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Probabilistic Sonar Scan Matching for Robust Localization.

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Abstract— **This paper presents a probabilistic framework to perform scan matching localization using standard time-of-flight ultrasonic sensors. Probabilistic models of the sensors as well as techniques to propagate the errors through the models are also presented and discussed. A method to estimate the most probable trajectory followed by the robot according to the scan matching and odometry estimations is also presented. Thanks to that, accurate robot localization can be performed without the need of geometric constraints. The experiments demonstrate the robustness of our method even in the presence of large amounts of noisy readings and odometric errors.**

I. INTRODUCTION

A crucial issue for a mobile robot to execute useful long-term missions is to determine and keep track of its position. Localization usually consists on matching recent sensory information against prior knowledge of the environment. It is a usual practice to introduce geometric constraints [1], such as the existence of lines and corners in the environment, in the localization process. Although this may reduce the computational complexity, it renders the process useful only for structured indoor environments. To confront this problem, some other methods use raw range sensor readings and make no assumption on the structure of the environment [2], [3], [4]. These techniques have been successfully applied to a wide range of subjects as a way to improve odometry. Also some SLAM and map building issues benefit from these ideas [5], [6]. An important approach to this kind of localization is the *scan matching*.

The idea behind most of the scan matching algorithms is as follows. Starting with two sets of range readings and an initial guess for the displacement between them, the algorithm iteratively refines the displacement estimation by generating pairs of corresponding points on the scans and minimizing an error metric. The most popular method to perform scan matching is the *Iterative Closest Point* (ICP). This algorithm was originally used in the computer vision community for fine registration of 2D and 3D point sets [7], [8], and subsequently used to match sets of range readings in robotics [9], [10]. Although this algorithm provides good estimates of the displacement between two scans, it does not when dealing with rotation. Other methods, such as the *Iterative Dual Correspondence* (IDC) or the *Metric-Based Iterative Closest Point* (mbICP) [11] deal with this problem. However, none of these methods take into account sensor imprecisions. The work by Pfister [2] do explicitly take into account the sensor imprecision. In this work, the contribution of each scan point to the overall matching estimate is weighted according to its uncertainty. Another interesting method to perform scan matching dealing with uncertainty is the *probabilistic Iterative Correspondence* (pIC) [12]. In this method statistical compatibility between scan points is computed by means of the Mahalanobis distance.

Laser sensors present some desirable properties in terms of accuracy and density of their readings. For this reason, they are extensively used to perform scan matching. Ultrasonic range sensors are not as accurate as laser nor provide dense sets of readings. Thus, they are not usually used in localization processes and only a few works deal with ultrasonic range sensors [13], [14], [15]. However, these sensors are more appealing than laser in terms of cost, weight and power consumption. Moreover, their basic features are shared with underwater sonar. In underwater robotics, sonar is more suitable than laser as sound suffers a lower absorption than light in this medium.

At the extent of the authors knowledge, there is only one work dealing with scan matching localization using standard time-of-flight ultrasonic range finders [10]. This method deals with the sparsity of the readings by grouping sonar readings along short robot trajectories and by correcting the whole set of odometric estimations performed during the grouping process. However, the low angular resolution and noisy behavior of sonar sensors are not taken into account.

This paper introduces the *sonar probabilistic Iterative Correspondence* (spIC). spIC is a sonar scan matching framework where both the sparseness and noise of the readings are taken into account. This is accomplished by means of probabilistic models of ultrasonic and odometric sensors, as well as a method to propagate the error through them. The matching process is accomplished by means of Mahalanobis distance [12]. Moreover, the error introduced by the matching process is estimated. Also, the correction of the whole robot trajectory involved in the scan matching process is presented as a constrained optimization problem.

This paper is structured as follows. Section II describes the scan building process, including error models and error propagation methods. In section III the probabilistic sonar scan matching is presented. Section IV deals with trajectory correction. Experiments demonstrating the validity of our approach are provided in section V. Finally, some conclusions are available in section VI

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II. SCAN BUILDING

A. Error models

The basic requirement for scan matching algorithms is the existence of two sets of readings where reliable point to point correspondences could be established. Because the number of readings provided by a ring of echo sounders is low, a process where sets of sonar readings are grouped is necessary. For simplicity, these groups of sonar readings will be referred to in this paper as *scans*. The two scans used in scan matching are named *current scan* (S_{cur}) and *reference scan* (S_{ref}), being the *current scan* the most recently gathered scan, and the *reference scan* a previously gathered scan.

The grouping process consists in moving the robot a certain distance and, using odometry, group the sonar returns. The traveled distance has to be long enough to acquire a large scan, but not too long so odometric error remains low. In practice traveled distances between one and two meters are good choices.

By performing the described grouping [10] process, the resulting scan is subject to two sources of error. On the one hand, there is an uncertainty in the position of the sonar readings with respect to the robot pose when they were taken. This uncertainty is mostly due to angular and range uncertainties of sonar sensors. On the other hand, the robot pose estimates used to build the scan are subject to odometric errors. The estimated position of a sonar return depends on the error of the sonar sensor and on the robot pose error. Because of that, both sources of error have to be taken into account simultaneously when building the scan. To accomplish this, the following sonar and odometry models are proposed.

A sonar reading taken by sensor i at time j is assumed to be a Gaussian random variable $x^{s_i}_{r_{i,j}} = N(\hat{x}^{s_i}_{r_{i,j}}, P^{s_i}_{r_{i,j}})$. The mean, $\hat{x}^{s_i}_{r_{i,j}}$, represents the translation and rotation of the reference frame $r_{i,j}$, located at the sonar reading coordinates, with respect to the reference frame s_i, located at the sensor position and aligned with the axis of the sonar beam. Thus, $\hat{x}^{s_i}_{r_{i,j}} = [x, y, \theta]^T$ has the form $[r, 0, 0]^T$, being r the current sonar range reading. $P^{s_i}_{r_{i,j}}$ is a covariance matrix of the form

$$P^{s_i}_{r_{i,j}} = \begin{bmatrix} \sigma^2_{xx} & 0 & 0 \\ 0 & \sigma^2_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (1)$$

where σ_{xx} models the range uncertainty. It has been experimentally set as $r/100$. The σ_{yy} models the sonar angular uncertainty of α degrees as follows

$$\sigma_{yy} = \frac{r}{2}\tan(\frac{\alpha}{2}). \qquad (2)$$

The sonar opening, α, has been set to 30^o, which is a usual configuration in Polaroid ultrasonic range finders. Fig. 1 shows the 2σ bound ellipses corresponding to the described sonar model for sonar measurements ranging from 0m to 4m at intervals of 10cm.

As the robot moves, its pose can be estimated by means of odometry. Our proposal is not to estimate the global robot pose but small displacements of the robot. In other words, we estimate the current robot pose with respect to the robot pose when the last estimation was performed. In this way, the j^{th} odometric estimation is modeled as a Gaussian random variable $x^{j-1}_j = N(\hat{x}^{j-1}_j, P^{j-1}_j)$. The mean, $\hat{x}^{j-1}_j = [x, y, \theta]^T$, represents the translation and rotation of the reference frame j, located at the current robot pose, with respect to the reference frame $j-1$, located at the pose of the robot in the previous estimation. P^{j-1}_j is its associated covariance. The obtention of both $\hat{x}^{j-1}_j$ and P^{j-1}_j depends on physical characteristics of the robot and is out of the scope of this paper.

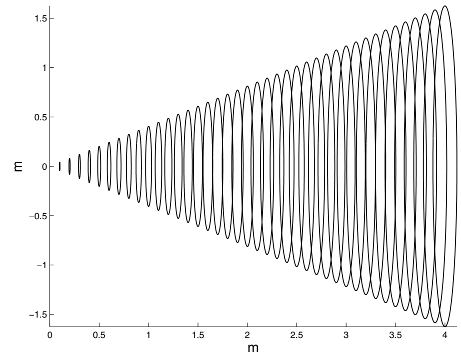


Fig. 1. 2σ bound ellipses corresponding to the sonar model

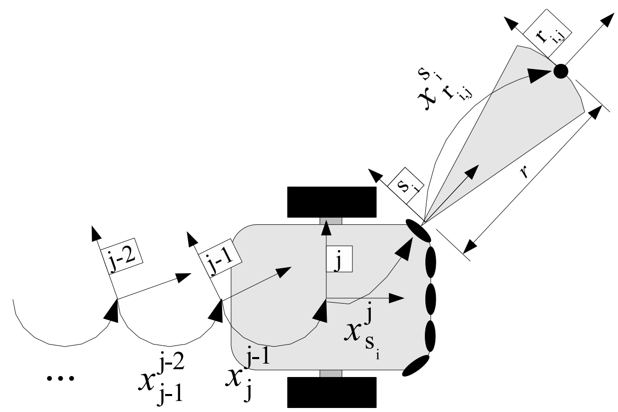


Fig. 2. Relations between the models. The grey circular sector represents the sonar beam.

Finally, the relative position $x^j_{s_i}$ of each ultrasonic sensor with respect to the robot reference frame also needs to be know. It is assumed that this relative position does not change over time and is perfectly known. Thus, it can be expressed as $x^j_{s_i} = N(\hat{x}^j_{s_i}, P^j_{s_i})$, where the covariance $P^j_{s_i}$ is set to zero. The mean, $\hat{x}^j_{s_i}$ either is provided by the manufacturer or can be measured on the robot. Fig. 2 summarizes the relation between the described models.

The set of the last N odometric estimations as well as the readings acquired at each of these positions is stored and will be referred to as the *Transformations History*.

B. Measurement grouping

The information in the Transformations History has to be processed in order to build the current and the reference scans. The position of the sonar readings belonging to the scan has to be expressed with respect to a common reference frame. In order to select the common reference frame, it has to be taken into account that the odometric error increases with distance. Thus, selecting the central position of the path followed by the robot when building the scan minimizes the global error of the scan. The mentioned central position has been chosen as the scan frame and is named A for the reference scan and B for the current scan.

In order to express the sonar readings with respect to a common reference frame, a mechanism to represent one reference frame with respect to another one is needed. A common approach in stochastic mapping and SLAM [13], where working with processes corrupted by Gaussian noises is a usual practice, is the use of operators $\oplus$ and $\ominus$. The former is the *composition* operator and the latter is the *inversion* operator, defined as follows.

Let $x_b^a = N(\hat{x}_b^a, P_b^a)$, where $\hat{x}_b^a = [x_1, y_1, \theta_1]^T$ represents the translation and rotation of the reference frame b with respect to the frame a and P_b^a is the associated covariance matrix. Let also define, in a similar way, $x_c^b = N(\hat{x}_c^b, P_c^b)$, where $\hat{x}_c^b = [x_2, y_2, \theta_2]^T$. The composition $x_c^a = x_b^a \oplus x_c^b$, where $x_c^a = N(\hat{x}_c^a, P_c^a)$, is calculated as follows.

$$\hat{x}_c^a = f(x_b^a, x_c^b) = \begin{bmatrix} x_1 + x_2\cos\theta_1 - y_2\sin\theta_1 \\ y_1 + x_2\sin\theta_1 + y_2\cos\theta_1 \\ \theta_1 + \theta_2 \end{bmatrix}. \quad (3)$$

Linearizing the system around the current estimates and using the first order Taylor approximation, the covariance can be expressed as follows

$$P_c^a = J_{\oplus 1} P_b^a J_{\oplus 1}^T + J_{\oplus 2} P_c^b J_{\oplus 2}^T, \quad (4)$$

where

$$J_{\oplus k} = \left.\frac{\partial f(x_b^a, x_c^b)}{\partial(x_k, y_k, \theta_k)}\right|_{(\hat{x}_b^a, \hat{x}_c^b)} \quad (5)$$

for k=1,2. The inversion is a transformation such that $\ominus x_b^a = x_a^b$, being $x_a^b = N(\hat{x}_a^b, P_a^b)$. It is calculated as follows.

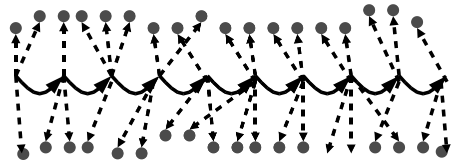
$$\hat{x}_a^b = g(x_b^a) = \begin{bmatrix} -x_1\cos\theta_1 - y_1\sin\theta_1 \\ x_1\sin\theta_1 - y_1 cos\theta_1 \\ -\theta_1 \end{bmatrix}. \quad (6)$$

Linearizing the system, the covariance P_a^b can be expressed as

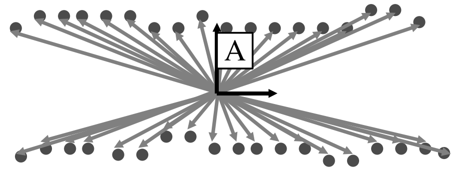
$$P_a^b = J_{\ominus} P_b^a J_{\ominus}^T \quad \text{, where} \quad J_{\ominus} = \left.\frac{\partial g(x_b^a)}{\partial(x_1, y_1, \theta_1)}\right|_{\hat{x}_b^a}. \quad (7)$$

Using these operators, the reference frame of a sonar reading $r_{i,j}$ with respect to a robot pose reference frame k could be obtained as follows

$$x_{r_{i,j}}^k = \begin{cases} x_{k+1}^k \oplus ... \oplus x_j^{j-1} \oplus x_{s_i}^j \oplus x_{r_{i,j}}^{s_i} & k < j \\ (\ominus x_k^{k-1}) \oplus ... \oplus (\ominus x_{j+1}^j) \oplus x_{s_i}^j \oplus x_{r_{i,j}}^{s_i} & k > j \\ x_{s_i}^j \oplus x_{r_{i,j}}^{s_i} & k = j \end{cases} \quad (8)$$



(a)



(b)

Fig. 3. The measurement grouping process. (a) Before applying the process. (b) After applying the process.

Using these operations, each of the readings in the Transformations History belonging to the scan being built (Fig. 3-a) can be represented with respect to the scan reference frame (Fig. 3-b).

After performing the mentioned process, the pose $[x, y, \theta]^T$ of each scan point frame with respect to the scan frame is generated. However, the orientation of the scan point frame does not provide useful information for scan matching. Thus, after building the scan, only the $[x, y]^T$ scan point coordinates and the corresponding 2x2 covariance matrix are held.

III. PROBABILISTIC SCAN MATCHING

The goal of a scan matching process is to estimate the relative displacement $\hat{x}_B^A = [x_x, x_y, x_\theta]^T$ between the reference scan frame A and the current scan frame B. This process is usually performed by means of an iterative process [9]. At each iteration k the algorithm establishes, for each point p_i in S_{new} a correspondence q_j in S_{ref} using the current estimate $\hat{x}_{cur_k}^{ref}$. Next, the new estimation $\hat{x}_{cur_{k+1}}^{ref}$ is computed as the one that minimizes the error of these correspondences. These two steps are repeated until convergence is achieved.

A usual practice is to use Euclidian distance to establish the correspondences, and, when established, minimize the squared sum of the distances between the pairs of corresponding points. This technique has proved to be useful, even with sparse sets of noisy sonar readings [10]. However, the recent work by Montesano et al. [12] proposes a probabilistic framework where sensor uncertainties can be explicitly taken into account. The idea is to pair a point in S_{new} with another one in S_{ref} if they are statistically compatible. When used with laser scans, the probabilistic scan matching lets the robot to deal with the small angular and range uncertainties of laser devices, as well as with uncertainties in the relative scan location.

As stated before, to perform scan matching with sonar poses some additional problems. Scans have to be built by grouping sonar readings, thus, odometric errors are also present inside the scan. Moreover, sonar range and angular uncertainties are much greater than laser ones. In this context, our proposal resides on the use of the statistical models described in section II in a probabilistic scan matching framework. Thanks to that, the robot is able to cope with odometric and sonar uncertainties.

A. Nearest neighbor data association

Let $p_i = N(\hat{p_i}, P_{p_i})$ and $q_j = N(\hat{q_j}, P_{q_j})$ be S_{new} and S_{ref} items respectively. These distributions have been obtained according to the process described in section II. Let $\hat{p_i} = [p_x, p_y]^T$ and $\hat{q_i} = [q_x, q_y]^T$.

To decide whether p_i is compatible or not with q_j, the Mahalanobis distance is used. The squared Mahalanobis distance between p_i and q_j is defined as follows

$$D^2(p_i, q_j) = h_{i,j}^T C_{i,j}^{-1} h_{i,j}, \quad (9)$$

where $h_{i,j} = h(\hat{x}_B^A, p_i, q_j)$ computes the difference between p_i and q_j. To calculate this difference, p_i has to be transformed to the reference frame A.

$$h_{i,j} = \begin{bmatrix} x_x + p_x \cos x_\theta - p_y \sin x_\theta \\ x_y + p_x \sin\ x_\theta + p_y \cos x_\theta \end{bmatrix} - \begin{bmatrix} q_x \\ q_y \end{bmatrix}. \quad (10)$$

By linearizing h around the current estimate $\hat{x}_{B_k}^A$ and the points $\hat{p_i}$ and $\hat{q_j}$, the covariance matrix $C_{i,j}$ can be computed as follows

$$C_{i,j} = J_{3_{i,j}} P_B^A J_{3_{i,j}}^T + J_{4_{i,j}} P_{p_i} J_{4_{i,j}}^T + P_{q_j}, \quad (11)$$

where

$$J_{3_{i,j}} = \left.\frac{\partial h(x,p,q)}{\partial x}\right|_{\hat{x}_{B_k}^A, \hat{p}_i, \hat{q}_j}, J_{4_{i,j}} = \left.\frac{\partial h(x,p,q)}{\partial p}\right|_{\hat{x}_{B_k}^A, \hat{p}_i, \hat{q}_j}. \quad (12)$$

The matrix P_B^A is the covariance associated to $\hat{x}_B^A$. The obtention of this matrix will be described in section III-C.

The Mahalanobis distance is, under Gaussian assumption, a chi-squared distribution with $dim(h_{i,j})$ degrees of freedom. Thus, p_i and q_j are compatible if and only if $D^2(p_i, q_j) < \chi_{2,p}^2$, where p is the desired confidence level. For each p_i, the set of compatible points in S_{ref} is built. Among them, the corresponding point q_j is selected as the one which is closer to p_i in the Mahalanobis sense.

As a result of this process, the set C of correspondences is generated.

B. Minimization

The second step in the scan matching process is to find the relative displacement x_B^A between the two scans that minimizes the error between pairs of corresponding points. The notation $C = \{< a_1, b_1 >, < a_2, b_2 > ... < a_n, b_n >\}$ will be used to denote correspondences between the a_i in S_{ref} and the b_i in S_{cur}. In this work, a Least Squares method has been used to minimize the sum of squared Mahalanobis distances between the points in C.

The criteria to be minimized is the following

$$\min_x \sum_{i=1}^{n} h_i^T C_{i,i}^{-1} h_i, \quad (13)$$

where $h_i = h(x, b_i, a_i)$ as defined in (10) and $C_{i,i}$ is the associated covariance matrix as defined in (11). Linearizing h using the first order Taylor approximation, a point a_i can be expressed by $J_{3_{i,i}} \hat{x}_B^A - h(\hat{x}_B^A, b_i, a_i)$, where $J_{3_{i,i}}$ is defined in (12) and $\hat{x}_B^A$ is the estimation obtained in the previous scan matching iteration. In the first iteration, x_B^A is obtained from odometry.

Thanks to this, (13) can be rewritten as follows

$$\min_x (Jx - A)^T Q^{-1} (Jx - A), \quad (14)$$

where

$$J = \begin{bmatrix} J_{3_{1,1}} \\ J_{3_{2,2}} \\ \dots \\ J_{3_{n,n}} \end{bmatrix}, A = \begin{bmatrix} J_{3_{1,1}} \hat{x}_B^A - h(\hat{x}_B^A, b_1, a_1) \\ J_{3_{2,2}} \hat{x}_B^A - h(\hat{x}_B^A, b_2, a_2) \\ \dots \\ J_{3_{n,n}} \hat{x}_B^A - h(\hat{x}_B^A, b_n, a_n) \end{bmatrix} \quad (15)$$

and Q is a block diagonal matrix containing the $C_{i,i}$. By using the orthogonality principle, the x that minimizes the previous Equation is

$$x_{min} = (J^T Q^{-1} J)^{-1} J^T Q^{-1} A. \quad (16)$$

This x_{min} is used in the next iteration as the $\hat{x}_B^A$ in order to find correspondences and minimize again.

C. Error estimation

More than finding the vector $\hat{x}_B^A$ that better explains the correspondences between scan points, we want also to estimate the error produced by the matching process. For this reason, the scan matching output is represented as a Gaussian distribution of the form $x_B^A = N(\hat{x}_B^A, P_B^A)$. This section describes a method to compute the covariance matrix P_B^A.

Let the function $F(x)$ be defined as follows

$$F(x) = \begin{bmatrix} h(x, b_1, a_1) \\ h(x, b_2, a_2) \\ \dots \\ h(x, b_n, a_n) \end{bmatrix}. \quad (17)$$

The covariance $C_{i,i}$ is known for each $h(\hat{x}_B^A, b_i, a_i)$. Thus, the block diagonal matrix Q containing the $C_{i,i}$ represents the covariance of $F(\hat{x}_B^A)$.

Linearizing $F(x)$ around $\hat{x}_B^A$ and using the first order Taylor approximation, Q can be written as follows

$$Q = J_5 P_B^A J_5^T \quad \text{, where} \quad J_5 = \left.\frac{\partial F(x)}{\partial x}\right|_{\hat{x}_B^A}. \quad (18)$$

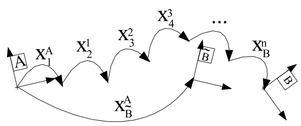
Thus, the scan matching covariance P_B^A can be computed as follows

$$P_B^A = J_5^+ Q (J_5^T)^+, \quad (19)$$

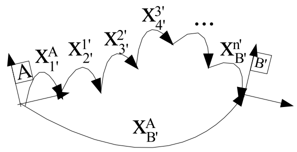
where $^+$ represents the Moore-Penrose pseudoinverse of a matrix.

IV. TRAJECTORY CORRECTION

The probabilistic scan matching described in the previous section estimates $x_{\hat{B}}^A = N(\hat{x}_{\hat{B}}^A, P_{\hat{B}}^A)$ as the relative displacement between A and B. If each scan has been gathered from a single robot pose, odometry can be rejected and the scan matching estimation used instead. This is how scan matching



(a)



(b)

Fig. 4. Trajectory correction. (a) Before correction (b) After correction, odometric estimations are corrected to agree with scan matching.

localization is usually performed with laser. However, the sonar scans are built using a number of readings acquired along a set of robot poses. Thus, it is desirable to correct the whole set of odometric estimations, that is, the robot trajectory, involved in the scan generation.

Let $X_o = \{x_1^A, x_2^1, ..., x_B^n\}$ be the set of odometric estimations between the reference frames and $x_{\tilde{B}}^A$ the estimation provided by the scan matching. In order to find the corrected set of estimations between the two scans, we impose the condition that the composition of the corrected odometric estimations must be equal to the one coming from scan matching. Being $x = \{x_{1'}^A, x_{2'}^{1'}, x_{3'}^{2'}, ..., x_{B'}^{n'}, \ominus x_{B'}^A\}$ the set of corrected estimations, this condition, depicted in Fig. 4, can be expressed as follows

$$h(x) = x_{1'}^A \oplus x_{2'}^{1'} \oplus x_{3'}^{2'} \oplus ... \oplus x_{B'}^{n'} \oplus (\ominus x_{B'}^A) = 0. \quad (20)$$

Because of the angular terms h(x) is nonlinear. To confront those nonlinearities, the problem is formulated as the obtention of the a posteriori maximum likelihood estimation of the relative motions during the scan building, given the scan matching constraint. In other words, our goal is to compute the most probable trajectory followed by the robot that agrees with the estimation provided by the scan matching. This can be expressed as the following constrained optimization problem

$$\begin{aligned} \min_x f(x) &= \min_x \frac{1}{2}(x - x_o)^T P^{-1}(x - x_o) \\ h(x) &= 0, \end{aligned} \quad (21)$$

where $x_o = \{x_1^A, x_2^1, ..., x_B^n, \ominus x_{\tilde{B}}^A\}$ and P is the block diagonal matrix containing the odometric estimations error covariances $P_1^A, P_2^1, ..., P_B^n$ and the scan matching error covariance $P_{\tilde{A}}^{\tilde{B}}$.

The *Sequential Quadratic Programming* (SQP) is a method vastly used to solve this kind of problems. This work presents the *Iterated Extended Kalman Filter* (IEKF) as an alternative method to solve this constrained optimization problem [10] because, in situations like those expressed with (21), using an IEKF with an exact measurement function is faster and leads to the same results that SQP.

The feasibility and formulation of the IEKF to correct the trajectory involved in the scan building process can easily be derived from the loop closing IEKF formulation in Hierarchical SLAM [16].

V. EXPERIMENTAL RESULTS

The method described in this paper has been implemented on a Pioneer 3-DX mobile robot equipped with 16 Polaroid ultrasonic range finders. For comparison, two methods not taking into account sonar and odometry errors have also been implemented. These methods are analogous to ICP and IDC, but grouping sonar readings prior to the scan matching and correcting the trajectory after the scan matching. These methods will be referred to, for simplicity, as sICP (sonar ICP) and sIDC (sonar IDC).

TABLE I

COMPARISON OF sICP, sIDC AND spIC ERRORS WHEN $\sigma_{odo} = 0.02$.

	μ_x	σ_x	μ_y	σ_y	μ_θ	σ_θ
spIC	0.0153	0.0122	0.0154	0.0103	0.0396	0.0304
sIDC	0.0526	0.1003	0.0425	0.0567	0.1645	0.2250
sICP	0.0789	0.0976	0.0784	0.0871	0.3102	0.2667

The first experiment consists on matching two scans gathered along the same robot trajectory. The robot moved in a non structured indoor environment. Two sources of error have been added to the scans. First, odometric noise while building the scan, producing distorted scans. Thus, both scans may present slightly different shapes. Odometric noise is generated by adding Gaussian random noise with zero mean and σ_{odo} variance to each wheel encoder reading. In the experiments presented in this paper, σ_{odo} ranges from 0 to 0.05 at 0.01 intervals. Notice that, since the encoder readings are provided in meters per second and sensor data is obtained every 100ms, a σ_{odo} of 0.05 represents a huge error. The second source of error is added to the initial estimate. Errors added to this initial estimate range from -0.2m to 0.2m in x and y, and from -45^o to 45^o in θ. Notice how large are the errors, specially in rotation. For each σ_{odo}, a set of tests has been performed by adding the described noise to the initial estimate. Convergence of the algorithms was achieved when the error ratio was below 0.001 during three consecutive iterations. The maximum number of iterations is set to 250.

Because the scans have been gathered along the same robot trajectory, the ground truth $[0, 0, 0]^T$ is perfectly known. Fig. 5-a summarizes the results obtained in this experiment. The x axis represents the σ_{odo}. The y axis represents the percentage of right estimates. An estimate is considered to be right if the x and y error is below 5 cm and the angular error is under 10^o. Results using spIC (solid line) are much better than the ones with sICP and sIDC. For σ_{odo} between 0 and 0.02, spIC has a 100% hit rate. Notice that a σ_{odo} of 0 does not mean error free odometry, but no odometric noise added to the original real scans. sIDC performs much better than sICP for low odometry noise because it deals better with angular errors. However, with higher σ_{odo} sIDC results are quite similar to the sICP ones. On the contrary, because all the errors involved in the matching process are taken into account in our probabilistic framework, spIC provides much better results even in the presence of huge odometric errors.

To illustrate this experiment, the following results concentrate on $\sigma_{odo} = 0.02$. Fig. 5-b depicts the final estimates of sIDC, sICP and spIC for the mentioned σ_{odo}. It can be observed how all the spIC estimates concentrate around the

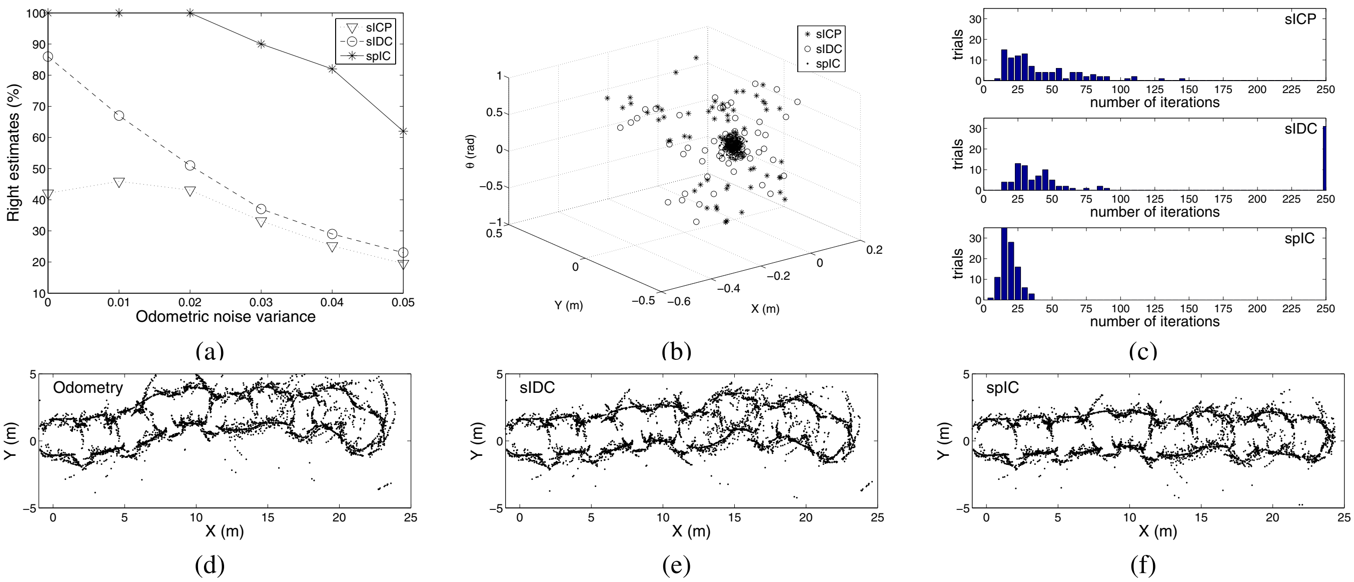


Fig. 5. Comparison of sonar scan matching methods. (a) Percentage of right estimates of sICP, sIDC and spIC with respect to odometry noise. (b) Estimations of the three methods for $\sigma_{odo} = 0.02$. spIC results concentrate around the ground truth. (c) Convergence rate. (d) Visual map using odometry. (e) Visual map using sIDC. (f) Visual map using spIC.

ground truth. On the contrary, sICP and sIDC results, both negatives and positives, may provide estimates far away from the ground truth. This fact can be observed on Table I, where the means and standard deviations of the errors in the estimates are shown. Only positives (both true and false) have been used.

Convergence rate is an important factor. Fig. 5-c shows the number of iterations for each trial using $\sigma_{odo} = 0.02$. It can be observed how spIC converges faster than sICP and sIDC. This is due to the accuracy of the correspondences established thanks to the described models and statistical framework. Similar results are obtained with other σ_{odo}.

The second experiment has been conducted while the robot moved in a corridor in our university. Additional noise has been added to odometric readings, providing the map depicted in Fig. 5-d. It can be observed how the visual result of spIC (Fig. 5-f) is better than the sIDC (Fig. 5-e), specially in the areas where lots of spurious readings, artifacts and specular reflections are present. At the end of the corridor, where an important angular error is present, spIC also performs much better than sIDC. An additional problem with sIDC has been detected in this experiment. Due to the large amount of spurious readings and distortion in the scans, sIDC was not able to achieve the convergence criteria. This is consistent with figure 5-c, where a significant part of sIDC trials reach the limit of 250 iterations. It is also consistent with Fig. 5-a, where sIDC performance significantly falls as odometric noise increases. On the contrary, spIC achieved convergence during all the experiment. Because of this, the convergence criteria for sIDC had to be reduced to 0.05.

Finally, the effects of the trajectory correction can be observed in Fig. 6. This data corresponds to a different run in our university. Trajectory corrections produced by the sonar scan matching produce a map with discontinuities, as

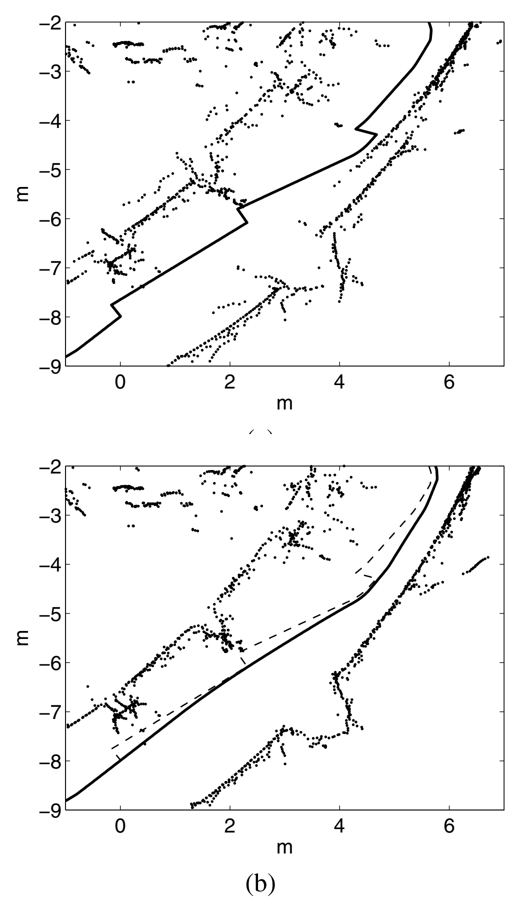


Fig. 6. After scan matching, (a) without trajectory correction and (b) with trajectory correction

depicted in Fig. 6-a. By applying the proposed trajectory correction method, the sonar readings, as well as the robot poses, are continuously located at their most probable position (Fig. 6-b).

VI. CONCLUSION

This paper presents a probabilistic framework to perform scan matching localization using standard time-of-flight ultrasonic sensors. Probabilistic models of the sensors as well as techniques to propagate the errors through the models are also presented and discussed. A method to estimate the most probable trajectory followed by the robot according to the scan matching and odometry estimations is also presented. Thanks to that, accurate robot localization can be performed without the need of geometric constraints.

The experiments demonstrate the robustness of our method even in the presence of large amounts of noisy readings and huge odometric errors. The tolerance to odometric errors is of crucial importance when performing localization with sonar, because the sparsity of the readings makes necessary to group them using dead reckoning.

Our method has been compared to sIDC and sICP, which are variations of the well known IDC and ICP to be used with sonar data. Results show important improvements in the accuracy and robustness of the estimates as well as in the convergence rate.

It has to be remarked that ICP was originally designed for fine registration. Thus, it assumes small errors in the initial estimate. However, when performing scan matching, large errors may appear in the initial estimate, specially in rotation, due to miss-calibrated odometry, slippery surfaces, etc. These errors in the initial estimate may be specially important when performing sonar scan matching, because the scan building process itself relies on odometry. Because of that, the large errors obtained with ICP in the experiments are not due to the algorithm itself, but to its use in the scan matching context. As ICP is an algorithm vastly used to perform scan matching, the previously mentioned fact constitutes a problem to be solved. The provided experiments demonstrate that, when dealing with the sonar scan matching problem, the probabilistic approach of spIC provides a robust and accurate solution.

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