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Fast identification method to control a flexible manipulator with parameter uncertainties

J. Becedas, J.R. Trapero, H. Sira-Ramírez and V. Feliu-Battle

Abstract— **In this article, we propose a fast on-line closed loop identification method of continuous-time combined with an output feedback controller of the Generalized Proportional Integral type (GPI), for the control of an uncertain flexible robotic arm with unknown mass at the tip, including a Coulomb friction term in the motor dynamics. A fast, non-asymptotic, algebraic identification method is used to identify the unknown system parameter and update the designed certainty equivalence GPI controller. In order to verify this method several informative experiments are shown.**

I. Introduction

Flexible arm manipulators span a wide range of applications: space robots, nuclear maintenance, micro-surgery, contouring control, pattern recognition and many others. A survey of the literature dealing with applications and challenging problems related to flexible manipulators may be found in [1]. The main problem of flexible manipulators is their vibration due to low stiffness. In order to cancel that vibration methodologies are required. These combine on-line identification techniques with control schemes in a suitable way. Therefore, we first introduce the background on identification and then the control scheme is proposed.

Identification of continuous-time system parameters has been studied from different points of view. The works led by Young in [2] and the books of Sinha and Unbehauen in [3], [4] describe most of the available techniques. The different approaches are usually classified into two categories: *i)* Indirect approaches: we need an equivalent discrete-time model to fit the data. After that, we transfer the estimated discrete-time parameters to continuous time. *ii)* Direct approaches: We try to estimate the original continuous-time parameters from the discrete-time data via approximations for the signals and operators in the continuous-time model. In the case of the indirect method a classical well-known theory is developed (see [11]). Nevertheless, these approaches often require a high numeric burden, without even guaranteeing convergence. Furthermore, the estimated parameters may not be correlated with the physical properties of the system.

Unfortunately, on the one hand, identification of robotic systems is generally focused on indirect approaches, see [12], [14]. On the other hand the existing identification techniques included in the direct approach suffer from poor speed performance. The objective of this paper is the fast on-line closed loop identification of continuous-time of the natural frequency of a one-degree-of-freedom flexible manipulator beam. This technique is combined with a generalized proportional integrator controller. This was first proposed by Marquez in [13]; but it was internally unstable although the closed loop system was asymptotically stable. We propose, by further manipulation of the integral reconstructor an internally stable control scheme. The authors have based their work on the fast identification techniques that were recently proposed by Fliess and his co-workers in [7]. (See also [8]) for the state and constant estimation of parameters in a fast and reliable way in feedback control systems. Let us recall that those techniques are not asymptotic, and do not need any statistical knowledge of the noises corrupting the data. Furthermore, this methodology has been successfully applied in [10] for signal processing.

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II. Identification

A. Model description

Consider the following simplified model of a very lightweight flexible link, with its entire mass concentrated at the tip, actuated by a DC motor, as shown in Fig. 1. The dynamics of the system are given by:

$$mL^2\ddot{\theta}_t = c(\theta_m - \theta_t) \tag{1}$$

$$ku = J\ddot{\hat{\theta}}_m + \nu\dot{\hat{\theta}}_m + \hat{\Gamma}_c + \hat{\Gamma}_{coup} \tag{2}$$

$$\hat{\Gamma}_{coup} = \frac{c}{n}(\theta_m - \theta_t) \tag{3}$$

where m is the unknown mass in the tip position. L and c are the length of the flexible arm and the stiffness of the bar respectively, and are assumed to be perfectly known, J is the inertia of the motor, ν the viscous friction coefficient, $\hat{\Gamma}_c$ is the unknown Coulomb friction torque, $\hat{\Gamma}_{coup}$ is the measured coupling torque between the motor and the link, k is the known electromechanical constant of the motor, u is the motor input voltage, $\ddot{\theta}_m$ stands for the acceleration of the motor gear, $\dot{\theta}_m$ is the velocity of the motor gear. The constant factor n is the reduction ratio of the motor gear; thus $\theta_m = \hat{\theta}_m/n$. $\hat{\theta}_m$ is the angular position of the motor and θ_t is the unmeasured angular position of the tip.

B. Algebraic estimation of the natural frequency

In order to make the equation deduction more understandable we suppose that signals are noise free. Consider

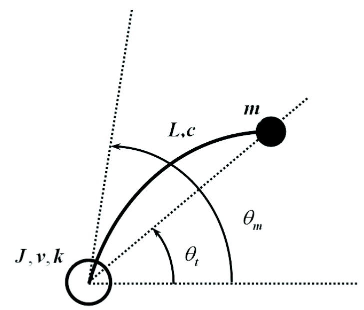

Fig. 1. Diagram of a single link flexible arm.

equation (3)

$$\theta_t = \theta_m - \frac{n}{c}\hat{\Gamma}_{coup} \tag{4}$$

If we substitute (4) in (1) we have:

$$\frac{d^2\theta_m}{dt^2} - \frac{n}{c}\ddot{\hat{\Gamma}}_{coup} = \frac{n}{c}\omega_0^2\hat{\Gamma}_{coup} \tag{5}$$

where $\omega_0 = \left(c/mL^2\right)^{1/2}$ is the unknown natural frequency of the bar due to the lack of precise knowledge of m. We assume, however, that c and L are perfectly known. The main goal is to obtain ω_0^2 as fast as possible. We only need to measure θ_m and $\hat{\Gamma}_{coup}$. The Laplace transform of (5) is:

$$\begin{aligned} &\frac{c}{n}\left(s^2\theta_m - s\theta_m(0) - \dot{\theta}(0)\right) \\ &-\left(s^2\hat{\Gamma}_{coup} - s\hat{\Gamma}_{coup}(0) - \dot{\hat{\Gamma}}_{coup}(0)\right) = \left[\omega_0^2\right]\hat{\Gamma}_{coup} \end{aligned}$$

Taking two derivatives with respect to the complex variable s, we cancel the initial conditions:

$$\frac{d^2(s^2\theta_m)}{ds^2} - \frac{n}{c}\frac{d^2(s^2\hat{\Gamma}_{coup})}{ds^2} = \omega_0^2\frac{n}{c}\frac{d^2(\hat{\Gamma}_{coup})}{ds^2} \tag{6}$$

If we define $\xi = \theta_m(t) - \frac{n}{c}\hat{\Gamma}_{coup}(t)$ we can rewrite (6) as follows:

$$\frac{d^2(s^2\xi)}{ds^2} = \omega_0^2\frac{n}{c}\frac{d^2(\hat{\Gamma}_{coup})}{ds^2} \tag{7}$$

Employing the chain rule, we obtain:

$$s^2\frac{d^2(\xi)}{ds^2} + 4s\frac{d\xi}{ds} + 2\xi = \omega_0^2\frac{n}{c}\frac{d^2(\hat{\Gamma}_{coup})}{ds^2} \tag{8}$$

Consequently, in order to avoid multiplications by positive powers of s, which in the time domain are translated as undesirable time derivatives, we multiply the above expression by s^{-2}. After rearrangement we obtain:

$$\omega_0^2 = \frac{c}{n}\frac{\dfrac{d^2(\xi)}{ds^2} + 4s^{-1}\dfrac{d\xi}{ds} + 2s^{-2}\xi}{s^{-2}\dfrac{d^2(\hat{\Gamma}_{coup})}{ds^2}} \tag{9}$$

Let $\mathscr{L}$ denote the usual operational calculus transform acting on exponentially bounded signals with bounded left support. Recall that $\mathscr{L}^{-1}s(\cdot) = \frac{d}{dt}(\cdot)$, $\mathscr{L}^{-1}\frac{d^\nu}{ds^\nu}(\cdot) = (-1)^\nu t^\nu(\cdot)$ and $\mathscr{L}^{-1}\frac{1}{s}(\cdot) = \int_0^t(\cdot)(\sigma)d\sigma$. Taking this into account we can translate (9) into the time domain, being:

$$\omega_0^2 = \frac{c}{n}\frac{\left[t^2\xi(t) - 4\int_0^t\sigma\xi(\sigma)d\sigma + 2\int_0^t\int_0^\sigma\xi(\lambda)d\lambda d\sigma\right]}{\int_0^t\int_0^\sigma\lambda^2\hat{\Gamma}(\lambda)d\lambda d\sigma} \tag{10}$$

The time realization of (10) can be written in a State Space framework via time variant linear (unstable) filters:

$$\begin{array}{rclrcl} n_e(t) &=& t^2\xi(t) + z_1 & d_e(t) &=& z_3 \\ \dot{z}_1 &=& z_2 - 4t\xi(t) & \dot{z}_3 &=& z_4 \\ \dot{z}_2 &=& 2\xi(t) & \dot{z}_4 &=& t^2\hat{\Gamma}_{coup}(t) \end{array} \tag{11}$$

The natural frequency estimator ω_0^2 is given by:

$$[\omega_0^2]_{est} = \begin{cases} \text{arbitrary} & \text{for } t \in [0,\Delta) \\ \frac{c}{n}\left[\frac{n_e(t)}{d_e(t)}\right] & \text{for } t \in [\Delta,+\infty) \end{cases} \tag{12}$$

where Δ is an arbitrary small real number. It should be noted that for the time $t = 0$, $n_e(t)$ and $d_e(t)$ both are zero. Therefore, the quotient is undefined for a small period of time. It is supposed that after a time $t = \Delta > 0$ the quotient is reliably computed. Notice that $t = \Delta$ depends on the arithmetic processor precision and on the data acquisition card.

Since the available signals θ_m and $\hat{\Gamma}_{coup}$ are noisy the estimation precision yielded by the estimator in (11)-(12) will depend on the Signal to Noise Ratio (SNR). We assume that θ_m and $\hat{\Gamma}_{coup}$ are perturbed by an added noise with unknown statistical properties. In order to enhance the SNR we filter the numerator and denominator, simultaneously, by the same low-pass filter. Taking advantage of the estimator rational form in (10), the quotient will not be affected by the filters. This invariance is emphasized with the use of the different notations in frequency and time domain, such as:

$$\omega_0^2 = \frac{F(s)n_e(t)}{F(s)d_e(t)} \tag{13}$$

with $F(s)$ given by the following transfer function:

$$\begin{aligned} F(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \\ n_e(t) &= t^2\xi(t) - 4\int_0^t\sigma\xi(\sigma)d\sigma + \ldots \\ &\quad \ldots + 2\int_0^t\int_0^\sigma\xi(\lambda)d\lambda d\sigma \\ d_e(t) &= \int_0^t\int_0^\sigma\lambda^2 y(\lambda)d\lambda d\sigma \end{aligned} \tag{14}$$

where identical low-pass filters were used, with cut-off frequency, ω_n, and the damping coefficient ζ. The filtered numerator and denominator, defined by $n_f(t)$ and $d_f(t)$ respectively, are given by the solution of the following system, being the signals $n_e(t)$ and $d_e(t)$ of (11) the inputs:

$$\begin{aligned} n_f(t) &= z_5 \\ d_f(t) &= z_7 \\ \dot{z}_5 &= z_6 \\ \dot{z}_6 &= -2\zeta\omega_n z_6 - \omega_n^2(z_5 - n_e(t)) \\ \dot{z}_7 &= z_8 \\ \dot{z}_8 &= -2\zeta\omega_n z_7 - \omega_n^2(z_7 - d_e(t)) \end{aligned} \tag{15}$$

Finally, ω_0^2 is obtained by:

$$\omega_0^2 = \begin{cases} \text{arbitrary} & t \in [0, \Delta] \\ \dfrac{n_f(t)}{d_f(t)} & t \in (\Delta, +\infty) \end{cases} \tag{16}$$

Remark 2.1: Invariant low-pass filtering requires *a priori* knowledge about the bandwidth of the system. If we do not have this we can use pure integrations of the form $1/s^p$, $p \geq 1$.

III. Generalized Proportional Integrator Controller

In Laplace transforms notation, the flexible bar transfer function, obtained from (1), can be written as follows,

$$Gb(s) = \frac{\theta_t(s)}{\theta_m(s)} = \frac{\omega_0^2}{s^2 + \omega_0^2} \tag{17}$$

As in [5] the coupling torque can be cancelled in the motor by means of a compensation term. In this case the voltage applied to the motor is of the form,

$$u = u_c + \frac{\hat{\Gamma}_{coup}}{k} \tag{18}$$

where u_c is the voltage applied before the compensation term. The system in (2) is then given by:

$$ku_c = J\ddot{\hat{\theta}}_m + \nu\dot{\hat{\theta}}_m + \hat{\Gamma}_c \tag{19}$$

where $\hat{\Gamma}_c$ is a perturbation produced by Coulomb's friction, depending only on the sign of the angular velocity. The controller to be designed will be robust with respect to these unknown piecewise constant torque disturbances affecting the motor dynamics. Thus the perturbation free system to be considered is the following:

$$Ku_c = J\ddot{\theta}_m + \nu\dot{\theta}_m \tag{20}$$

To simplify developments, let $A = K/J$, $B = \nu/J$. The DC motor transfer function is then written as:

$$\frac{\theta_m(s)}{u_c(s)} = \frac{A}{s(s+B)} \tag{21}$$

The regulation of the load position $\theta_t(t)$ to track a given smooth reference trajectory $\theta_m^*(t)$ is desired. For the synthesis of the feedback law we use only the measured motor position θ_m and the measured coupling torque $\hat{\Gamma}_{coup}$. One of the prevailing restrictions throughout our treatment of the problem is our desire *not* to measure, or compute on the basis samplings, the angular velocities of the motor shaft or of the load.

A. Outer loop controller

Consider the model of the flexible link, given in (1) and suppose for a moment that we know the value of the unknown parameter ω_0. This subsystem is flat, with flat output given by θ_t. (See [9]). The parametrization of θ_m in terms of θ_t is given, in reduction gear terms, by:

$$\theta_m = \frac{mL^2}{c}\ddot{\theta}_t + \theta_t = \frac{1}{\omega_0^2}\ddot{\theta}_t + \theta_t \tag{22}$$

System (22) is a second order system in which we wish to regulate the tip position of the flexible bar, θ_t, towards a given smooth reference trajectory $\theta_t^*(t)$ with θ_m acting as an *auxiliary* control input. Clearly, if an auxiliary open loop control input exist, $\theta_m^*(t)$ that ideally achieves the tracking of $\theta_t^*(t)$ for suitable initial conditions, it thus satisfies the second order dynamics, in reduction gear terms (23).

$$\theta_m^*(t) \quad = \quad \frac{1}{\omega_0^2}\ddot{\theta}_t^{\,*}(t) + \theta_t^*(t) \tag{23}$$

Subtracting (23) from (22), we obtain an expression in terms of the angular tracking errors:

$$\ddot{e}_{\theta_t} = \omega_0^2\left(e_{\theta_m} - e_{\theta_t}\right) \tag{24}$$

where $e_{\theta_m} = \theta_m - \theta_m^*(t)$, $e_{\theta_t} = \theta_t - \theta_t^*(t)$. Suppose for a moment that we are able to measure the angular position velocity tracking error, e_{θ_t}, then the outer loop feedback incremental controller could be proposed to be the following PID controller,

$$e_{\theta_m} = e_{\theta_t} + \frac{1}{\omega_0^2}\left[-k_2\dot{e}_{\theta_t} - k_1 e_{\theta_t} - k_0\int_0^t e_{\theta_t}(\sigma)d\sigma\right] \tag{25}$$

In such a case, the closed loop tracking error e_{θ_t} evolves governed by,

$$e_{\theta_t}^{(3)} + k_2\ddot{e}_{\theta_t} + k_1\dot{e}_{\theta_t} + k_0 e_{\theta_t} = 0 \tag{26}$$

The design parameters $\{k_2, k_1, k_0\}$, are then chosen so as to render the closed loop characteristic polynomial, a Hurwitz polynomial with desirable roots. However, in order to avoid tracking error velocity measurements, we propose to obtain an *integral reconstructor* for the angular velocity error signal $\dot{e}_{\theta_t}$. We proceed by integrating the expression (24) once; and, later, by disregarding the constant error due to the tracking error velocity initial conditions. The estimated error velocity $\hat{\dot{e}}_{\theta_t}$ can be computed in the following manner:

$$\hat{\dot{e}}_{\theta_t} = \dot{e}_{\theta_t}(t) - \dot{e}_{\theta_t}(0) = \omega_0^2\int_0^t \left(e_{\theta_m}(\sigma) - e_{\theta_t}(\sigma)\right)d(\sigma) \tag{27}$$

The integral reconstructor neglects the possibly nonzero initial condition $\dot{e}_{\theta_t}(0)$ and, hence, it exhibits a constant estimation error. When the reconstructor is used in the derivative part of the PID controller, the constant error is suitably compensated thanks to the integral control action of the PID controller. By substituting the integral reconstructor $\hat{\dot{e}}_{\theta_t}$ (27) for $\dot{e}_{\theta_t}$ in the PID controller (25), and after various rearrangements we obtain:

$$(\theta_m - \theta_m^*) = \left[\frac{\gamma_1 s + \gamma_0}{s + \gamma_2}\right](\theta_t^* - \theta_t) \tag{28}$$

The tip angular position can not be measured, but it can certainly be computed from the expression relating to the tip position with the motor position and the coupling torque (Γ):

$$\Gamma = c(\theta_m - \theta_t) = mL^2\ddot{\theta}_t = n\hat{\Gamma}_{coup} \tag{29}$$

Thus, the angular position θ_t is readily expressed as,

$$\theta_t = \theta_m - \frac{1}{c}\Gamma \tag{30}$$

Fig. 2 depicts the feedback control scheme under which the outer loop controller would actually be implemented in practise. The outer loop system in Fig. 2 is asymptotically exponentially stable. To specify the parameters, $\{\gamma_2, \gamma_1, \gamma_0\}$, we can choose to locate the closed loop poles in the left half of the complex plane. All three poles can be located at the same point of the real axis, $s = -a$, using the following polynomial equation,

$$s^3 + 3as^2 + 3a^2 s + a^3 = 0 \tag{31}$$

where the parameter a represents the desired location of the poles. The characteristic equation of the closed loop system is,

$$s^3 + \gamma_2 s^2 + \omega_0^2(1+\gamma_1)s + \omega_0^2(\gamma_2+\gamma_0) = 0 \tag{32}$$

By identifying each term of the expression (31) with those of (32), the design parameters $\{\gamma_2, \gamma_1, \gamma_0\}$ can be uniquely specified.

B. Inner loop controller

The angular position θ_m, generated as an auxiliary control input in the previous controller design step, is now regarded as a reference trajectory for the motor controller. We denote this reference trajectory as $\hat{\theta}_{mr}$.

The dynamics of the DC motor, including the Coulomb friction term, is given by (19). It is desirable to design the controller to be robust with respect to this torque disturbance.

The following feedback controller is proposed,

$$\begin{aligned} e_\nu \;=\; & \frac{\nu}{K}\hat{\dot{e}}_{\theta_m} + \frac{J}{K}\Bigg[-k_3\hat{\dot{e}}_{\theta_m} - k_2 e_{\theta_m} - k_1\int_0^t e_{\theta_m}(\sigma)d(\sigma) \\ & -k_0\int_0^t\int_0^t (e_{\theta_m}(\sigma_2))\,d(\sigma_2)d(\sigma_1)\Bigg] \end{aligned} \tag{33}$$

The following *integral reconstructor* for the angular velocity error signal $\hat{\dot{e}}_{\theta_m}$ is obtained.

$$\hat{\dot{e}}_{\theta_m} = \frac{K}{J}\int_0^t e_\nu(\sigma)d(\sigma) - \frac{\nu}{J}e_{\theta_m} \tag{34}$$

By replacing $\hat{\dot{e}}_{\theta_m}$ (34) in (30), and after various rearrangements, the feedback control law is obtained as:

$$(u_c - u_c^*) = \left[\frac{\alpha_2 s^2 + \alpha_1 s + \alpha_0}{s(s+\alpha_3)}\right](\theta_m^* - \theta_m) \tag{35}$$

The open loop control $u_c^*(t)$ that ideally achieves the open loop tracking of the inner loop is given by

$$u_c^*(t) = \frac{1}{A}\ddot{\theta}_m^*(t) + \frac{B}{A}\dot{\theta}_m^*(t) \tag{36}$$

The inner loop system in Fig. 2 is asymptotically exponentially stable. To design the parameters $\{\alpha_3, \alpha_2, \alpha_1, \alpha_0\}$ we can choose to place the closed loop poles in a desired location of the left half of the complex plane. As with the outer loop, all poles can be located at the same real value and $\alpha_3, \alpha_2, \alpha_1, \alpha_0$ can be uniquely obtained by equalizing the terms of the following two polynomials:

$$s^4 + 4ps^3 + 6p^2s^2 + 4p^3s + p^4 = 0 \tag{37}$$

$$s^4 + (\alpha_3 + B)s^3 + (\alpha_3 B + \alpha_2 A)s^2 + \alpha_1 As + \alpha_0 A = 0 \tag{38}$$

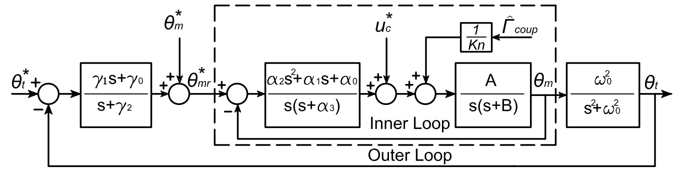

Fig. 2. Flexible link dc motor system controlled by a two stage GPI controller design.

TABLE I

DATA OF THE MOTOR-GEAR SET

J (kgm^2)	ν (Nms)	k ($\frac{Nm}{V}$)	n	A ($\frac{N}{Vkgs}$)	B($\frac{Ns}{kgm}$)
6.87×10^{-5}	1.041×10^{-3}	0.21	50	61.14	15.15

where the parameter p represents the common location of all the closed loop poles.

IV. EXPERIMENTATION

In this section the experimental platform and the experiment design are briefly explained. The identification and control method previously described are applied here to a one degree of freedom flexible robot.

A. Experimental platform description

Fig. 3 depicts a picture of the experimental platform constituted by a three legged metallic structure to support an Harmonic Drive mini servo DC motor RH-8D-6006-E036AL-SP(N) which has a reduction relation characterized by $n = 50$. The frame makes the stable and free rotation of the motor around the vertical axis of the platform possible. The motor shaft is capable of turning either right or left around the Z axis. A servo amplifier is used to supply voltage to the DC motor. This amplifier accepts control inputs from the computer in the range of $[-10, 10]$ (V). A carbon fiber flexible beam with a diameter of 3 (mm), a length of 500 (mm) and a stiffness of 1.6 (Nm) is embedded in the motor shaft. At the other end of the beam there is a load in the shape of a disc with a diameter of 90 (mm). The load freely rotates with respect to its vertical axis. This means that the torque produced by the load inertia does not influence the tip of the beam. The load floats on the surface of an air table, so the gravity effect and the friction of the load with the surface of the table are canceled. The sensor system is integrated by an encoder embedded in the motor which allows us to know the motor position with a precision of 7×10^{-5} (rad) and a pair of strain gauges with a gauge factor of 2.16 and a resistance of 120.2 Ω. The sample time for the processing of the signals was of 2×10^{-3} (s).

B. Results

The values of the motor parameters are shown in Table I. These numerical values are used to implement the inner loop controller on the actual physical platform.

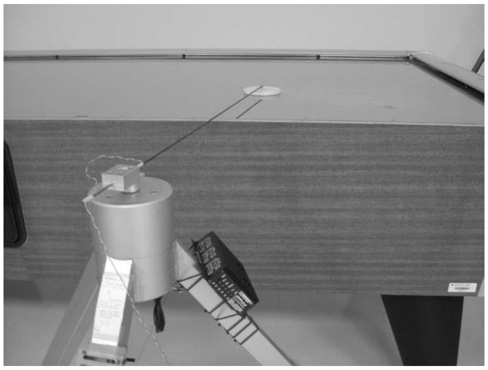

Fig. 3. Flexible robot prototype.

The system should be as fast as possible, but care should be taken of possible saturations of the motor which occur at 10 (V). The poles can be located in a reasonable location of the negative real axis. If closed loop poles are located in, say, -95, the transfer function of the controller from (35), which depends on the location of the poles in closed loop of the inner loop and the values of the motor parameters A and B as is shown in (37) and (38) respectively, results in the following expression,

$$\frac{u_c - u_c^*}{\theta_m^* - \theta_m} = \frac{798s^2 + 5.6 \cdot 10^4 s + 1.3 \cdot 10^6}{s(s+365)} \tag{39}$$

The feed-forward term in (36), which depends on the values of the motor parameters is computed in accordance with,

$$u_c^* = 0.02\dddot{\theta}_m^*(t) + 0.25\ddot{\theta}_m^*(t) \tag{40}$$

The parameter used for the flexible arm are $c = 1.6$ (Nm) and $L = 0.5$ (m), the mass, "m" being unknown. The poles for the outer loop design are located at -10 in the real axis, in order to assure that the outer loop is slower than the inner loop, with a natural frequency of the bar given by an initial, arbitrary, estimate of $\omega_{0i} = 9$ (rad/s). The transfer function of the controller (28), which depends on the location of the closed loop poles of the outer loop and the natural frequency of the bar as1 is shown in (31) and (32) respectively, is given by the following expression,

$$\frac{\theta_m - \theta_m^*}{\theta_t^* - \theta_t} = \frac{2.7s - 17.7}{s+30} \tag{41}$$

The open loop reference control input from (23) in terms of the initial, arbitrary estimate of ω_{0i} is given by,

$$\theta_m^*(t) = \frac{1}{\omega_{0i}^2}\ddot{\theta}_t^*(t) + \theta_t^*(t) = 12.3 \cdot 10^{-3}\ddot{\theta}_t^*(t) + \theta_t^*(t) \tag{42}$$

The desired reference trajectory used for the tracking problem of the flexible arm is specified as a Bezier's eighth order polynomial. The on-line algebraic estimation of the unknown parameter ω_0, in accordance with (11), (15) and (16), is carried out in $\Delta = 0.5$ seconds (see Fig. 4). At the end of this small time interval, the controller is immediately replaced, or updated, with the accurate parameter estimate, given by $\omega_{0e} = 15.2$ (rad/s). Fig. 5 depicts the update of the controller and how, after the update of the controller (after the dashed line) the tip position θ_t tracks the desired trajectory θ_t^* with no steady state error. The corresponding transfer function of this new controller is then found to be,

$$\frac{\theta_m - \theta_m^*}{\theta_t^* - \theta_t} = \frac{0.3s - 25.7}{s+30} \tag{43}$$

The open loop reference control input $\theta_m^*(t)$ from (23) in terms of the new estimate ω_{0e} is given by:

$$\theta_m^*(t) = \frac{1}{\omega_{0e}^2}\ddot{\theta}_t^*(t) + \theta_t^*(t) = 4.3 \cdot 10^{-3}\ddot{\theta}_t^*(t) + \theta_t^*(t) \tag{44}$$

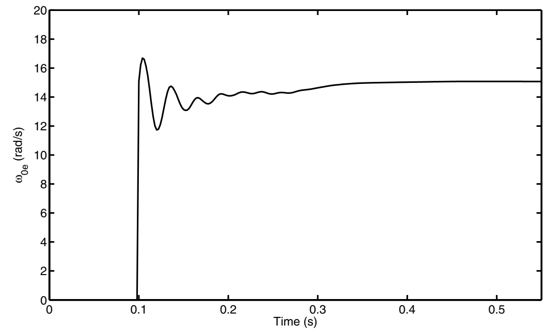

Fig. 4. On-line estimation of ω_0.

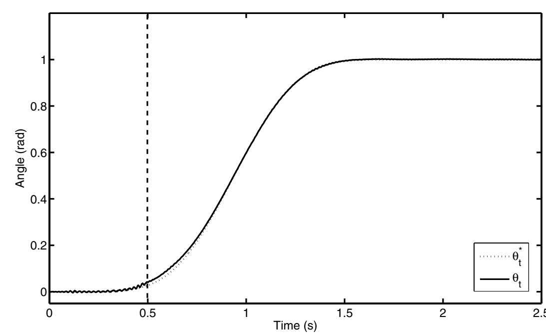

Fig. 5. Trajectory tracking with on-line estimate ω_0.

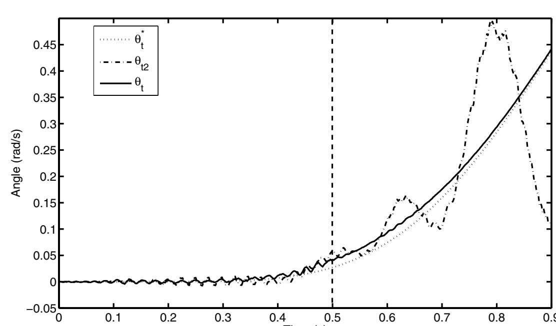

Fig. 6. Comparison between trajectory trackings.

Fig. 6 shows the commanded trajectory θ_t^* and the response of the closed loop system θ_{t2} when the feedback controller uses a wrong, arbitrary estimate of ω_0 given by the specification: $\omega_{0i} = 9$ (rad/s). The controlled arm response clearly shows a highly oscillatory response. Nevertheless, the controller tries to track the trajectory and locate the arm in the required steady state position. However, with the adaptive control, the tip position θ_t does not follows the commanded trajectory until time 0.5 (s) because of the initial, value of the natural frequency of the bar ω_{0i}. After 0.5 (s),when the feedback controller incorporates the on-line estimate ω_{0e} the error produced up to 0.5 (s) rapidly converges to zero and thus a quite precise tracking of the desired trajectory is achieved, Fig. 6, (see also Fig. 5).

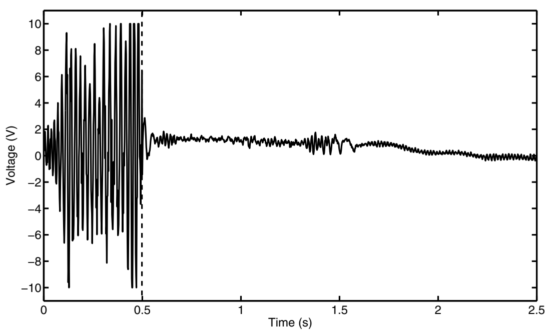

Fig. 7. Control input voltage to the DC motor.

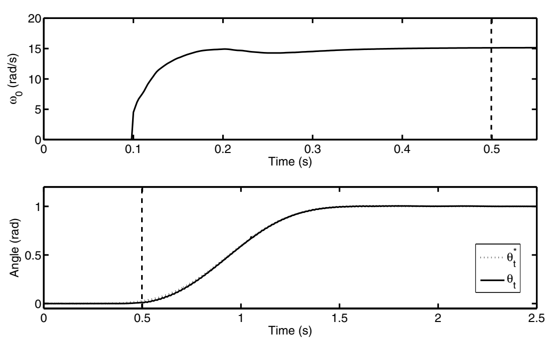

Fig. 8. Results of the second experiment.

Fig. 7 depicts the input control voltage to the DC motor. It may be noted that the controller does not work well and saturates the amplifier at [10,-10] (V) before 0.5 (s). This is the time when the controller is updated with the on-line estimate ω_{0e}. After this time, the controller rapidly eliminates the tracking error, and therefore the input control voltage is smoothed and does not saturate the amplifier.

A new experiment is carried out to validate the previous results. Now, a new initial, arbitrary estimate natural frequency $\omega_{0i} = 20$ (rad/s) is introduced. The real natural frequency is estimated with the algebraic method in approximately 0.5 seconds and the value estimated is $\omega_{0e} = 15.2$ (rad/s) (see the first picture of Fig. 8). The controller is updated at this time with this new accurate parameter estimate as took place in the experiment above. The second picture of Fig. 8 depicts the good trajectory tracking. These results are similar to those obtained in the previous experiment with $\omega_{0i} = 9$ (rad/s).

V. CONCLUSIONS

A fast on-line closed loop continuous-time estimator of natural frequency of a flexible robot is proposed in connection with a two stage GPI controller design scheme. This methodology only requires the measurement of the angular position of the motor and the coupling torque. Among the advantages of this technique we find that: *i)* A direct estimation of the parameters without an undesired translation between discrete and continuous time domains is achieved. *ii)* Independent statistical hypothesis of the signal is not required, so closed loop operation is easier to implement. Finally, after the identification has been carried out we are capable of controlling the system via a GPI feedback control scheme. This methodology is well-suited to the important problem of control degradation in flexible arms as consequence of payload changes.

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