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Stabilization of a Small Unmanned Aerial Vehicle Model without Velocity Measurement

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Abstract— **This paper presents a method to design guidance and control laws for small Vertical Take Off and Landing Unmanned Aerial Vehicles when no measurement of linear velocity nor angular velocity is available. The control strategy is based on the introduction of virtual states in the state equation of the system and allows the design of stabilizing feedback controllers without using any observer. Simulation results are provided for a six degrees of freedom model of a small rotorcraft-based Unmanned Aerial Vehicle.**

I. Introduction

Miniature rotorcraft-based Unmanned Aerial Vehicles (UAVs) have received a growing interest in both industrial and academic research. Thanks to their hover capability, they are prone to be useful for many civil missions such as video supervision of road traffic, surveillance of urban districts or building inspection for maintenance.

Design of guidance navigation and control algorithms for the autonomous flight of small rotorcraft-based UAVs is a challenging research area because of their nonlinear dynamics and their high sensitivity to aerodynamic perturbations. Various control strategies such as backstepping [12], adaptive backstepping [14], nonlinear model predictive control [11] or the combination of adaptive and model predictive control [1] have been successfully applied to UAV models. Nevertheless most of them require full state knowledge for feedback control design.
For robotic systems it may be useful, for cost or payload reasons, to limit the number of embedded sensors. Therefore measurement of the full state may not be available. Classical solution to overcome consists in using observer. Such a solution was adopted in [6] where the problem of trajectory tracking for a planar Vertical Take Off and Landing (VTOL) aircraft with only positions and attitude angle measurement is treated by designing a full-order observer. Another approach that can be used to avoid computational burden or complexity due to the introduction of an observer is partial state feedback. Early work on partial state feedback has been done in the context of rigid-link robot manipulators when no velocity measurement is available. In [3] the velocity measurement is replaced by a velocity-related signal generated by a linear filter based only on link position measurement. An extension of this work can be found in [5] where a nonlinear filter is used. The same method has been applied to solve the problem of attitude tracking of rigid bodies with unknown inertia. A linear filter is employed in [17] to generate a velocity-related signal. In this work, a kinematic representation using modified Rodrigues parameters has been chosen. In [4], a unit quaternion based representation is adopted and a nonlinear filter generates a signal replacing the angular velocity measurement in the feedback controller. A unit quaternion representation is also used in [15] where a linear feedback controller depending on an estimation error quaternion is designed to solve the problem of a rigid spacecraft attitude control.
As can be seen in the simulation or experimental results provided in some of the aforementioned works, oscillating closed loop behavior can be considered as the main drawback of partial state feedback strategies.

In this paper we deal with the problem of guidance and control of a six degrees of freedom UAV model when no measurement of the linear velocity nor of the angular velocity is available. The method we present in this paper is based on the introduction of virtual states in the state equation of the system; no observer design is required. In addition, the closed loop oscillations are sensibly attenuated by the proposed approach. Contrary to the previous works, the kinematic representation we use exploits the $SO(3)$ group and its manifold.

Section 2 presents the system dynamics of a VTOL UAV and the cascaded structure of the controller. The design of the position controller is detailed in section 3 whereas the attitude controller is presented in section 4. Stability analysis of the closed loop system is given in section 5 and simulations results are provided in section 6. Concluding remarks are finally given in the last part of this paper.

II. UAV model and control strategy

A. VTOL UAV model

The VTOL UAV model is represented by a rigid body of mass m and of tensor of inertia $I = \text{diag}(I_1, I_2, I_3)$ with I_1, I_2 and I_3 strictly positive. We define an inertial reference frame $(\mathcal{I})$ associated with the vector basis $(e_1,\ e_2,\ e_3)$ and a body frame $(\mathcal{B})$ attached to the UAV and associated with the vector basis $(e_1^b,\ e_2^b,\ e_3^b)$ (see Fig. 1). The position and the linear velocity of the UAV in $(\mathcal{I})$ are respectively denoted

$\xi = [x\ y\ z]^T$ and $v = [v_x\ v_y\ v_z]^T$. The orientation of the UAV is given by the orientation matrix $R \in SO(3)$ from $(\mathcal{I})$ to $(\mathcal{B})$, usually parameterized by Euler's pseudo angles ψ, θ, ϕ (yaw, pitch, roll):

$$R = \begin{bmatrix} c_\theta\, c_\psi & s_\phi\, s_\theta\, c_\psi - c_\phi\, s_\psi & c_\phi\, s_\theta\, c_\psi + s_\phi\, s_\psi \\ c_\theta\, s_\psi & s_\phi\, s_\theta\, s_\psi + c_\phi\, c_\psi & c_\phi\, s_\theta\, s_\psi - s_\phi\, c_\psi \\ -s_\theta & s_\phi\, c_\theta & c_\phi\, c_\theta \end{bmatrix} \quad (1)$$

with the trigonometric shorthand notations $c_\alpha = \cos(\alpha)$ and $s_\alpha = \sin(\alpha)$, $\forall \alpha \in \mathbb{R}$. Let $\Omega = [\omega_p\ \omega_q\ \omega_r]^T$ be the angular velocity of the UAV defined in $(\mathcal{B})$.

We assume that a translational force F and a control torque Γ are applied to the UAV. The translational force combines thrust, lift, drag and gravity components. In quasi-stationary flight we can reasonably assume that the aerodynamic forces are always in direction e_3^b, since the lift force predominates the other components [8]. The gravity component mge_3 can be separated from the combined aerodynamic forces and the dynamics of the VTOL UAV can be written as[1]:

$$\begin{cases} \dot{\xi} = v \\ m\dot{v} = -\mathcal{T}\, Re_3 + mge_3 \\ \dot{R} = R\,\Omega_\times \\ I\dot{\Omega} = -\Omega \times I\Omega + \Gamma \end{cases} \quad (2)$$

where the inputs are the scalar $\mathcal{T} \in \mathbb{R}$ representing the magnitude of the external forces applied in direction e_3^b, and the control torque $\Gamma = [\Gamma_1\ \Gamma_2\ \Gamma_3]^T$ defined in $(\mathcal{B})$.

B. Control strategy

Designing a controller for the UAV model (2) can be realized by a classical backstepping approach applied to the whole dynamical system. In that case, the input vector $\frac{-\mathcal{T}}{m} Re_3$ must be dynamically extended [7], [13]. To avoid such a dynamical extension, we take advantage of the cascaded structure of the system to design separate controllers for the two following connected subsystems: the translational dynamics and the orientation dynamics. Such a hierarchical control strategy is frequently used in guidance and control algorithms [14], [16]. It leads to time scale separation between the translational dynamics and the orientation dynamics. The position controller computes the input $\mathcal{T}$ and the desired orientation R^d of the UAV. The attitude controller determines the control torque Γ such that the orientation R converges to the desired one R^d.

To get position and linear velocity measurements, the vehicle may be equipped with a GPS. Note however that the use of a GPS can be useful in outdoor flight and in a free environment. In constrained environment, the preferred approach consists in using a vision sensor to estimate the position of the vehicle with respect to the environment, and therefore no measurement of the linear velocity v is available.

We also assume that the UAV is equipped with inclinometers and a compass to provide orientation measurements. We assume that no measurement of the angular velocity Ω is available.

Since the use of observers may increase complexity and introduce computation delay, especially for the orientation dynamics, we will design feedback control laws without using any observer.

[1]For any vector $a \in \mathbb{R}^3$ we denote $a_\times$ its associated skew symmetric matrix verifying $\forall b \in \mathbb{R}^3, a_\times b = a \times b$, for the vector cross-product $\times$.

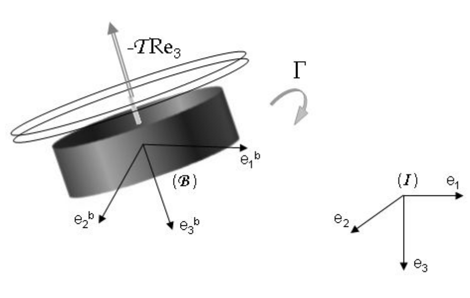

Fig. 1. Reference frames

III. Position controller

Let us consider the translational dynamics. Assume that $R = R^d$. We introduce two virtual states $q,\ w \in \mathbb{R}^3$ and a virtual control $\delta \in \mathbb{R}^3$ such that:

$$\begin{cases} \dot{\xi} = v \\ \dot{v} = -\dfrac{\mathcal{T}}{m} R^d e_3 + ge_3 \\ \dot{q} = -w \\ \dot{w} = \delta \end{cases} \quad (3)$$

Lemma 1

Consider the system dynamics (3). The following control vector

$$\mathcal{T} R^d e_3 = mge_3 + \frac{m}{k_v}\left\{k_x\ \xi + k_1(\xi - q) + k_2(\xi - q + w)\right\} \quad (4)$$

along with the virtual control

$$\delta = -\frac{1}{k_2}\left\{k_2 w + k_1(\xi - q) + k_2(w + \xi - q)\right\} \quad (5)$$

where k_x, k_v, k_1 and k_2 are strictly positive gains, exponentially stabilizes the translational dynamics (3).

Proof

Let us introduce the following coefficients:

$$\alpha = -\frac{k_x + k_1 + k_2}{k_v} \qquad \beta = \frac{k_1 + k_2}{k_v}$$
$$\gamma = -\frac{k_2}{k_v} \qquad \epsilon = \frac{k_1 + k_2}{k_2} \quad (6)$$

The closed loop system can be written in matrix form

$$\begin{bmatrix} \dot{\xi} \\ \dot{v} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha & 0 & \beta & \gamma \\ 0 & 0 & 0 & -1 \\ -\epsilon & 0 & \epsilon & -2 \end{bmatrix}}_{\Phi} \begin{bmatrix} \xi \\ v \\ q \\ w \end{bmatrix} \quad (7)$$

Let us show that Φ is Hurwitz. Its characteristic polynomial is given by:

$$P(s) = s^4 + 2s^3 + (\epsilon - \alpha)s^2 + (\epsilon\gamma - 2\alpha)s - \epsilon(\alpha + \beta) \quad (8)$$

Using Routh's criterion, Φ is Hurwitz if and only if the three following conditions simultaneously hold:

$$\epsilon - \alpha - \frac{\epsilon\gamma - 2\alpha}{2} > 0 \quad (9)$$

$$\epsilon\gamma - 2\alpha + 4\frac{\alpha + \beta}{2 - \gamma} > 0 \quad (10)$$

$$-\epsilon(\alpha + \beta) > 0 \quad (11)$$

Using $\epsilon\gamma = -\beta$ and writing these conditions in terms of gains k_x, k_v, k_1 and k_2, it yields:

$$\frac{k_1 + k_2}{k_2} + \frac{1}{2}\frac{k_1 + k_2}{k_v} > 0 \quad (12)$$

$$-\frac{k_1 + k_2}{k_v} + 2\frac{k_x + k_1 + k_2}{k_v} + 4\frac{-\frac{k_x}{kv}}{2 + \frac{k_2}{k_v}} > 0 \quad (13)$$

$$(-\frac{k_1 + k_2}{k_2})(-\frac{k_x}{k_v}) > 0 \quad (14)$$

Since k_x, k_v, k_1 and k_2 are strictly positive, conditions (12) and (14) are immediately satisfied. Condition (13) can be restated as:

$$\frac{2k_x + k_1 + k_2}{k_v} > 4\frac{k_x}{2k_v + k_2}$$

Using the positivity of the gains, it becomes:

$$2k_v(k_1 + k_2) + k_2(2k_x + k_1 + k_2) > 0$$

which is always satisfied, each term being strictly positive.

We can then conclude that Φ is Hurwitz and therefore, from (7), that the control (4) along with the virtual control (5) exponentially stabilizes the translational dynamics (3). ■

Remark 1

The control input $\mathcal{T}$ can be directly computed by taking the norm of the right hand side of (4):

$$\mathcal{T} = \left\| mge_3 + \frac{m}{k_v}\left\{k_x\xi + k_1(\xi - q) + k_2(\xi - q + w)\right\}\right\| \quad (15)$$

The desired orientation R^d can be obtained using (1) and

$$R^d e_3 = \frac{1}{\mathcal{T}}(mge_3 + \frac{m}{k_v}\left\{k_x\xi + k_1(\xi - q) + k_2(\xi - q + w)\right\}) \quad (16)$$

and solving for (ψ, θ, ϕ) for a given specified yaw trajectory $\psi^d(t)$ [9].

To ensure that equation (16) is well defined, let us introduce the following Lyapunov function for the translational dynamics:

$$\begin{aligned} \mathcal{S} =& \frac{1}{2}k_x\|\xi\|^2 + \frac{1}{2}k_v\|v\|^2 + \frac{1}{2}k_1\|\xi - q\|^2 \\ &+ \frac{1}{2}k_2\|\xi - q + w\|^2 \end{aligned} \quad (17)$$

We also introduce $k_{min} = \min(k_x, k_v, k_1, k_2)$ and $k_{max} = \max(k_x, k_1, k_2)$.

Lemma 2

For any initial condition $\xi(0)$, $v(0)$, with $q(0) = \xi(0)$ and $w(0) = 0$ verifying

$$\mathcal{S}(0) < \frac{1}{18}\frac{g^2\, k_v^2\, k_{min}}{k_{max}^2} \quad (18)$$

the input $\mathcal{T}$ is strictly positive.

Proof

From equation (15) and triangular inequality we get:

$$\mathcal{T} \geq mg - \frac{m}{k_v}k_{max}\left\{\|\xi\| + \|\xi - q\| + \|\xi - q + w\|\right\}$$

$$\begin{aligned} \mathcal{T} \geq& mg - 3\frac{m}{k_v}k_{max}\left\{\ \|\xi\|^2 + \|v\|^2 + \|\xi - q\|^2 \right. \\ &\left. + \|\xi - q + w\|^2\ \right\}^{\frac{1}{2}} \end{aligned} \quad (19)$$

Using the definitions of $\mathcal{S}$ and k_{min}, we have:

$$\frac{1}{2}k_{min}\left\{\ \|\xi\|^2 + \|v\|^2 + \|\xi - q\|^2 + \|\xi - q + w\|^2\ \right\} \leq \mathcal{S} \quad (20)$$

It can be easily verified that the time derivative of S along the trajectories of the closed loop translational dynamics is negative. Therefore S is decreasing and we have $\mathcal{S} \leq \mathcal{S}(0)$, where, taking $q(0) = \xi(0)$ and $w(0) = 0$, the initial value $\mathcal{S}(0)$ is defined by $\mathcal{S}(0) = \frac{1}{2}k_x\|\xi(0)\|^2 + \frac{1}{2}k_v\|v(0)\|^2$. Using (19) along with (20) and $\mathcal{S} \leq \mathcal{S}(0)$, we get:

$$\mathcal{T} \geq mg - 3\frac{m}{k_v}k_{max}\sqrt{\frac{2\mathcal{S}(0)}{k_{min}}} \quad (21)$$

Using condition (18), we finally obtain $\mathcal{T} > 0$. ■

Therefore, the relation (16) which is used to compute R^d is well defined for any initial condition $\xi(0)$, $v(0)$, with $q(0) = \xi(0)$ and $w(0) = 0$ verifying (18).

Due to the position controller we developed, the closed-loop translational dynamics is exponentially stable for $R = R^d$. However, since the orientation R will not converge instantaneously to the desired value R^d, an orientation error term is introduced in the translational dynamics:

$$m\dot{v} = -\mathcal{T}R^d e_3 + mge_3 - \mathcal{T}(R - R^d)e_3 \quad (22)$$

Therefore we have to design an attitude controller allowing at least asymptotic convergence of R to R^d.

IV. Attitude Controller

Similarly to the translational dynamics, we introduce two virtual states $Q \in SO(3)$, $W \in \mathbb{R}^3$ and a virtual control $\Delta \in \mathbb{R}^3$ for the orientation dynamics, such that:

$$\begin{cases} \dot{R} = R\,\Omega_\times \\ I\dot{\Omega} = -\Omega \times I\Omega + \Gamma \\ \dot{Q} = -Q\,W_\times \\ \dot{W} = \Delta \end{cases} \quad (23)$$

For a given desired orientation R^d we define

$$\tilde{R} = (R^d)^T R \tag{24}$$

$$\tilde{Q} = Q^T \tilde{R} \tag{25}$$

Due to the cascaded structure of the controller, the attitude controller will have to be tuned so that the deviation $\tilde{R}$ converges to the identity matrix I_d of $\mathbb{R}^{3\times 3}$ faster than the position stabilization. Therefore, we assume for control design that the desired orientation can be considered constant, i.e. $\dot{R}^d = 0$ [2], and that the desired angular velocity is zero, i.e. $\Omega^d = 0$.
Using (24) and (25), we rewrite (23) as:

$$\begin{cases} \dot{\tilde{R}} = \tilde{R}\,\Omega_\times \\ I\dot{\Omega} = -\Omega \times I\Omega + \Gamma \\ \dot{\tilde{Q}} = W_\times \tilde{Q} + \tilde{Q}\Omega_\times \\ \dot{W} = \Delta \end{cases} \tag{26}$$

We denote respectively $\boldsymbol{P}_a(A) = \frac{A - A^T}{2}$ and $\boldsymbol{P}_s(A) = \frac{A + A^T}{2}$ the anti symmetric part and the symmetric part of a given matrix $A \in \mathbb{R}^{n\times n}$. We define by $\boldsymbol{V}$ the operator from $SO(3)$ to $\mathbb{R}^3$ verifying $\forall b \in \mathbb{R}^3, \boldsymbol{V}(b_\times) = b$ and $\forall B \in SO(3), \boldsymbol{V}(B)_\times = B$.

Let us introduce the following Lyapunov function:

$$\begin{aligned} \mathcal{L} =& \frac{1}{2}k_r \mathrm{tr}(I_d - \tilde{R}) + \frac{1}{2}k_\omega \Omega^T I \Omega + \frac{1}{2}k_3 \mathrm{tr}(I_d - \tilde{Q}) \\ &+ \frac{1}{2}k_4 \mathrm{tr}\left\{(W_\times + \boldsymbol{P}_a(\tilde{Q}))^T (W_\times + \boldsymbol{P}_a(\tilde{Q}))\right\} \end{aligned} \tag{27}$$

where k_r, k_ω, k_3 and k_4 are strictly positive scalars. We denote $\mathcal{L}(0)$ the initial value of $\mathcal{L}$.

Lemma 3

$$\begin{aligned} \Gamma = \frac{1}{k_\omega}\{ &-k_r \boldsymbol{V}(\boldsymbol{P}_a(\tilde{R})) - k_3 \boldsymbol{V}(\boldsymbol{P}_a(\tilde{Q})) \\ &+ k_4 \boldsymbol{V}(\boldsymbol{P}_a(M)) + k_4 \boldsymbol{V}(\boldsymbol{P}_a(N)) \} \end{aligned} \tag{28}$$

and the virtual control

$$\begin{aligned} \Delta = -\frac{1}{k_4}\boldsymbol{V}(&\frac{1}{2}k_3 \boldsymbol{P}_a(\tilde{Q}) + \frac{1}{2}k_4(W_\times \tilde{Q} + \tilde{Q}^T W_\times) \\ &+ \frac{1}{2}k_5(W_\times + \boldsymbol{P}_a(\tilde{Q}))) \end{aligned} \tag{29}$$

where

$$M = (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T \tilde{Q} \tag{30}$$

$$N = \tilde{Q}^T (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T \tag{31}$$

and k_r, k_ω, k_3, k_4 and k_5 are strictly positive gains with

$$k_r < k_3 \tag{32}$$

Consider the control Lyapunov function candidate (27). Then, for any initial condition $\tilde{R}(0)$, $\Omega(0)$, with $Q(0) = \tilde{R}(0)$ and $W(0) = 0$, such that

$$\mathcal{L}(0) < 2k_r \tag{33}$$

the control torque (28) along with the virtual control (29) asymptotically stabilizes the orientation dynamics (26).

Proof
Consider the Lyapunov function $\mathcal{L}$ defined by (27). It's time derivative along the trajectories of (26) is given by:

$$\begin{aligned} \dot{\mathcal{L}} =& -\frac{1}{2}k_r \mathrm{tr}(\dot{\tilde{R}}) + k_\omega \Omega^T\{-\Omega \times I\Omega + \Gamma\} - \frac{1}{2}k_3 \mathrm{tr}(\dot{\tilde{Q}}) \\ &+ k_4 \mathrm{tr}\left\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T (\Delta_\times + \overbrace{\boldsymbol{P}_a(\tilde{Q})}^{\cdot}) \right\} \end{aligned}$$

where

$$\overbrace{\boldsymbol{P}_a(\tilde{Q})}^{\cdot} = \frac{1}{2}\left\{ W_\times \tilde{Q} + \tilde{Q}\Omega_\times + \tilde{Q}^T W_\times + \Omega_\times \tilde{Q}^T \right\} \tag{34}$$

Using (34) and the fact that $\Omega^T(\Omega \times I\Omega) = 0$, it yields:

$$\begin{aligned} \dot{\mathcal{L}} =& -\frac{1}{2}k_r \mathrm{tr}(\tilde{R}\Omega_\times) + k_\omega \Omega^T \Gamma - \frac{1}{2}k_3 \mathrm{tr}(W_\times \tilde{Q} + \tilde{Q}\Omega_\times) \\ &+ k_4 \mathrm{tr}\Big\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T (\Delta_\times + \frac{1}{2}\Big\{ W_\times \tilde{Q} + \tilde{Q}\Omega_\times \\ &+ \tilde{Q}^T W_\times + \Omega_\times \tilde{Q}^T \ \}) \ \} \end{aligned}$$

Recalling that for any pair of matrices A, $B \in \mathbb{R}^{n\times n}$, $\mathrm{tr}(\boldsymbol{P}_a(A)\boldsymbol{P}_s(B)) = 0$, we get:

$$\begin{aligned} \dot{\mathcal{L}} =& -\frac{1}{2}k_r \mathrm{tr}(\Omega_\times \boldsymbol{P}_a(\tilde{R})) + k_\omega \Omega^T \Gamma - \frac{1}{2}k_3 \mathrm{tr}(W_\times \boldsymbol{P}_a(\tilde{Q})) \\ &- \frac{1}{2}k_3 \mathrm{tr}(\Omega_\times \boldsymbol{P}_a(\tilde{Q})) \\ &+ k_4 \mathrm{tr}\Big\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T (\Delta_\times + \frac{1}{2}\Big\{ W_\times \tilde{Q} + \tilde{Q}\Omega_\times \\ &+ \tilde{Q}^T W_\times + \Omega_\times \tilde{Q}^T \ \}) \ \} \end{aligned}$$

Using the fact that for two given antisymmetric matrices $A_a, B_a \in \mathbb{R}^{n\times n}$ we have $\frac{1}{2}\mathrm{tr}(A_a\, B_a) = \boldsymbol{V}(A_a^T)^T \boldsymbol{V}(B_a)$, we obtain:

$$\begin{aligned} \dot{\mathcal{L}} =& \Omega^T \left\{ k_r \boldsymbol{V}(\boldsymbol{P}_a(\tilde{R})) + k_\omega \Gamma + k_3 \boldsymbol{V}(\boldsymbol{P}_a(\tilde{Q})) \right\} \\ &- \frac{1}{2}k_3 \mathrm{tr}(W_\times \boldsymbol{P}_a(\tilde{Q})) + \frac{1}{2}k_4 \mathrm{tr}\left\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T \tilde{Q}\Omega_\times \right\} \\ &+ \frac{1}{2}k_4 \mathrm{tr}\left\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T \Omega_\times \tilde{Q}^T \right\} \\ &+ k_4 \mathrm{tr}\left\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T (\Delta_\times + \frac{1}{2}(W_\times \tilde{Q} + \tilde{Q}^T W_\times)) \right\} \end{aligned}$$

Using (30) we have:

$$\begin{aligned} \frac{1}{2}k_4 \mathrm{tr}\left\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T \tilde{Q}\Omega_\times \right\} &= \frac{1}{2}k_4 \mathrm{tr}\{M\Omega_\times\} \\ &= -k_4 \Omega^T \boldsymbol{V}(\boldsymbol{P}_a(M)) \end{aligned}$$

In the same way, we use (31) to get:

$$\frac{1}{2}k_4 \mathrm{tr}\left\{ (W_\times + \boldsymbol{P}_a(\tilde{Q}))^T \Omega_\times \tilde{Q}^T \right\} = -k_4 \Omega^T \boldsymbol{V}(\boldsymbol{P}_a(N))$$

Therefore, the time derivative of $\mathcal{L}$ can be simplified:

$$\begin{aligned}\dot{\mathcal{L}} = \Omega^T \{ & k_r V(P_a(\tilde{R})) + k_\omega \Gamma + k_3 V(P_a(\tilde{Q})) \\ & - k_4 V(P_a(M)) - k_4 V(P_a(N)) \} - \frac{1}{2}k_3 \mathrm{tr}(W_\times P_a(\tilde{Q})) \\ & + k_4 \mathrm{tr}\left\{ (W_\times + P_a(\tilde{Q}))^T (\Delta_\times + \frac{1}{2}(W_\times \tilde{Q} + \tilde{Q}^T W_\times)) \right\}\end{aligned}$$

Choosing Γ according to (28) leads to:

$$\begin{aligned}\dot{\mathcal{L}} = & -\frac{1}{2}k_3 \mathrm{tr}(W_\times P_a(\tilde{Q})) \\ & + k_4 \mathrm{tr}\left\{ (W_\times + P_a(\tilde{Q}))^T (\Delta_\times + \frac{1}{2}(W_\times \tilde{Q} + \tilde{Q}^T W_\times)) \right\}\end{aligned}$$

As $W_\times^T = -W_\times$, and introducing $P_a(\tilde{Q})$ in the first term:

$$\begin{aligned}\dot{\mathcal{L}} = & \frac{1}{2}k_3 \mathrm{tr}((W_\times - P_a(\tilde{Q}) + P_a(\tilde{Q}))^T P_a(\tilde{Q})) \\ & + k_4 \mathrm{tr}\left\{ (W_\times + P_a(\tilde{Q}))^T (\Delta_\times + \frac{1}{2}(W_\times \tilde{Q} + \tilde{Q}^T W_\times)) \right\} \\ \dot{\mathcal{L}} = & -\frac{1}{2}k_3 \mathrm{tr}(P_a(\tilde{Q})^T P_a(\tilde{Q})) \\ & + \mathrm{tr}\{ (W_\times + P_a(\tilde{Q}))^T (\frac{1}{2}k_3 P_a(\tilde{Q}) + k_4 \Delta_\times \\ & + \frac{1}{2}k_4 (W_\times \tilde{Q} + \tilde{Q}^T W_\times)) \}\end{aligned}$$

Taking Δ as defined in (29), one has:

$$\begin{aligned}\dot{\mathcal{L}} = & -\frac{1}{2}k_3 \mathrm{tr}\left\{ P_a(\tilde{Q})^T P_a(\tilde{Q}) \right\} \\ & -\frac{1}{2}k_5 \mathrm{tr}\left\{ (W_\times + P_a(\tilde{Q}))^T (W_\times + P_a(\tilde{Q})) \right\}\end{aligned}$$

Using the fact that for any antisymmetric matrix $A_a \in \mathbb{R}^{n\times n}$, $\frac{1}{2}\mathrm{tr}(A_a^T A_a) = \|V(A_a)\|^2$, we finally have:

$$\dot{\mathcal{L}} = -k_3 \left\| V(P_a(\tilde{Q})) \right\|^2 - k_5 \left\| V(W_\times + P_a(\tilde{Q})) \right\|^2 \quad (35)$$

ensuring that $\mathcal{L}$ is strictly decreasing until $P_a(\tilde{Q}) \to 0$ and $W_\times \to -P_a(\tilde{Q})$, i.e. $W_\times \to 0$.
Denote by $(\gamma_{\tilde{Q}}, \mathbf{n}_{\tilde{Q}})$ the angle-axis coordinates of $\tilde{Q}$. One has:

$$k_3(1 - \cos(\gamma_{\tilde{Q}})) = \frac{1}{2}k_3 \mathrm{tr}(I_d - \tilde{Q}) \le \mathcal{L}$$

Since $\mathcal{L}$ is decreasing, we have $\mathcal{L} \le \mathcal{L}(0)$. Using (33) it yields:

$$k_3(1 - \cos(\gamma_{\tilde{Q}})) \le \mathcal{L} \le \mathcal{L}(0) < 2k_r$$

Using (32), we get:

$$1 - \cos(\gamma_{\tilde{Q}}) < 2\frac{k_r}{k_3} < 2 \quad (36)$$

From $P_a(\tilde{Q}) \to 0$, we have $\gamma_{\tilde{Q}} = 0$ or $\gamma_{\tilde{Q}} = \pm\pi$. The second possibility is excluded by (36). Therefore we have $\tilde{Q} \to I_d$. By (25), it yields $\tilde{R} \to Q$. By continuity and using La Salle's principle, we get $\dot{\tilde{R}} \to \dot{Q}$. Using the first equation of (26) and the third equation of (23) , one has $\tilde{R}\Omega_\times \to -QW_\times$. Since $\tilde{R}$ is orthogonal, we get $\Omega_\times \to -\tilde{R}^T Q W_\times$. Using $W_\times \to 0$ it yields $\Omega_\times \to 0$ and then $\Omega \to 0$. Therefore, using the first equation of (26), we get $\dot{\tilde{R}} \to 0$. By continuity $\dot{\Omega} \to 0$ and then, by the second equation of system (26), $\Gamma \to 0$. Knowing that $P_a(\tilde{Q})$ and $W_\times$ converge to zero, one can ensure that, respectively from (30) and (31), M and N converge to zero. Combining the above discussion with the fact that $\Gamma \to 0$, equation (28) ensures that $P_a(\tilde{R}) \to 0$. Similarly to the previous analysis on $\tilde{Q}$, let us denote $(\gamma_{\tilde{R}}, \mathbf{n}_{\tilde{R}})$ the angle-axis coordinates of $\tilde{R}$. One has:

$$k_r(1 - \cos(\gamma_{\tilde{R}})) = \frac{1}{2}k_r \mathrm{tr}(I_d - \tilde{R}) \le \mathcal{L} \le \mathcal{L}(0) < 2k_r$$

It yields

$$1 - \cos(\gamma_{\tilde{R}}) < 2 \quad (37)$$

From $P_a(\tilde{R}) \to 0$, we have $\gamma_{\tilde{R}} = 0$ or $\gamma_{\tilde{R}} = \pm\pi$. The second possibility is excluded by (37). Therefore, we finally have $\tilde{R} \to I_d$ and $R \to R_d$. ■

V. Stability analysis

We consider the full dynamics of the system along with virtual states and with the orientation error term in the translational dynamics:

$$\left\{ \begin{aligned} \dot{\xi} &= v \\ \dot{v} &= -\frac{\mathcal{T}}{m} R^d e_3 + g e_3 - \frac{\mathcal{T}}{m}(R - R^d)e_3 \\ \dot{q} &= -w \\ \dot{w} &= \delta \\ \dot{\tilde{R}} &= \tilde{R}\Omega_\times \\ I\dot{\Omega} &= -\Omega \times I\Omega + \Gamma \\ \dot{\tilde{Q}} &= W_\times \tilde{Q} + \tilde{Q}\Omega_\times \\ \dot{W} &= \Delta \end{aligned} \right. \quad (38)$$

Proposition 1
Consider the system dynamics (38). Under the conditions (32) and (33), the control laws (4) and (28) along with the virtual controls (5) and (29) asymptotically stabilize the system (38).

Sketch of the proof (see [2] *for the detailed proof)*
By Lemma 3, under the conditions (32) and (33), the closed loop orientation dynamics is asymptotically stable when (28) and (29) are respectively used as control and virtual control. Therefore, the orientation error term $(R - R^d)$ asymptotically converges to zero.
Since, from Lemma 1, the control of the translational dynamics is exponentially stabilizing for $R = R^d$, we can use [10] to conclude that the control of the translational dynamics is asymptotically stabilizing in presence of the orientation error term.
Therefore, the system (38) is asymptotically stable when the control laws (4) and (28) are used along with the virtual control laws (5) and (29). ■

By introducing virtual states, we have been able to design position and attitude control laws which stabilize the VTOL UAV model using no measurement of the linear velocity v nor of the angular velocity Ω.

VI. Simulation results

The VTOL UAV is described by the following parameters: $m = 2.5$ kg, $I_1 = I_2 = 0.13$ kg.m^2 and $I_3 = 0.16$ kg.m^2. The gravitational acceleration is $g = 9.81$ m.s^{-2}. Simulation results are provided for stabilization at hover around the origin starting from the initial condition $\xi_0 = [2\ \ -2\ \ 3]^T$ (m), $[\psi_0\ \theta_0\ \phi_0] = [0\ \ -10\ \ -8]^T$ (deg), $v_0 = 0$ and $\Omega_0 = 0$. The desired yaw ψ^d was chosen to be equal to zero. The values of the gains are: $k_x = 0.2$, $k_v = 3.0$, $k_1 = 0.8$, $k_2 = 0.8$, $k_r = 0.74$, $k_\omega = 3.3$, $k_3 = 12$, $k_4 = 0.25$, $k_5 = 6.1$.
Fig. 2 presents position coordinates and attitude angles. Stabilization of the UAV model is achieved from the given initial condition with satisfying behavior performances. The evolution of the angular deviation terms $\tilde{\phi} = \phi - \phi^d$, $\tilde{\theta} = \theta - \theta^d$ and $\tilde{\psi} = \psi - \psi^d$ are plotted in Fig. 3. It can be verified that these terms converge faster than the closed loop translation, hence validating the time scale separation approach used for the design of the controllers.

VII. Conclusion

We have presented a method to design guidance and control feedback laws for a small rotorcraft-based Unmanned Aerial Vehicle when no measurement of the linear velocity nor of the angular velocity is available.
The proposed approach, based on the introduction of virtual states in the state equation of the system, allows to design a stabilizing feedback controller and provides satisfying behavior of the closed loop system, without using any observer. Closed loop stability using the partial state feedback position and attitude controllers we designed has been proved. To illustrate the performances of the proposed approach, simulation results have been presented.

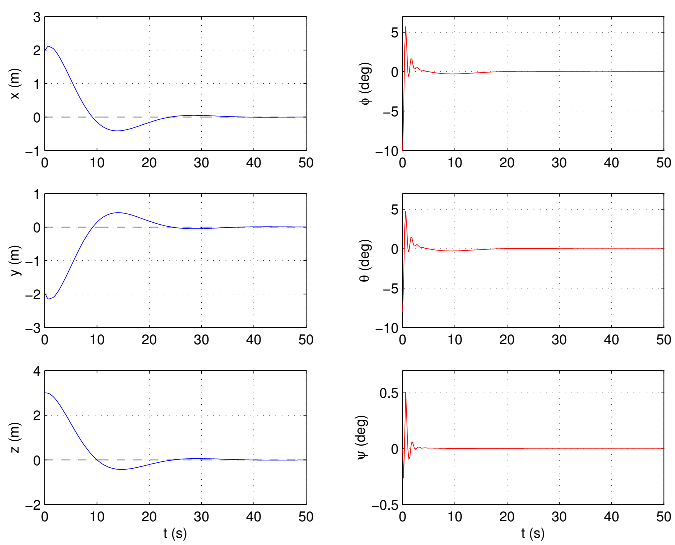

Fig. 2. Position and attitude angles

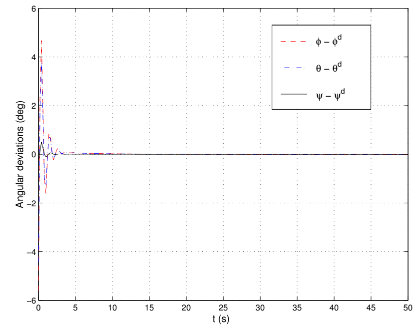

Fig. 3. Angular deviation terms

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