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Friction Independent Dynamic Capturing Strategy for a 2D Stick-shaped Object

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Abstract— **This paper proposes dynamic capturing strategies where a 2D stick-shaped object with both translational and rotational velocities is completely stopped by two robotic fingers. We first show the fingertip position and the object orientation for generating a desired velocity of the object under the friction independent collision. Once the object results in a pure translational motion whose direction is perpendicular to the longitudinal axis of object, it is guaranteed that two fingers can always capture the object irrespective of friction coefficient. By using this nature, we show both 2-step and 3-step capturing strategies for a 2D stick-shaped object whose width is negligibly small. The 3-step capturing strategy can guide the object in an arbitrary direction, while the 2-step one can do it only in a particular direction. The 3-step capturing strategy is demonstrated by experiment for verifying our idea.**

I. INTRODUCTION

With the increase of sensing speed, it has become possible for a robot system to chase or manipulate a moving object [1]–[6]. For example, Hong and Slotine [1] have proposed a catching algorithm by which they succeeded in real time catching of free-flying spherical balls tossed by hand from random locations. Namiki et al. [2] have realized the dynamic catching of a ball by the combination of the 1ms-vision and a high-speed hand. Higashimori et al. [3] have discussed the design of the 100G capturing robot capable of capturing a ball with its moving speed of 4[m/s]. These works, however, have treated simple objects, such as a sphere or a circle. Furukawa et al. [6] have shown a regrasping strategy where a hand throws up a stick and catches it with the help of high speed vision. This work allows a weak rotational motion of object but no rigorous capturing strategy under a large rotational velocity is discussed. Also, this work implicitly supposes that a hand can support an enough moment by utilizing three fingers with point contact, so that the hand can quickly stop the object even under a direct catching.

Now, suppose that a two-fingered robot hand with point contact tries to capture a 2D stick-shaped object with both translational and rotational velocities $(\boldsymbol{v}_{B,0}, \omega_{B,0})$, as shown in Fig.1(a-1). Intuitively, it seems to be difficult to quickly stop the rotational motion of object with two fingers unless the fingertips can produce an extremely large moment, which is normally difficult for such robot fingers. On the other hand, two fingers can stop the object with a pure translational velocity by gradually closing the fingers. This consideration suggests a capturing strategy where a fingertip first stops

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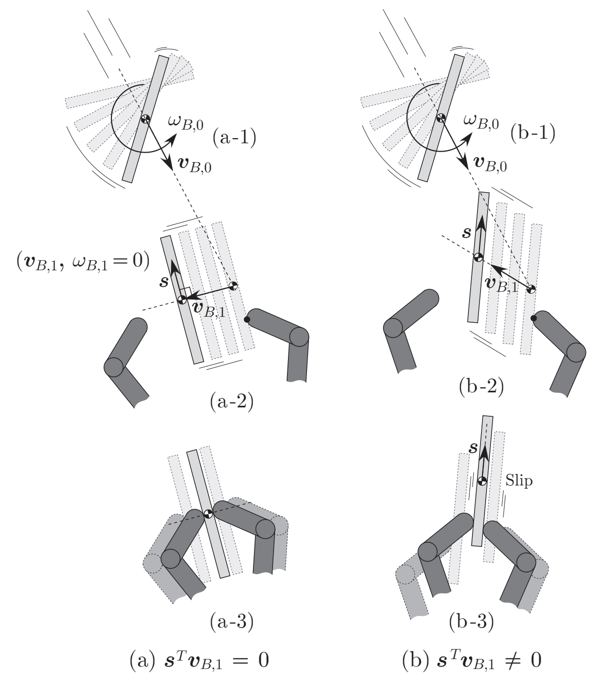


Fig. 1. Basic concept of the proposed capturing strategy.

the rotational motion of object by choosing an appropriate fingertip position for the first collision between the fingertip and the object, as shown in Fig.1(a-2) and (b-2), and then the two fingers start to approach the object for completely stopping it, as shown in Fig.1(a-3) and (b-3). For achieving such a capturing strategy, we have to be careful for both the direction of the translational motion of object and the object posture. For a stick-shaped object, for example, if the longitudinal velocity component of the object is large as shown in Fig.1(b-2), the object may slip away from the fingertip under a small friction coefficient, as shown in Fig.1(b-3). In order for two fingers to successfully stop the object even with slippery surface, we determine the fingertip position so that the object may result in a pure translational motion whose direction is perpendicular to the longitudinal axis of object, as shown in Fig.1(a-2). This condition can be obtained by setting $\omega_{B,1} = 0$ and $\boldsymbol{s}^T\boldsymbol{v}_{B,1} = 0$, where $\boldsymbol{v}_{B,1}$, $\omega_{B,1}$, and $\boldsymbol{s}$ are the translational velocity after the collision, the rotational velocity after the collision, and the unit vector indicating the longitudinal direction of object, respectively. Once the object results in a pure translational motion whose direction is perpendicular to the longitudinal axis of object, it can be always captured by simply closing the two fingers, as shown in Fig.1(a-3), even under an extremely small friction coefficient. This is because there is no tangential velocity component at the fingertip surface.

Based on the above consideration, we have proposed the basic concept of two-step strategy for capturing a stick-shaped object in our former work [7]. However, the strategy has two disadvantages; one is that the strategy is based on the assumption where the friction coefficient between the fingertip and the object is zero ($\mu = 0$), and the other is that the strategy cannot guide the object to an arbitrary direction. This paper extends the strategy to cope with the above two points. For a stick-shaped object with both translational and rotational velocities as shown in Fig.1(a-1), we introduce the friction independent collision where the contact friction does not influence the object's motion under an arbitrary friction coefficient ($\mu > 0$). By introducing such a particular collision for generating the pure translational motion of object as shown in Fig.1(a-2), we can determine the set of fingertip positions in the new strategy, irrespective of the friction coefficient μ. This means that the strategy can be applied even under unknown friction condition. Also, we newly proposed the 3-step capturing strategy where we can choose an arbitrary direction of the object's motion after two collisions, so that we can guide the object to an appropriate direction to be captured, for example, when kinematic constraints of the fingers need to be considered.

In Section II, we introduce the friction independent collision for a stick-shaped object under the collision model based on the restitution coefficient. After deriving the relationship between velocities before and after the collision, we show the fingertip position for controlling the rotational or the translational motion of the object after the collision. In Section III, we show two capturing strategies. By utilizing the friction independent collision, we first reformulate the 2-step capturing strategy. In this strategy, the translational motion of object is limited on the line of the initial moving direction. We then proposed the 3-step capturing strategy where we can generate an arbitrary direction of the object's motion after two collisions. In Section IV, we show an experimental result for confirming the validity of the 3-step capturing strategy by using a two-fingered robot hand with the assistance of a high-speed vision system.

II. Friction Independent Collision

A. Model for collision

Before describing the model for the capturing problem, we show the model for collision used in this work. Let us consider a collision between two rigid bodies in a plane, as shown in Fig.2, where the simplifying assumptions are given as follows:

Assumption 1: The contact between the bodies is modeled as a point-contact.

Assumption 2: The collision process is instantaneous with an impulsive force, and translational and rotational velocities of the bodies change discontinuously.

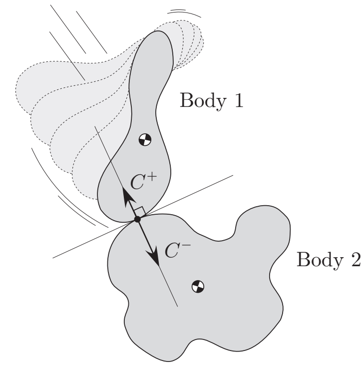


Fig. 2. The two-dimensional model for collision between two rigid bodies.

Assumption 3: No displacements occur during the collision.

Assumption 4: The friction coefficient based on Coulomb's law is given between the bodies, where static friction and dynamical one are not distinguished.

For such a case, the restitution coefficient has been often adopted to describe the energy dissipation by the collision and to analyze the behaviors of the objects. There are two major laws governing the restitution coefficients; Poisson's hypothesis and Newton's law of restitution [8], [9]. In Poisson's hypothesis, the restitution coefficient is given by

$$e_P = \frac{P_r}{P_c} \quad (0 \le e_P \le 1), \tag{1}$$

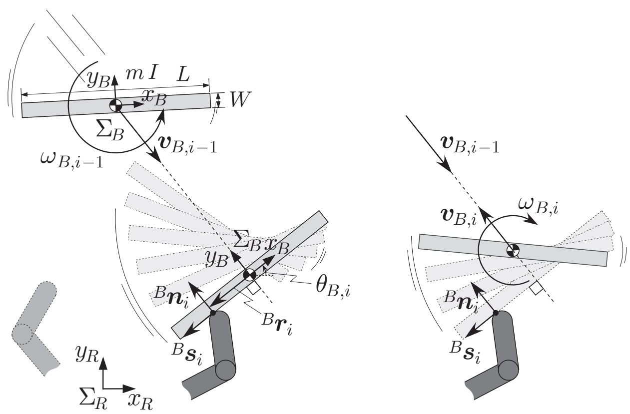
where P_c and P_r are the normal component of impulse accumulated during the compression phase and that accumulated during the restitution phase, respectively. This e_P is assumed to be constant and to depend solely on the materials of the bodies. On the other hand, the restitution coefficient in Newton's law of restitution is given by

$$e_N = -\frac{C^+}{C^-} \quad (0 \le e_N \le 1), \tag{2}$$

where C^- and C^+ are the normal components of relative velocity at the contact point before and after the collision, respectively. Generally, when we take the contact friction into account at the collision, Newton's law of restitution can not express the collision phenomenon for the tangential velocity component adequately and yields the increase of the system energy by the collision [8]. While Poisson's hypothesis is superior to Newton's law of restitution, it is known that the same motion for both restitution coefficients is obtained under the condition where either (i) the contact friction between the two bodies is zero or (ii) a particular geometrical condition is satisfied for the object. We try to find a friction independent collision based on (ii). Therefore, we utilize the restitution coefficient of Newton's low ($e = e_N$) for simplicity.

B. Model for robot-object system

Consider a two-dimensional robot-object system as shown in Fig.3, where Fig.3(a) and (b) show the situation before the



(a) Before collision (b) After collision

Fig. 3. Friction independent collision.

i-th collision ($i = 1, 2, \ldots$) between an object and a fingertip and that after the collision, respectively. For simplifying the analysis, we set the additional following assumptions:

Assumption 5: A stick-shaped object whose width is small enough to be neglected and a two-fingered robot hand are supposed.

Assumption 6: The object has both translational and rotational motion at the initial phase and these velocities stay constant before the object makes a collision with the robot hand.

Assumption 7: The mass, the moment of inertia, the position of the center of mass of the object are known. The position and the orientation of the object can be observed.

Assumption 8: The hand-arm system is sufficiently rigid (or the mass of the arm-hand system is large enough) so that no displacement of the fingertip occurs during each collision.

Assumption 9: The motions of finger links and arm, and interference among them and the object can be neglected (we focus only on the fingertips).

Assumption 10: The value of restitution coefficient e is known and the friction coefficient is given by μ ($\mu > 0$).

The meanings of the symbols in Fig.3 are as follows:

- Σ_R : The reference coordinate system.
- Σ_B : The object coordinate system fixed at the center of mass of the object.
- m : The mass of the object.
- I : The moment of inertia of the object around its center of mass.
- L : The length of the object.
- W : The width of the object.
- $\boldsymbol{v}_{B,i}$: The translational velocity vector of the object after the i-th collision with respect to Σ_R. $\boldsymbol{v}_{B,i} \in \Re^2$.
- $\omega_{B,i}$: The rotational angular velocity of the object after the i-th collision.
- ${}^B\boldsymbol{r}_i$: The position vector expressing the i-th contact point with respect to Σ_B. ${}^B\boldsymbol{r}_i \in \Re^2$.
- ${}^B\boldsymbol{n}_i$: The normal unit vector at the i-th contact point on the object surface with respect to Σ_B. ${}^B\boldsymbol{n}_i \in \Re^2$.
- ${}^B\boldsymbol{s}_i$: The tangential unit vector at the i-th contact point on the object surface with respect to Σ_B. ${}^B\boldsymbol{s}_i \in \Re^2$.
- $\theta_{B,i}$: The orientation of Σ_B with respect to Σ_R at the moment of the i-th collision.

Furthermore, $\boldsymbol{v}_{B,0}$ and $\omega_{B,0}$ are the initial velocities of the object and Σ_B is fixed on the center of mass of object so that x_B-axis corresponds to the longitudinal direction with the length of L. Now, suppose a stick-shaped object where $W \approx 0$ and that each contact with a fingertip on the side with length of L, the fingertip position for the i-th collision can be expressed as follows:

$$
{}^B\boldsymbol{r}_i \triangleq \left[{}^Br_{x,i},\ {}^Br_{y,i}\right]^T
$$
$$
(-L/2 \le {}^Br_{x,i} \le L/2,\ {}^Br_{y,i} = \pm W/2).
$$

The unit vectors indicating the tangential direction and the normal one at the contact point can be expressed as follows:

$$
{}^B\boldsymbol{s}_i = \begin{cases} [1,0]^T & \text{if } {}^Br_{y,i} = W/2, \\ [-1,0]^T & \text{if } {}^Br_{y,i} = -W/2, \end{cases} \tag{3}
$$

$$
{}^B\boldsymbol{n}_i = \begin{cases} [0,-1]^T & \text{if } {}^Br_{y,i} = W/2, \\ [0,1]^T & \text{if } {}^Br_{y,i} = -W/2, \end{cases} \tag{4}
$$

where ${}^Br_{y,i} = W/2$ and ${}^Br_{y,i} = -W/2$ mean that the fingertip makes contact with the object from $y_B \ge 0$ and $y_B \le 0$, respectively.

C. Change of Velocity on Friction Independent Collision

We can obtain the relationship among the momentum, the angular momentum, and the impulse $\boldsymbol{F}_i \in \Re^2$ applied to the object at the i-th collision, as follows:

$$
\boldsymbol{M}\Delta\boldsymbol{u}_{B,i} = \boldsymbol{H}_i^T\boldsymbol{F}_i, \tag{5}
$$

where

$$
\begin{aligned}
\boldsymbol{M} &\triangleq \begin{bmatrix} m\boldsymbol{E}_2 & \boldsymbol{0} \\ \boldsymbol{0} & I \end{bmatrix} \in \Re^{3\times3}, \\
\boldsymbol{H}_i &\triangleq \begin{bmatrix} \boldsymbol{E}_2 & \left[\boldsymbol{R}_B(\theta_{B,i}){}^B\boldsymbol{r}_i\otimes\right]^T \end{bmatrix} \in \Re^{2\times3}, \\
\Delta\boldsymbol{u}_{B,i} &\triangleq \boldsymbol{u}_{B,i} - \boldsymbol{u}_{B,i-1} \in \Re^3, \\
\boldsymbol{u}_{B,i} &\triangleq \begin{bmatrix} \boldsymbol{v}_{B,i} \\ \omega_{B,i} \end{bmatrix} \in \Re^3,
\end{aligned}
$$

$\boldsymbol{E}_n \in \Re^{n\times n}$ and $\boldsymbol{R}_B(\theta_{B,i}) \in \Re^{2\times2}$ denote the $n \times n$ unit matrix and the matrix expressing the posture of Σ_B with respect to Σ_R, respectively, and $[\boldsymbol{a}\otimes] \triangleq [-a_y, a_x] \in \Re^{1\times2}$ for an arbitrary vector $\boldsymbol{a} \triangleq [a_x, a_y]^T \in \Re^2$. Translational velocities at the contact point just before and after the collision are respectively given by

$$
\boldsymbol{v}_{C,i-1} = \boldsymbol{H}_i\boldsymbol{u}_{B,i-1}, \tag{6}
$$
$$
\boldsymbol{v}_{C,i} = \boldsymbol{H}_i\boldsymbol{u}_{B,i}. \tag{7}
$$

From the above equations, the relationship between the change of velocity of object $\Delta\boldsymbol{u}_{B,i}$ and that of translational velocity of the contact point $\Delta \boldsymbol{v}_{C,i} \triangleq \boldsymbol{v}_{C,i} - \boldsymbol{v}_{C,i-1} \in \Re^2$ can be expressed by following:

$$\Delta \boldsymbol{v}_{C,i} = \boldsymbol{H}_i \Delta \boldsymbol{u}_{B,i}. \quad (8)$$

From (5) and (8), we can obtain the following equation:

$$\boldsymbol{F}_i = \boldsymbol{K}_i^{-1} \Delta \boldsymbol{v}_{C,i}, \quad (9)$$

where

$$\boldsymbol{K}_i \triangleq \boldsymbol{H}_i \boldsymbol{M}^{-1} \boldsymbol{H}_i^T \in \Re^{2\times 2}.$$

Based on (2), the restitution coefficient e is given for the normal velocity component at the contact point, as follows:

$$e = -\frac{\boldsymbol{n}_i^T \boldsymbol{v}_{C,i}}{\boldsymbol{n}_i^T \boldsymbol{v}_{C,i-1}}, \quad (10)$$

where $\boldsymbol{n}_i \triangleq \boldsymbol{R}_B(\theta_{B,i})^B\boldsymbol{n}_i \in \Re^2$. From (10), we can obtain the following equation:

$$\boldsymbol{n}_i^T \Delta \boldsymbol{v}_{C,i} = -(1+e)\boldsymbol{n}_i^T \boldsymbol{H}_i \boldsymbol{u}_{B,i-1}. \quad (11)$$

Now, suppose the collision as shown in Fig.3(a), where the following equation is satisfied between the object orientation and the translational direction:

$$\boldsymbol{s}_i^T \boldsymbol{v}_{B,i-1} = 0, \quad (12)$$

where $\boldsymbol{s}_i \triangleq \boldsymbol{R}_B(\theta_{B,i})^B\boldsymbol{s}_i \in \Re^2$. By considering that ${}^Br_{y,i} = \pm W/2 \approx 0$ leads to $\boldsymbol{s}_i^T[\boldsymbol{R}_B(\theta_{B,i})^B\boldsymbol{r}_i\otimes]^T = 0$, we can obtain, after some transformations on (6) and (7), the following relationship:

$$\boldsymbol{s}_i^T \boldsymbol{v}_{C,i-1} = \boldsymbol{s}_i^T \boldsymbol{v}_{B,i-1}, \quad (13)$$

$$\boldsymbol{s}_i^T \boldsymbol{v}_{C,i} = \boldsymbol{s}_i^T \boldsymbol{v}_{B,i}. \quad (14)$$

From (12) and (13), we can obtain

$$\boldsymbol{s}_i^T \boldsymbol{v}_{C,i-1} = 0. \quad (15)$$

Equation (15) means that the tangential velocity component of the object at the contact point is zero just before the collision. By considering that any rotational velocity does not contribute to the tangential velocity at the contact point during the collision ((13) and (14)), we can see that the frictional force is not generated at the moment of the contact. Hereafter, we call such a collision expressed by (12) the friction independent collision for a stick-shaped object. Since the frictional force is not generated, we can easily derive $\boldsymbol{s}_i^T \boldsymbol{F}_i = 0$. From the tangential components between the momentum and the impulse just before and after collision being expressed by (5), we can obtain

$$m\boldsymbol{s}_i^T(\boldsymbol{v}_{B,i} - \boldsymbol{v}_{B,i-1}) = \boldsymbol{s}_i^T \boldsymbol{F}_i. \quad (16)$$

By substituting (12) and $\boldsymbol{s}_i^T \boldsymbol{F}_i = 0$ to (16), for the translational velocity of the object after the collision, we can obtain the following equation:

$$\boldsymbol{s}_i^T \boldsymbol{v}_{B,i} = 0. \quad (17)$$

Equation (17) means that the translational direction after the collision is also limited to be perpendicular to the orientation of object at the moment of collision, as shown in Fig.3(b). From (14) and (17), we can obtain

$$\boldsymbol{s}_i^T \boldsymbol{v}_{C,i} = 0. \quad (18)$$

From (15) and (18), we can obtain

$$\boldsymbol{s}_i^T \Delta \boldsymbol{v}_{C,i} = 0. \quad (19)$$

By substituting (11) and (19) to (9), we can obtain the following equation:

$$\boldsymbol{F}_i = \boldsymbol{K}_i^{-1}(\boldsymbol{n}_i\boldsymbol{n}_i^T \Delta \boldsymbol{v}_{C,i} + \boldsymbol{s}_i\boldsymbol{s}_i^T \Delta \boldsymbol{v}_{C,i}) \quad (20)$$

$$= -(1+e)\boldsymbol{K}_i^{-1}\boldsymbol{n}_i\boldsymbol{n}_i^T \boldsymbol{H}_i \boldsymbol{u}_{B,i-1}. \quad (21)$$

By substituting (21) into (5), we can express the relationship between the velocity before the collision $\boldsymbol{u}_{B,i-1}$ and that after the collision $\boldsymbol{u}_{B,i}$ as follows:

$$\boldsymbol{u}_{B,i} = \boldsymbol{J}_i \boldsymbol{u}_{B,i-1}, \quad (22)$$

where

$$\boldsymbol{J}_i \triangleq \boldsymbol{E}_3 - (1+e)\boldsymbol{M}^{-1}\boldsymbol{H}_i^T\boldsymbol{K}_i^{-1}\boldsymbol{n}_i\boldsymbol{n}_i^T\boldsymbol{H}_i \in \Re^{3\times 3}.$$

Thus, under the friction independent collision where (12) is satisfied, we can clearly express the relationship between the velocities before and after the collision irrespective of the friction coefficient μ. We would note that this particular collision can be classified into a *generalized central impact* besides a *direct impact* [8].

D. Controlling Object Motion by Friction Independent Collision

The parameters which can be controlled at the i-th collision are the orientation object $\theta_{B,i}$ and the fingertip position ${}^B\boldsymbol{r}_i$. In order to achieve the friction independent collision for the object originally moving with $\boldsymbol{v}_{B,i-1}$, $\theta_{B,i}$ is given by

$$\theta_{B,i} = \alpha(\boldsymbol{v}_{B,i-1}), \quad (23)$$

where

$$\alpha(\boldsymbol{a}) \triangleq \begin{cases} \tan^{-1}(-a_x/a_y) & \text{if } a_y \neq 0, \\ \pi/2 & \text{if } a_y = 0, \end{cases}$$

for an arbitrary vector $\boldsymbol{a} \triangleq [a_x, a_y]^T \in \Re^2$. When the set of the orientation of object $\theta_{B,i}$ and the contact direction of the fingertip ${}^Br_{y,i}$ is given, $\boldsymbol{J}_i$ on (22) can be regarded as the function of the fingertip position on the longitudinal direction ${}^Br_{x,i}$. Now, let us consider to generate a desired rotational angular velocity by the friction independent collision. By extracting the rotational velocity component from (22), we obtain

$$\omega_{B,i} = \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix}^T \boldsymbol{J}_i({}^Br_{x,i})\boldsymbol{u}_{B,i-1}. \quad (24)$$

Equation (24) is the quadratic equation with respect to ${}^Br_{x,i}$. By solving (24) with respect to ${}^Br_{x,i}$, the fingertip position for generating the desired rotational angular velocity $\omega_{B,i}^d$ is given by

$${}^Br_{x,i} = \mathcal{F}_{\omega,i}(\omega_{B,i}^d), \quad (25)$$

where

$$\mathcal{F}_{\omega,i}(\omega^d_{B,i}) \triangleq \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1},$$
$$a_1 \triangleq m(\omega^d_{B,i} + e\omega_{B,i-1}),$$
$$b_1 \triangleq -(1+e)m\boldsymbol{n}_i^T\boldsymbol{v}_{B,i-1},$$
$$c_1 \triangleq I(\omega^d_{B,i} - \omega_{B,i-1}).$$

The sign expressing the positive or negative value in $\mathcal{F}_{\omega,i}(\omega^d_{B,i})$ correspond the following; one is the solution where the normal velocity component $\mathcal{C}_i$ at the contact point is toward the opposite direction to the fingertip, and another is the solution where the component is toward the inner direction of the fingertip. Since the valid solution is the former one, we choose the solution of ${}^B r_{x,1}$ satisfying the following equation:

$$\mathcal{C}_i({}^B r_{x,i}) > 0, \tag{26}$$

where

$$\mathcal{C}_i({}^B r_{x,i}) \triangleq \boldsymbol{n}_i^T\boldsymbol{H}_i({}^B r_{x,i})\boldsymbol{J}_i({}^B r_{x,i})\boldsymbol{u}_{B,i-1}.$$

On the other hand, the translational direction of the object after the friction independent collision is limited to the positive or negative translational direction before the collision, as shown in Fig.3(b). By letting $V_{B,i}$ be the speed of the translational velocity after the i-th collision, from (22), we can obtain

$$V_{B,i} = \begin{bmatrix} -\frac{\boldsymbol{v}_{B,i-1}}{\|\boldsymbol{v}_{B,i-1}\|} \\ 0 \end{bmatrix}^T \boldsymbol{J}_i({}^B r_{x,i})\boldsymbol{u}_{B,i-1}. \tag{27}$$

By solving (27) with respect to ${}^B r_{x,i}$, the fingertip position for generating the desired translational speed $V^d_{B,i}$ is given by

$${}^B r_{x,i} = \mathcal{F}_{V,i}(V^d_{B,i}), \tag{28}$$

where

$$\mathcal{F}_{V,i}(V^d_{B,i}) \triangleq \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2c_2}}{2a_2},$$
$$a_2 \triangleq m\left(V^d_{B,i} + \|\boldsymbol{v}_{B,i-1}\|\right),$$
$$b_2 \triangleq (1+e)I\omega_{B,i-1}\boldsymbol{n}_i^T\frac{\boldsymbol{v}_{B,i-1}}{\|\boldsymbol{v}_{B,i-1}\|},$$
$$c_2 \triangleq I\left(V^d_{B,i} - e\|\boldsymbol{v}_{B,i-1}\|\right).$$

The determination of the sign in $\mathcal{F}_{V,i}(V^d_{B,i})$ is done by testing (26).

III. Capturing Strategies Based on Friction Independent Collision

A. Towards Capturing Strategy

Suppose a stick-shaped object with the initial velocities $\boldsymbol{v}_{B,0}$ and $\omega_{B,0}$, as shown in Fig.4(a). Let us now consider the capturing strategy, where we can determine the contact condition expressed by

$$\left[\theta_{B,i}, {}^B\boldsymbol{r}_i^T\right]^T \in \Re^3 \quad (i = 1, 2, \ldots),$$

which is composed of controllable parameters.

Now, suppose the situation where the object has a pure translational motion whose direction is perpendicular to the longitudinal axis of object, as shown in Fig.4(e), as the final phase of capturing motion. This condition can be expressed by

$$(\boldsymbol{s}_i^T\boldsymbol{v}_{B,i} = 0) \quad \cap \quad (\omega_{B,i} = 0). \tag{29}$$

Under this condition, we can always capture the object irrespective of the contact friction by simply closing both fingertips along the line passing the center of mass of object and perpendicular to the longitudinal axis of object. Based on this consideration, we determine the final phase as shown in Fig.4(e), and construct the capturing strategy with achieving the condition given by (29).

In Section II, we showed how to compute the fingertip position for controlling the object motion after the collision under the friction independent collision. The commands controlling the object motion at the i-th collision can be classified into the following three groups;

- Eliminating the rotational motion: $\omega_{B,i} = 0$ (Fig.4(b)),
- Keeping both the translational and the rotational motion: $(\boldsymbol{v}_{B,i} \neq \boldsymbol{0}) \cap (\omega_{B,i} \neq 0)$,
- Eliminating the translational motion: $\boldsymbol{v}_{B,i} = \boldsymbol{0}$ (Fig.4(c)).

Among the above three commands, the second command does not produce a particular effect for capturing an object, since it is equivalent to the initial phase as shown in Fig.4(a). Therefore, we construct the capturing strategy so that each collision can eliminate either the translational motion or the rotational one. From the next section, we explain the 2-step capturing strategy and the 3-step capturing strategy. These strategies are separated according to eliminating either the rotational motion ($\omega_{B,1} = 0$) or the translational motion ($\boldsymbol{v}_{B,1} = \boldsymbol{0}$), respectively, at the first collision.

B. The 2-step Capturing Strategy

The 2-step capturing strategy is based on the idea in our former work [7]. We are reformulating the strategy with the friction independent collision in this paper.

Step 1: We completely stop the rotational motion at the first collision ($\omega_{B,1} = 0$), as shown in Fig.4(b). Under the friction independent collision, $\boldsymbol{s}_1^T\boldsymbol{v}_{B,1} = 0$ is guaranteed by (17) and we can utilize (25) for achieving $\omega_{B,1} = 0$. Therefore, the condition given by (29) can be satisfied at the first collision. The contact condition for this collision can be expressed by

$$\begin{bmatrix} \theta_{B,1} \\ {}^B\boldsymbol{r}_1 \end{bmatrix} = \begin{bmatrix} \alpha(\boldsymbol{v}_{B,0}) \\ \mathcal{F}_{\omega,1}(0)|_{\mathcal{C}_1({}^B r_{x,1})>0} \\ W/2 \ \text{ or } \ -W/2 \end{bmatrix}. \tag{30}$$

From $\boldsymbol{s}_1^T\boldsymbol{v}_{B,0} = \boldsymbol{s}_1^T\boldsymbol{v}_{B,1} = 0$, the direction of $\boldsymbol{v}_{B,1}$ is limited to the positive or the negative direction of $\boldsymbol{v}_{B,0}$. This relationship can be expressed by

$$\boldsymbol{v}_{B,1}/\|\boldsymbol{v}_{B,1}\| = p\boldsymbol{v}_{B,0}/\|\boldsymbol{v}_{B,0}\| \quad (p = 1 \text{ or } -1), \tag{31}$$

where p depends upon the contact direction of fingertip ${}^B r_{y,1}$. We would note that it is desirable to choose ${}^B r_{y,1}$ leading

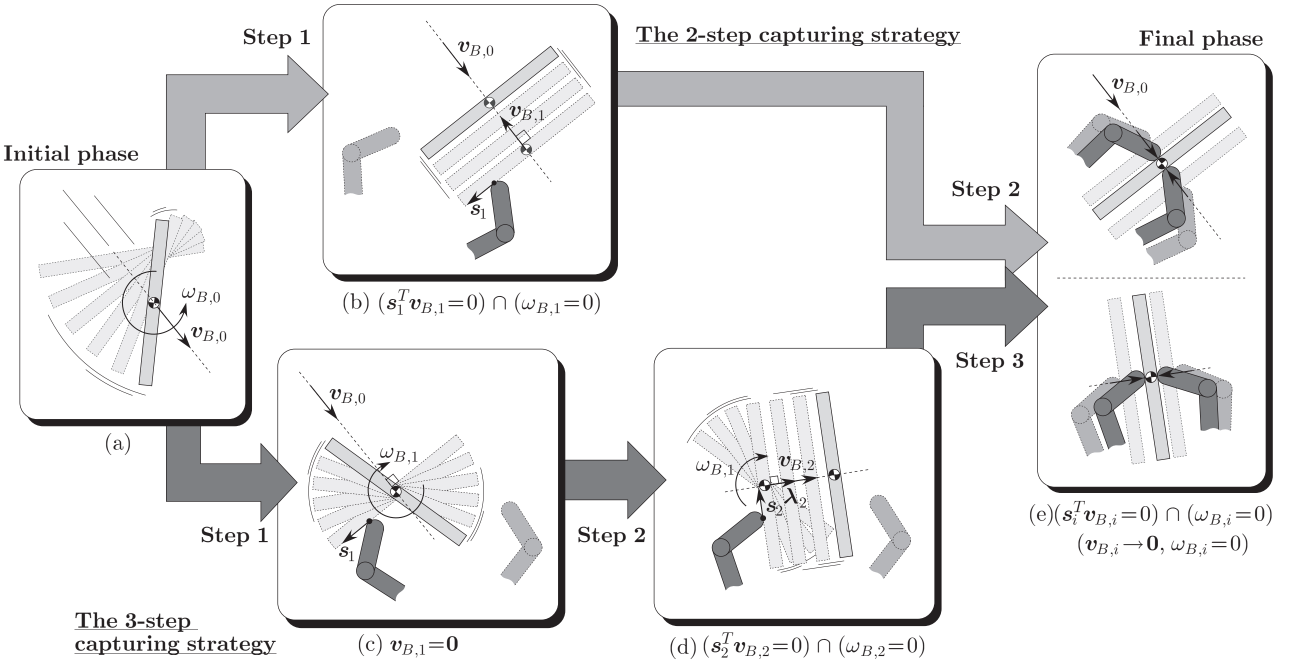


Fig. 4. Friction independent dynamic capturing strategies. The 2-step capturing strategy guides the object along the line coinciding with that of the initial translational motion with its posture perpendicular to the line, as shown in (e). Since the object posture after removing rotational motion is limited in the perpendicular direction to the initial translational motion in this strategy, the fingertips may be obliged to move quickly enough to capture the object within the workspace. On the other hand, since the 3-step capturing strategy can eventually guide the object in an arbitrary direction after two times collisions, the constraint on workspace required for robot fingers will be greatly relaxed.

to $p = -1$ by testing (22), in order to decrease the speed of object after the collision.

Step 2: From the second collision, we determine the fingertip position so that the normal direction at the contact point may pass through the center of mass of object, as shown in Fig.4(e), for avoiding any rotational motion. The contact condition for such a collision can be expressed by

$$\begin{bmatrix} \theta_{B,i} \\ {}^{B}\boldsymbol{r}_i \end{bmatrix} = \begin{bmatrix} \theta_{B,1} \\ 0 \\ W/2 \ \text{ or } \ -W/2 \end{bmatrix} \quad (i \geq 2), \quad (32)$$

where the sign of ${}^{B}r_{y,i}$ is given alternatively so that $\boldsymbol{n}_i = -\boldsymbol{n}_{i-1}$ is satisfied. In this case, the following relationship is satisfied:

$$\boldsymbol{v}_{B,i}/\left\|\boldsymbol{v}_{B,i}\right\| = -\boldsymbol{v}_{B,i-1}/\left\|\boldsymbol{v}_{B,i-1}\right\| \qquad (i \geq 2). \qquad (33)$$

Equation (33) means that the object repeats only a translational motion continuously. Under the condition shown by (32), the remaining translational velocity can be gradually eliminated ($\boldsymbol{v}_{B,i} \rightarrow \mathbf{0}$) by simply closing the distance between two fingertips.

As discussed in the above, we can determine the fingertip positions for both Step 1 and Step 2 irrespective of the friction coefficient μ in the 2-step capturing strategy. This is the advantage where the proposed strategy can be applied to the case under unknown frictional condition.

In the 2-step capturing strategy, the direction of $\boldsymbol{v}_{B,i}$ is limited to the positive or negative direction of $\boldsymbol{v}_{B,0}$ as shown in the dotted line in Fig.4(e). In robot application, there may exist the case where the fingertip for the second collision cannot reach the desired position due to either kinematical or mechanical limitation, and as a result, the fingertips may miss to capture the object. To cope with this issue, we introduce the 3-step capturing strategy in the next section.

C. The 3-step Capturing Strategy

In the 3-step capturing strategy, we generate a pure rotational motion by eliminating the translational motion of object at the first collision. Once the object motion results in a pure rotational one as shown in Fig.4(c), we can generate a pure translational motion whose direction is perpendicular to the longitudinal axis of object for an arbitrary direction at the second collision, since we can choose any fingertip positions at a certain radius with respect to the rotational center. Thus, we can achieve the capturing motion in an arbitrary direction with this strategy as shown in Fig.4(d), to cover the lack of workspace, degrees of freedom, and speed of robot.

Step 1: We completely stop the translational motion at the first collision ($\boldsymbol{v}_{B,1} = \mathbf{0}$), as shown in Fig.4(c). Based on the friction independent collision, the contact condition can be given by

$$\begin{bmatrix} \theta_{B,1} \\ {}^{B}\boldsymbol{r}_1 \end{bmatrix} = \begin{bmatrix} \alpha(\boldsymbol{v}_{B,0}) \\ \mathcal{F}_{V,1}(0)|_{\mathcal{C}_1({}^{B}r_{x,1})>0} \\ W/2 \ \text{ or } \ -W/2 \end{bmatrix}, \qquad (34)$$

where it is desirable for ${}^{B}r_{y,1}$ to be chosen so that it satisfies

$$\omega_{B,1}/\left|\omega_{B,1}\right| = -\omega_{B,0}/\left|\omega_{B,0}\right|, \qquad (35)$$

by testing (22), in the same way as the first collision of the 2-step grasping strategy.

Step 2: We completely stop the rotational motion at the second ($\omega_{B,2} = 0$), as shown in Fig.4(d), and covert it to the translational motion. Since $\boldsymbol{v}_{B,1} = \boldsymbol{0}$ by the first collision, $\boldsymbol{s}_2^T \boldsymbol{v}_{B,1} = 0$ is satisfied under all possible orientation of the object at the second collision. Therefore, the friction independent collision occurs for an arbitrary object orientation $\theta_{B,2}$, and $\boldsymbol{s}_2^T \boldsymbol{v}_{B,2} = 0$ after the second collision is guaranteed by (17). Since we can utilize (25) for $\omega_{B,2} = 0$, the condition given by (29) can be achieved at the second collision. The contact condition for the second collision can be given by

$$\begin{bmatrix} \theta_{B,2} \\ {}^B\boldsymbol{r}_2 \end{bmatrix} = \begin{bmatrix} \alpha(\boldsymbol{\lambda}_2) \\ \mathcal{F}_{\omega,2}(0)|_{\mathcal{C}_2({}^Br_{x,2})>0} \\ W/2 \quad \text{or} \quad -W/2 \end{bmatrix}, \tag{36}$$

where $\boldsymbol{\lambda}_2 \triangleq [\lambda_{x,2}, \lambda_{y,2}]^T \in \Re^2$ is the selection vector for determining the translational direction of object after the collision. We can give an arbitrary $\boldsymbol{\lambda}_2$ based on the consideration of the fingertip position waiting for the third collision, as shown in Fig.4(d). By testing (22), ${}^Br_{y,2}$ is determined so that it satisfies

$$\boldsymbol{v}_{B,2} / \|\boldsymbol{v}_{B,2}\| = \boldsymbol{\lambda}_2. \tag{37}$$

Step 3: After the third collision, we can apply the same procedure as taken after the second collision of the 2-step capturing strategy as shown in Fig.4(e). According to (32), such conditions are given by

$$\begin{bmatrix} \theta_{B,i} \\ {}^B\boldsymbol{r}_i \end{bmatrix} = \begin{bmatrix} \theta_{B,2} \\ 0 \\ W/2 \quad \text{or} \quad -W/2 \end{bmatrix} \quad (i \geq 3). \tag{38}$$

The 3-step capturing strategy can be executed under unknown friction condition as well.

Suppose that a robot is quick enough for moving the fingertip to the designated position and has sufficient degrees of freedom for avoiding any interference between finger link and the object. Under such a robot, the 2-step capturing strategy will work appropriately. However, if this is not the case, the 3-step strategy will work better, since it can guide the object with an arbitrary direction and cover the lack of workspace, degrees of freedom, and speed of robot.

IV. Experiments

A. Experimental System

Fig.5 shows the experimental system. The hand is composed of two fingers where each finger has two joints. It can produce the maximum rotational speed of 14.0[rad/s] for each joint. Each joint angle is measured by an encoder integrated in the motor for driving robot finger. The sliders for moving the hand can work within 400[mm]×400[mm] with the maximum speed of 1.0[m/s] in each axis. The position of the base of the hand is measured by encoders integrated in the motors for driving the sliders. The hand is not equipped with a degree of freedom to control its orientation. A high-speed vision [10] which detects the position and the orientation of object with 300[fps] is implemented in the height of 1[m] from the table. A stick-shaped object with $L = 100.0$[mm], $W = 10.0$[mm], $m = 22.5$[g], and

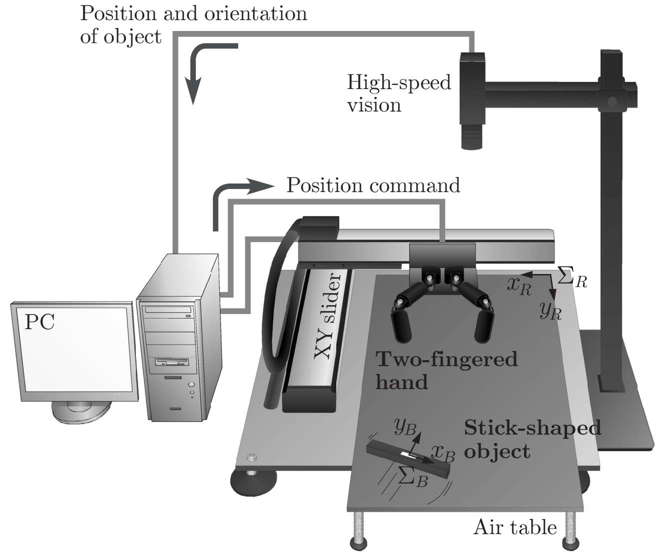


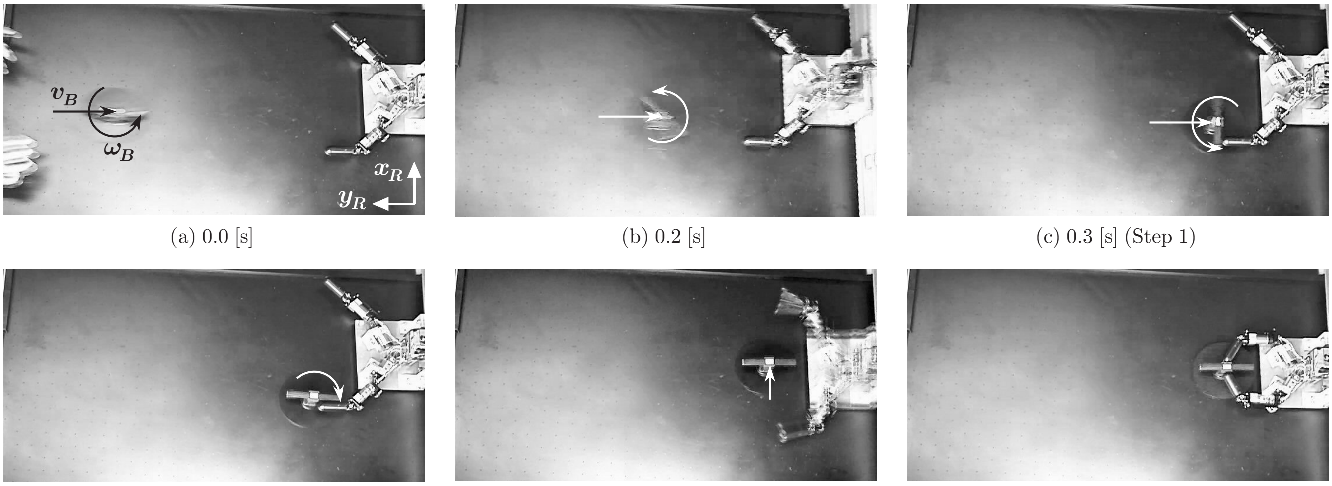
Fig. 5. Experimental setup.

$I = 32.0 \times 10^3$[g·mm^2] is used. Approximate friction and restitution coefficient between the object and the fingertip are $\mu = 0.3$ and $e = 0.19$, respectively. Air is regularly supplied from the surface of table so that the friction between the object and the table is negligible. We intentionally attach a rectangular shaped white marker to the object so that the vision system can recognize both position and orientation of object. By using this information, PC estimates the velocity and predicts the trajectory of object. The hand and the slider are controlled by PC, so that the fingertip is placed at the target position to wait for the collision.

B. Experimental Results

Fig.6 shows a series of photos where the 3-step capturing strategy is executed[1]. At the first collision for Step 1, the translational motion is converted into a pure rotational one in the clockwise direction, as shown in Fig.6(c). At the second collision for Step 2, the rotational motion is converted into a pure translational one where the object posture is perpendicular to the palm, as shown in Fig.6(d). Since we obtain the desired fingertip position of ${}^Br_{x,2} = 48.7$[mm] under $L/2 = 50.0$[mm], we simply give the second collision to the tip of object. By these two collisions, the object is led to the center of two fingertips, as shown in Fig.6(e), and finally captured by Step 3, as shown in Fig.6(f). Fig.7 shows the velocities of the object with respect to time during the capturing motion. From Fig.7, we can see that the translational velocity of 1.1[m/s] is eliminated and the object motion is converted into the pure rotational one, by the first collision at $t = 0.31$[s]. Then, the rotational angular velocity of -10.0[rad/s] is eliminated by the second collision at $t = 0.51$[s] and the translational velocity of object is

[1] The video attachment media file for this paper shows both the 2-step and the 3-step capturing strategies in experiments in short version. Full version including demonstrations of the experimental system can be seen in our web site [11].



(a) 0.0 [s] (b) 0.2 [s] (c) 0.3 [s] (Step 1)

(d) 0.5 [s] (Step 2) (e) 0.9 [s] (f) 1.0 [s] (Step 3)

Fig. 6. A series of photos during the 3-step capturing strategy.

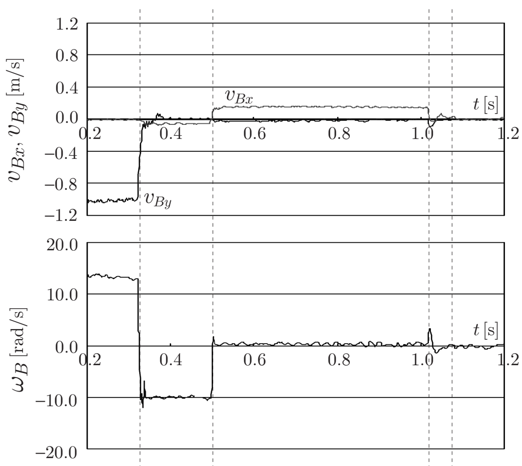


Fig. 7. Translational and rotational velocities with respect to time during the 3-step capturing strategy.

finally eliminated around $t = 1.24$[s]. Thus, the hand can manipulate and lead the object to being captured by utilizing the 3-step capturing strategy, under the condition where the hand does not have the degree of freedom for orientation.

V. Conclusions

We discussed dynamic capturing strategy for a stick-shaped object with both translational and rotational velocities. The main results are summarized as follows:

(1) Under the friction independent collision, we obtain the fingertip position for controlling the object motion after the collision.
(2) By utilizing the friction independent collision, we proposed the capturing strategies independent upon the contact friction between the fingertip and the object.
(3) The 2-step capturing strategy can guide the object along the line coinciding with that of the initial translational velocity vector with its posture perpendicular to the line.
(4) The 3-step capturing strategy can eventually guide the object in an arbitrary direction after two times collisions.
(5) We confirmed the validity of the 3-step capturing strategy by utilizing the high-speed hand with the assistant of the high-speed vision.

The results of this paper may be applicable for catching a stick-shaped object in space or dynamic parts sorting in industry. We would like to extend the strategy to 3D version in the future.

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