^{\dot{x}_w}, \theta_Q^{\dot{y}_w}, \theta_Q^{\dot{\varphi}_w}]$$

Obviously, we have

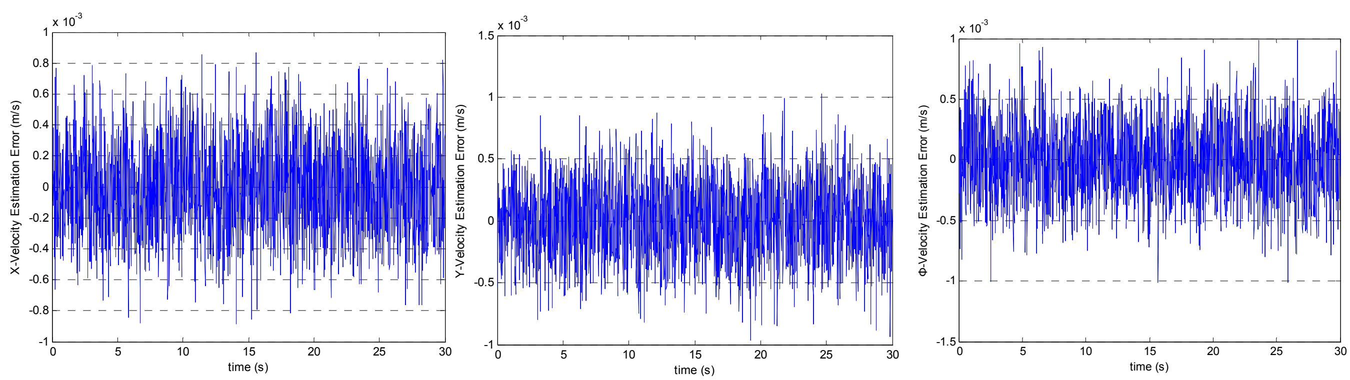
$$\theta_Q^{x_w} = \theta_Q^{\dot{x}_w} \cdot T_s^2, \theta_Q^{y_w} = \theta_Q^{\dot{y}_w} \cdot T_s^2, \theta_Q^{\varphi_w} = \theta_Q^{\dot{\varphi}_w} \cdot T_s^2$$

And the states $\theta_Q^{\dot{x}_w}, \theta_Q^{\dot{y}_w}, \theta_Q^{\dot{\varphi}_w}$ can be modelled as a non-correlated random drift vector as (7). The observation model of the slave UKF can be described as (8) with the measurement of (9).

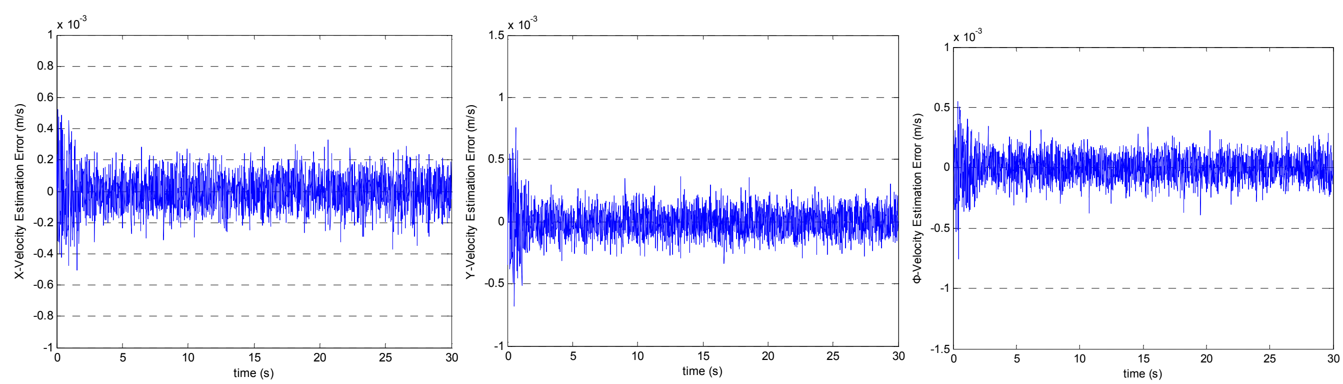
The parameters of the slave UKF are set to:

$$\begin{cases} \hat{\theta}_{Q0} = diag\{Q_0^x\} \\ \hat{P}_{\theta_{Q0}} = diag\{10^{-16}, 10^{-16}, 10^{-16}, 10^{-16}, 10^{-16}, 10^{-16}\} \\ Q^{\theta_Q} = diag\{10^{-24}, 10^{-24}, 10^{-24}, 10^{-21}, 10^{-21}, 10^{-21}\} \\ R^{\theta_Q} = diag\{2\times10^{-16}, 2\times10^{-16}, 2\times10^{-16}\} \\ \alpha = 1 \\ \beta = 2 \end{cases}$$

Fig.3-a) demonstrates the velocity estimation errors in *X*-, *Y*-, and *Φ*-direction by standard UKF and Fig.3-b) are those by the adaptive UKF. We can see that, with the incorrect priori knowledge of the process covariance, the standard UKF almost fails to estimate the true value from the noise-corrupted measurement. On the other hand, the adaptive UKF successfully reduces the noises to about one third of those by standard UKF.



a) Standard UKF



b) Adaptive UKF

Fig. 3 State Estimation Errors with the Incorrect Process Noise Covariance

C. Estimation with Incorrect Measurement Noise Covariance

The performance of the adaptive UKF with incorrect measurement noise covariance is tested in this section.

In the simulation, the measurement noise covariance is set as $R_T^x = diag\{10^{-6}, 10^{-6}, 10^{-6}\}$, and process noise covariance is $Q_T^x = diag\{10^{-12}, 10^{-12}, 10^{-12}, 10^{-8}, 10^{-8}, 10^{-8}\}$.

In the master UKF, the priori measurement noise covariance is selected as

$$Q^x = Q_T^x,\ R_0^x = diag\{10^{-8}, 10^{-8}, 10^{-8}\}.$$

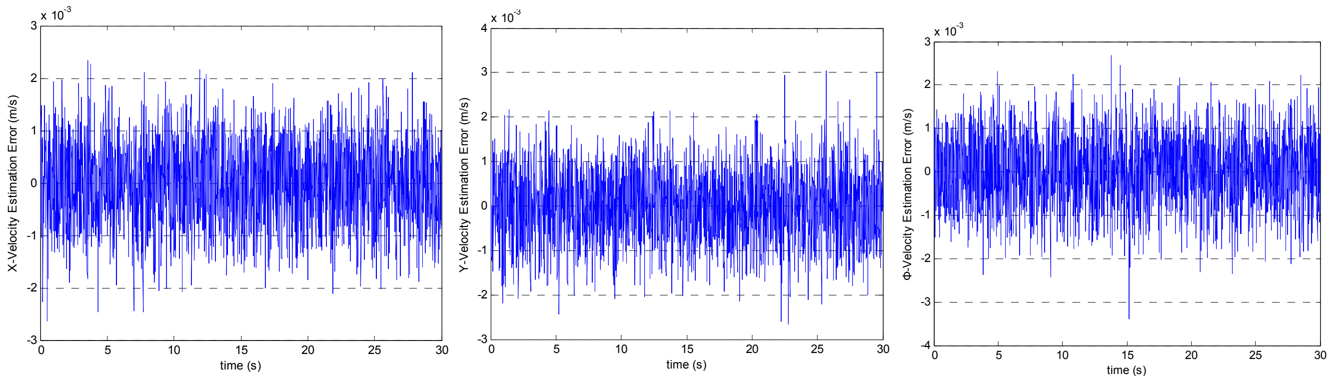
The state vector of the slave UKF is designed as:

$$\theta_R = diag\{R^x\} = \left[\theta_R^{\dot{x}_w}, \theta_R^{\dot{y}_w}, \theta_R^{\dot{\varphi}_w}\right]$$

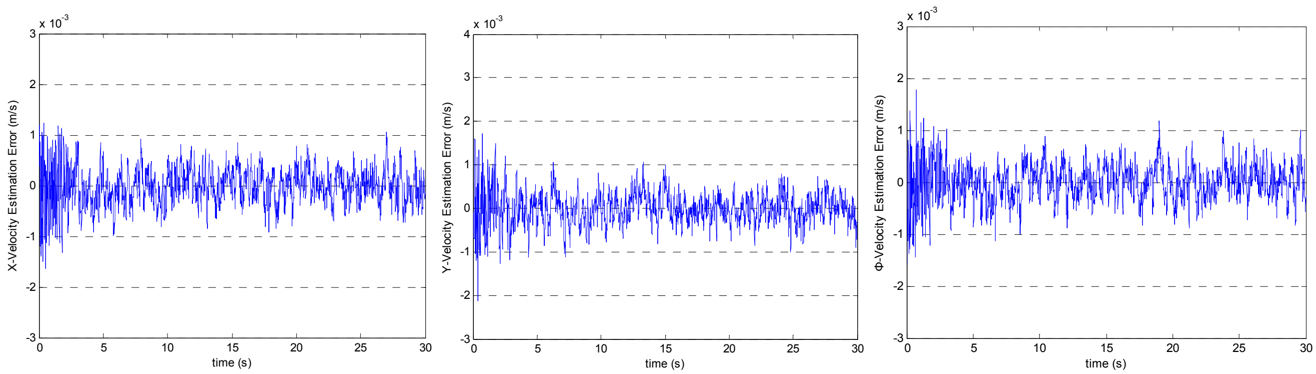
The parameters of the slave UKF are set to:

$$\begin{cases} \hat{\theta}_{R0} = diag\{R_0^x\} \\ \hat{P}_{\theta_{R0}} = diag\{10^{-16}, 10^{-16}, 10^{-16}\} \\ Q^{\theta_R} = diag\{10^{-21}, 10^{-21}, 10^{-21}\} \\ R^{\theta_R} = diag\{2\times 10^{-16}, 2\times 10^{-16}, 2\times 10^{-16}\} \\ \alpha = 1 \\ \beta = 2 \end{cases}$$

The simulation results are shown in Fig. 4. Similar to Fig.3, we can see that by online estimating the measurement noise covariance the adaptive UKF successfully reject the influence caused by the incorrect priori covariance of measurement noise and achieve better estimations.



a) Standard UKF



b) Adaptive UKF

Fig. 4 Velocity Estimation Errors with the Incorrect Measurement Noise Covariance

D. Estimation with Changing Process Noise Covariance

The proposed adaptive filter is tested by the situation with changing process noise covariance. Such situation might occur while the vehicle system suffering fault or damage. We assume that there occurs a abrupt change with the process noise covariance at *t*=10s, i.e.,

$$Q_T^x = \begin{cases} diag\{10^{-12}, 10^{-12}, 10^{-12}, 10^{-8}, 10^{-8}, 10^{-8}\} & t < 10s \\ diag\{10^{-10}, 10^{-10}, 10^{-10}, 10^{-6}, 10^{-6}, 10^{-6}\} & t \geq 10s \end{cases}$$

The measurement noise covariance is assumed as:

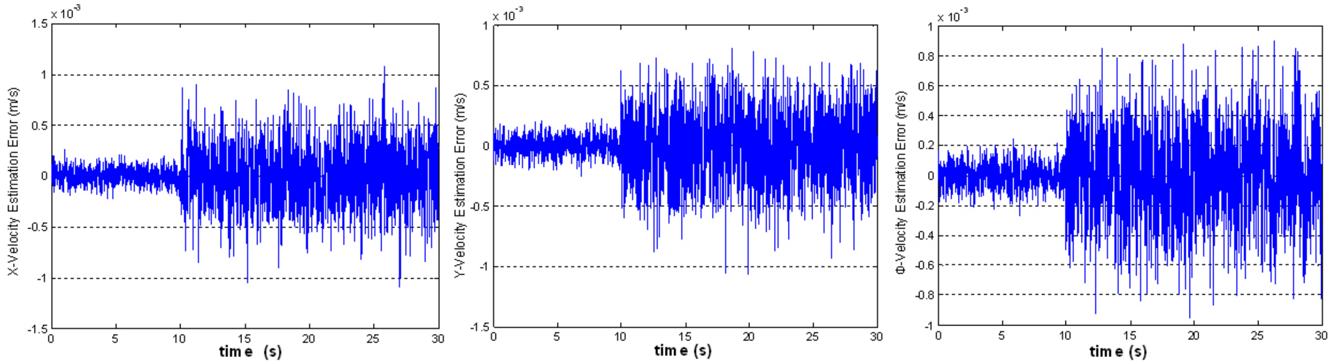
$$R_T^x = diag\{10^{-8}, 10^{-8}, 10^{-8}\}$$

The priori knowledge of the master UKF is designed as:

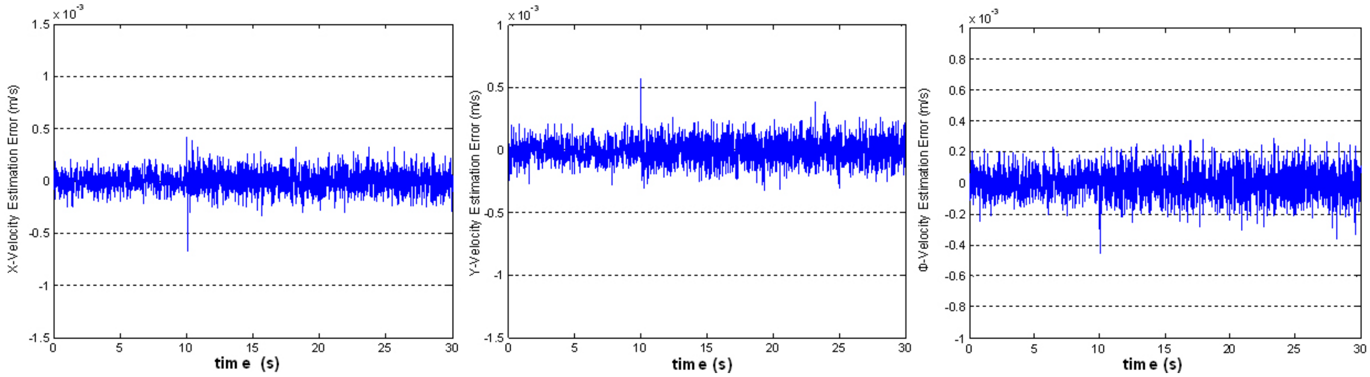
$$Q_0^x = diag\{10^{-12}, 10^{-12}, 10^{-12}, 10^{-8}, 10^{-8}, 10^{-8}\}$$

$$R_0^x = R_T^x$$

The other settings of the master and slave UKF are the same as those in Section V-B). Fig.5 shows the simulation results. We can see that, by the standard UKF, the estimations occur significant changes at the time while the priori covariance setting of UKF fails to meet the true values. By the adaptive UKF, on the other hand, the estimations remain same as before even though the process noise covariance has changed.



a) Standard UKF



b) Adaptive UKF

Fig.5 Velocity Estimation Errors with the Changing Process Noise Covariance

VI. CONCLUSION

In this paper, a novel adaptive filter based on two-UKF structure is proposed for the state estimation of nonlinear systems. An additional UKF, called slave UKF, is integrated into a standard UKF, named master UKF, to actively estimate the noise statistics. The estimated statistics are further used by the master UKF in order to adaptively compensate the influence caused by inaccurate priori knowledge and changing statistics of system noise. The proposed algorithm is presented and analyzed in details, and extensive simulations are carried out to perform state estimation of an omni-directional mobile robot and to make comparisons between the standard UKF and the proposed adaptive filter. It has been demonstrated that the proposed method successfully reduces the dependency of the estimation performance on the accurate knowledge of system noise statistics.

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