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Quality Evaluation in Flexible Machining Systems: A Flexible Fixture Case Study

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Abstract— **Most of the studies in flexible manufacturing systems address the issues of flexibility, productivity, cost, etc. The impact of flexible lines on product quality is less studied. This paper presents a quantitative model based on Markov chain analysis to evaluate quality performance of a flexible machining system. A case study of flexible fixture is provided to illustrate the applicability of the method.**

I. INTRODUCTION

To satisfying the rapidly changing markets and various customer demands, manufacturing systems are becoming more and more flexible. For example, in automotive industry, flexible manufacturing is "becoming ever more critical" ([1]). Substantial amount of research and practices have been devoted to flexible manufacturing systems (FMS), and it has taken an explicit role in production system design. Much of the work related to flexibility addresses the issues of investment cost, flexibility measurement, and the tradeoffs between productivity and flexibility, etc. However, interactions not only exist between flexibility and productivity, but also between flexibility and quality (as suggested by [2]). The latter one is much less studied.

For example, in many flexible machining systems, a flexible fixture restricts and is the core enabler to flexibility of the whole system, and the cost of designing and fabricating fixtures can amount to 10-20% of the total manufacturing system cost ([3], [4]). A flexible fixture often is a programmable fixture designed to support multiple distinguished parts being manufactured (assembled or machined) on the same line. With the flexible fixture, system flexibility can be achieved with little or no loss of production. In automotive industry, a flexible fixture might be clamps/locators held by robots or other "smart" mobile apparatus. The challenge, however, with the flexible fixture is the accuracy of the locator measured by the variance. Whenever there is a product change, the fixture needs to adapt itself to the desired corresponding location. The quality of the manufacturing operation heavily depends on the fixture. The discrepancy of the fixture location from its "ideal" one, in many cases, dominates the quality of the products. For instance, consider a production line producing two products, A and B. Assuming that the fixture is located in a "good" position, i.e., within the nominal tolerance, for product A, then, if the subsequent parts belong to product A, it is more likely good quality parts can be produced. Analogously, if the fixture is in a "bad" location, then more defective parts can be produced. However, when the subsequent part is switched to product B, then the fixture needs to readjust its location and either good quality or defective parts may be produced (more detailed description is introduced in Section IV). Therefore, the quality characteristic of the current part is dependent on the part type and product sequence. A study to evaluate the quality performance in flexible machining environment is valuable, however, has been missing in current literature.

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Additional examples can be found in painting, welding, assembly operations, etc., as well. For example, he number of available paint colors can significantly impact product quality ([1]). The paint quality may temporarily decline after color switch. Therefore, vehicles with the same colors are usually grouped into batches without sacrificing much on vehicle delivery. These examples suggest that flexibility and quality are tightly coupled and much more work is needed to fully understand this coupling. Such an issue is very important but almost neglected. We believe that quality should be integrated into the considerations in designing production systems as well as objectives of productivity and flexibility. In this paper, we limit our work to quality performance only. The main contribution of this paper is the development of a simple Markovian model to analyze the quality performance of a flexible machining system and the application of it in a flexible fixture case study. In addition, it suggests a possible approach for more complex modeling of quality and productivity in flexible manufacturing systems.

The remaining of the paper is structured as follows: Section II reviews the related literature. Analytical formulas to evaluate quality performance are derived in Section III. Using the method developed, a case study of quality evaluation of a flexible fixture is introduced in Section IV. Finally, Section V concludes the paper. Due to space limitation, all proofs are omitted and can be found in [5].

II. LITERATURE REVIEW

Flexibility has attracted a significant amount of research in the last two decades. Most of the work related to flexibility focus on the definition, meaning and measurement of manufacturing flexibility, and productivity modeling of flexible manufacturing systems, etc. Representative monographs and review papers on flexibility manufacturing systems can be found in [6]-[8] and [9]-[16], respectively.

In spite of the above efforts, only a few publications are found which investigate the impact of manufacturing flexibility on product quality. A measure of productivity, quality and flexibility for production systems is presented in [17]. Paper [18] studies the issues of flexibility, productivity, and quality from an extensive search and analysis of empirical studies. In [19], an aggregation model to measure the manufacturing flexibility using crisp and fuzzy numbers is presented and a method is developed to model the fuzzy flexibility elements such as quality level, efficiency, versatility and availability. In addition, paper [20] surveys the existing literature related to mass customization. In particular, it points out that quality control issues should be taken into account and current literature lacks in-depth study on how to assure quality in mass-customized products.

As pointed out in [3], most of the flexibility studies assume that quality related issues, such as, rejects, rework, etc., have minimal impact and that only products of acceptable quality are produced. The production of parts with high quality in a FMS requires significant efforts and investments. Since the flexibility of a whole FMS is typically restricted by the flexibility of its components, such as fixtures, and the cost of designing and fabricating fixtures can amount to 10-20% of the total manufacturing system cost, paper [3] develops a Fixture Repeatability and Reproducibility measure (FR-R) to evaluate the performance of machining fixtures using the degree of variability of a part dimension off the nominal in flexible manufacturing systems. A review of flexible fixture design and automation can be found in [4].

Therefore, the coupling or interactions between flexible manufacturing system design and product quality is still not fully understood. An in-depth analytical study of the impact of flexibility on quality is necessary and important. This paper is intended to contribute to this end.

III. ANALYTICAL MODELS

A. Two Product Types

Consider a flexible machining system producing two types of products, types 1 and 2. Let g_i and d_i, $i = 1, 2$, denote the states that the system is producing a good quality part type i or a defective part type i, respectively. Note that here we only study the working or production period of the system. In other words, machine breakdowns are not considered. Clearly states g_i and d_i are similar to the up- and down states in throughput analysis.

Introduce $P(g_i,t)$ and $P(d_i,t)$, $i = 1, 2$, as the probabilities to produce a good part type i or defective part type i at cycle t, respectively. In terms of the steady states, $P(g_i)$ and $P(d_i)$, $i = 1, 2$, are used to represent the probabilities to produce a good or a defective part during a cycle in steady states, respectively, i.e.,

$$\lim_{t\to\infty} P(g_i,t) =: P(g_i), \quad \lim_{t\to\infty} P(d_i,t) =: P(d_i), \quad i = 1,2.$$

If $P(g)$ and $P(d)$ are used to denote the good or defective part probabilities (of both products), respectively, we obtain

$$\begin{aligned} P(g_1) + P(g_2) &= P(g), \\ P(d_1) + P(d_2) &= P(d). \end{aligned} \tag{1}$$

In addition, introduce the following assumptions:

(i) A flexible machining system has four states: producing good part type 1, type 2, and producing defective part type 1 and type 2, denoted as g_1, g_2, d_1 and d_2, respectively.

(ii) The transition probabilities from good states g_i, $i = 1, 2$, to defective states d_j, $j = 1, 2$, are determined by λ_{ij}. The machining system has probabilities ν_{ij} to stay in good states g_j, $j = 1, 2$. Similarly, when machining system is in defective state d_i, $i = 1, 2$, it has probabilities μ_{ij} to transit to good states g_j, $j = 1, 2$, and probabilities η_{ij} to stay in defective states d_j, $j = 1, 2$.
Remark 1: λ_{ii} and μ_{ii}, $i = 1, 2$, can be viewed as *non-switching quality failure and repair probabilities*, respectively (i.e., product types are not switched). Analogously, λ_{ij} and μ_{ij}, $i, j = 1, 2$, $i \neq j$, can be viewed as *switching quality failure and repair probabilities*, respectively.

(iii) When incoming parts are in random order without correlations (non-sequenced), the part flow is identically and uniformly distributed with probabilities $P(1)$ and $P(2)$ for part types 1 and 2, respectively. In other words, every cycle the system has probability $P(1)$ or $P(2)$ to work on part types 1 and 2, respectively.
Remark 2: Assumptions (ii) and (iii) imply that probabilities $P(1)$ and $P(2)$ are embedded in the transition probabilities λ_{ij}, μ_{ij}, ν_{ij} and η_{ij}, $i, j = 1, 2$. For example, λ_{ij} defines the transition probability that the incoming part is type j and the machining system produces a defective part at $t + 1$ given that the machining system produces a good type i part at t.

Based on above assumptions, we can describe the system using a discrete Markov chain illustrated in Figure 1. In addition, since total probabilities equal to 1, we have

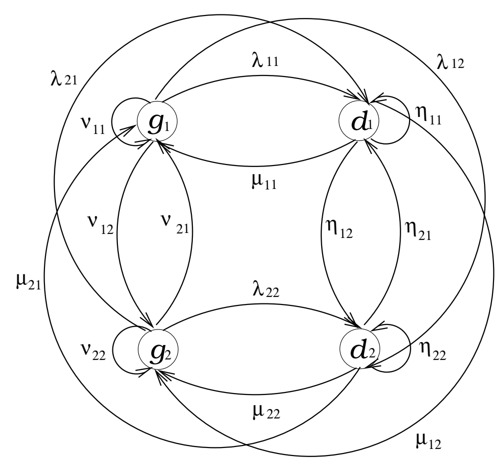

Fig. 1. State transition diagram of a flexible machining system with two product types

$$\begin{aligned} P(1) + P(2) &= 1, \\ P(g_1) + P(d_1) &= P(1), \\ P(g_2) + P(d_2) &= P(2), \end{aligned}$$

$$\lambda_{11}+\lambda_{12}+\nu_{11}+\nu_{12}=1, \quad (2)$$
$$\lambda_{22}+\lambda_{21}+\nu_{22}+\nu_{21}=1,$$
$$\mu_{11}+\mu_{12}+\eta_{11}+\eta_{12}=1,$$
$$\mu_{22}+\mu_{21}+\eta_{22}+\eta_{21}=1.$$

The transitions to state g_1 can be described as

$$\begin{aligned}P(g_1,t+1) &= P(\text{produce a good part type 1 at } t+1| \\ &\quad \text{produce a good part type 1 at } t)P(g_1,t) \\ &+P(\text{produce a good part type 1 at } t+1| \\ &\quad \text{produce a defective part type 1 at } t)P(d_1,t) \\ &+P(\text{produce a good part type 1 at } t+1| \\ &\quad \text{produce a good part type 2 at } t)P(g_2,t) \\ &+P(\text{produce a good part type 1 at } t+1| \\ &\quad \text{produce a defective part type 2 at } t)P(d_2,t) \\ &= P(g_1,t+1|g_1,t)P(g_1,t)+P(g_1,t+1|d_1,t)P(d_1,t) \\ &+P(g_1,t+1|g_2,t)P(g_2,t)+P(g_1,t+1|d_2,t)P(d_2,t) \\ &= \nu_{11}P(g_1,t)+\nu_{21}P(g_2,t)+\mu_{11}P(d_1,t)+\mu_{21}P(d_2,t).\end{aligned}$$

Considering the steady state probability $P(g_1)$, we have

$$P(g_1)=\nu_{11}P(g_1)+\nu_{21}P(g_2)+\mu_{11}P(d_1)+\mu_{21}P(d_2). \quad (3)$$

Similarly,

$$P(g_2)=\nu_{12}P(g_1)+\nu_{22}P(g_2)+\mu_{12}P(d_1)+\mu_{22}P(d_2), \quad (4)$$
$$P(d_1)=\lambda_{11}P(g_1)+\lambda_{21}P(g_2)+\eta_{11}P(d_1)+\eta_{21}P(d_2), \quad (5)$$
$$P(d_2)=\lambda_{12}P(g_1)+\lambda_{22}P(g_2)+\eta_{12}P(d_1)+\eta_{22}P(d_2). \quad (6)$$

Solving the above equations, we obtain a closed formula to calculate the probability of good quality part, $P(g)$.

Theorem 1: Under assumptions (i)-(iii), the good part probability $P(g)$ can be calculated as

$$P(g)=\frac{\mathcal{F}}{\mathcal{F}+\mathcal{G}}, \quad (7)$$

where

$$\begin{aligned}\mathcal{F} &= (\lambda_{11}-\lambda_{21})(\mu_{12}\mu_{21}-\mu_{11}\mu_{22}) \\ &\quad +(1-\nu_{22}+\nu_{12})[(1-\eta_{11})\mu_{21}+\eta_{21}\mu_{11}] \\ &\quad +(1-\nu_{11}+\nu_{21})[(1-\eta_{11})\mu_{22}+\eta_{21}\mu_{12}], \quad (8) \\ \mathcal{G} &= [(1-\nu_{11})(1-\nu_{22})-\nu_{12}\nu_{21}](1-\eta_{11}+\eta_{21}) \\ &\quad -(\mu_{12}-\mu_{22})[(1-\nu_{11})\lambda_{21}+\lambda_{11}\nu_{21}] \\ &\quad +(\mu_{21}-\mu_{11})[(1-\nu_{22})\lambda_{11}+\nu_{12}\lambda_{21}]. \quad (9)\end{aligned}$$

In case of "equal part types", i.e., two part types are equally composed (i.e., 50% each) and have identical transition probabilities, i.e.,

$$\begin{aligned}&\mu_{11}=\mu_{22}, \quad \nu_{11}=\nu_{22}, \quad \lambda_{11}=\lambda_{22}, \quad \eta_{11}=\eta_{22}, \\ &\mu_{12}=\mu_{21}, \quad \nu_{12}=\nu_{21}, \quad \lambda_{12}=\lambda_{21}, \quad \eta_{12}=\eta_{21}, \quad (10)\end{aligned}$$

which implies that the transitions from one product type to another are equivalent in terms of quality, we obtain

Corollary 1: Under assumptions (i)-(iii), the good part probability $P(g)$ for equal part type case is described by

$$P(g)=\frac{\mu_{12}+\mu_{11}}{\lambda_{11}+\lambda_{12}+\mu_{12}+\mu_{11}}. \quad (11)$$

In addition, $P(g)$ is monotonically increasing and decreasing with respect to μ_{1i} and λ_{1i}, $i=1,2$, respectively.

Remark 3: When only single product is processed, expression (11) is simplified to

$$P(g)=\frac{\mu_{11}}{\lambda_{11}+\mu_{11}},$$

which is an analogy to the machine "efficiency" in throughput analysis.

B. Multiple Product Types

Now consider a flexible machining system producing multiple products. Same assumptions and notations in Subsection III-A will be used with the exception that now $i=1,\ldots,n$, denoting n product types. Analogously to Subsection III-A, we obtain the following transition equations:

$$\begin{aligned}P(g_1) &= \sum_{i=1}^{n}\nu_{i1}P(g_i)+\sum_{i=1}^{n}\mu_{i1}P(d_i), \\ &\cdots \quad \cdots \quad \cdots \\ P(g_n) &= \sum_{i=1}^{n}\nu_{in}P(g_i)+\sum_{i=1}^{n}\mu_{in}P(d_i), \\ P(d_1) &= \sum_{i=1}^{n}\lambda_{i1}P(g_i)+\sum_{i=1}^{n}\eta_{i1}P(d_i), \\ &\cdots \quad \cdots \quad \cdots \\ P(d_{n-1}) &= \sum_{i=1}^{n}\lambda_{i,n-1}P(g_i)+\sum_{i=1}^{n}\eta_{i,n-1}P(d_i), \\ 1 &= \sum_{i=1}^{n}P(g_i)+\sum_{i=1}^{n}P(d_i).\end{aligned}$$

Rewrite into a matrix form we have

$$AX=B, \quad (12)$$

where A is defined in (13) on next page and

$$X = \Big(P(g_1),\ldots,P(g_n),P(d_1),\ldots,P(d_n)\Big)^T, \quad (14)$$
$$B = \Big(0,0,\ldots,1\Big)^T. \quad (15)$$

Therefore, we obtain

Theorem 2: Under assumptions (i)-(iii), the good part probability $P(g)$ can be calculated from

$$P(g)=\sum_{i=1}^{n}P(g_i)=\sum_{i=1}^{n}x_i, \quad (16)$$

where $x_i=P(g_i)$, $i=1,\ldots,n$, are the elements in X and can be solved from

$$X=A^{-1}B, \quad (17)$$

and A, B are defined in (13) and (15), respectively.

Note that the inverse of matrix A exists due to the fact that an irreducible Markov chain with finite number of states has a unique stationary distribution.

$$A = \begin{pmatrix} \nu_{11}-1 & \nu_{21} & \dots & \nu_{n1} & \mu_{11} & \mu_{21} & \dots & \mu_{n-1,1} & \mu_{n1} \\ \nu_{12} & \nu_{22}-1 & \dots & \nu_{n2} & \mu_{12} & \mu_{22} & \dots & \mu_{n-1,2} & \mu_{n2} \\ & \dots & & \dots & & \dots & & \dots & \\ \nu_{1n} & \nu_{2n} & \dots & \nu_{nn}-1 & \mu_{1n} & \mu_{2n} & \dots & \mu_{n-1,n} & \mu_{nn} \\ \lambda_{11} & \lambda_{21} & \dots & \lambda_{n1} & \eta_{11}-1 & \eta_{21} & \dots & \eta_{n-1,1} & \eta_{n1} \\ \lambda_{12} & \lambda_{22} & \dots & \lambda_{n2} & \eta_{12} & \eta_{22}-1 & \dots & \eta_{n-1,2} & \eta_{n2} \\ & \dots & & \dots & & \dots & & \dots & \\ \lambda_{1,n-1} & \lambda_{2,n-1} & \dots & \lambda_{n,n-1} & \eta_{1,n-1} & \eta_{2,n-1} & \dots & \eta_{n-1,n-1}-1 & \eta_{n,n-1} \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & & 1 \end{pmatrix}, \quad (13)$$

In the case of equal product types, we have

$$\mu_{11}=\mu_{ii}, \quad \nu_{11}=\nu_{ii}, \quad \lambda_{11}=\lambda_{ii}, \quad \eta_{11}=\eta_{ii},$$
$$\mu_{12}=\mu_{ij}, \quad \nu_{12}=\nu_{ij}, \quad \lambda_{12}=\lambda_{ij}, \quad \eta_{12}=\eta_{ij},$$
$$i,j=1,\dots,n, \qquad i\neq j.$$

Corollary 2: Under assumptions (i)-(iii), the good part probability $P(g)$ for n equal product types is described by

$$P(g)=\frac{\mu_{11}+(n-1)\mu_{12}}{\lambda_{11}+\mu_{11}+(n-1)(\lambda_{12}+\mu_{12})}. \quad (18)$$

In addition, $P(g)$ is monotonically increasing and decreasing with respect μ_{1i} and λ_{1i}, $i=1,2$, respectively.

In order to avoid messy notations, the following discussions are limited to equal product types only.

C. Discussions

Similar to throughput analysis, let $e_{1i}=\frac{\mu_{1i}}{\lambda_{1i}+\mu_{1i}}$, $i=1,2$, denote the "switching ($i=2$) or non-switching ($i=1$) quality efficiencies", respectively. In other words, e_{1i} represents the efficiency to produce a good quality part if product type is kept constant ($i=1$) or changed ($i=2$).

1) Less vs. more product types: Now we consider how the number of product types may affect quality. We can show that the good part probability $P(g)$ is monotonically decreasing or increasing with respect to number of product types n if $e_{11} > e_{12}$ or $e_{11} < e_{12}$, respectively (see [5] for details). This result suggests that when the switching quality efficiency is not as good as non-switching efficiency, introducing more products may be harmful for overall quality performance of the system. Therefore, to ensure maintaining desired quality performance, every effort has to be made to achieve $e_{12} \geq e_{11}$. In addition, it implies that when $e_{11} > e_{12}$, single product case has better quality than multiple products case, which suggests that batch production may be more preferable.

2) Random vs. sequence part flow: To further investigate this phenomenon, consider the following two systems, A and B, both producing n equal part types. System A adopts a sequencing policy with part types 1 to n being mixed randomly with uniform distribution (as described in assumption (iii)), while system B keeps strict alternative sequence $1,2,\dots,n,1,2,\dots,n,1,2,\dots$, i.e., product type changes at the end of every cycle. Let $P(g)^A$ and $P(g)^B$ define the good job probability of systems A and B, respectively. For system B, product type is changed at every cycle, therefore, $P(g)^B=e_{12}$.

Comparing $P(g)^A$ and $P(g)^B$, it can be shown that $P(g)^A > P(g)^B$ if $e_{11} > e_{12}$ ([5]). It implies that when quality efficiency is decreased for changing products, using randomly mixed sequence has better quality performance than using strictly alternating sequence policy, since the former one has less transitions among products. It again indicates that using batch processing may lead to a better quality performance than the sequencing policy. A thorough investigation of batch production is important and is a topic in future work.

Using the models described above, we apply them to a flexible fixture case study in a flexible machining system.

IV. A FLEXIBLE FIXTURE CASE STUDY

A. Problem Description

Consider a machining operation that drills a hole on part type A and part type B. The machine has a flexible fixture. When a job comes in, the fixture can adapt itself to pre-designed locations (referred to as L_a and L_b for part types A and B, respectively) in order to hold the part, then the drilling will take place. Now assuming incoming parts are in a random order mixed with types A and B (assumption (iii)), then the fixture may move to location L_a when part type A is coming, moves to L_b when B is coming, and returns to L_a after some time to processes A again. Since the fixture is not perfect, the L_as (correspondingly, L_bs) may not be the same as the designed L_a (correspondingly, L_b). One way of evaluating it is to measure the distance between the real L_a (correspondingly, L_b) and the ideal location. Figure 2 shows discrepancy of a locator from its nominal position, assuming the locator can be anywhere between the "ideal" location 0 and distance Δ_a or Δ_b with uniform distribution for part A and B, respectively. It is clear that when the locator (e.g., L_a) is too far from the designed (ideal) location, the hole will be drilled on a wrong place, which will cause a quality defect. On the other hand, when the locator is within the designed tolerance (shown in Figure 2 as ϵ), it will not hurt the hole drilling.

For simplicity and illustration purpose, now we assume that the flexible fixture is the only factor that causes quality defects. (It is common that the locating error is much larger than the tooling error.) Then the probability of a part with good quality is ϵ/Δ_a for part type A (correspondingly,

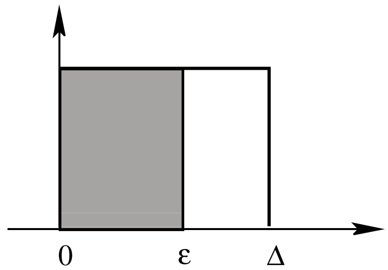

Fig. 2. Locator discrepancy and tolerance range

ϵ/Δ_b for part type B), denoted as δ_a (correspondingly, δ_b), indicating the probability that the locator moves to a satisfactory location.

The problem to be addressed in this case study is: Given the flexible fixture described above, develop a model to evaluate the quality performance as a function of system parameters.

B. Analytical Expression

Assuming δ_a and δ_b are independent of the locator's starting location, then the transition matrix of the states of this problem (making part A and part B) becomes

$$P_{transition} = \begin{pmatrix} \nu_{11} & \lambda_{11} & \nu_{12} & \lambda_{12} \\ \mu_{11} & \eta_{11} & \mu_{12} & \eta_{12} \\ \nu_{21} & \lambda_{21} & \nu_{22} & \lambda_{22} \\ \mu_{21} & \eta_{21} & \mu_{22} & \eta_{22} \end{pmatrix}$$
$$= \begin{pmatrix} P(\overline{\mathrm{m}},\mathrm{g})P_a & P(\overline{\mathrm{m}},\mathrm{d})P_a & P(\mathrm{m},\mathrm{g})P_b & P(\mathrm{m},\mathrm{d})P_b \\ P(\overline{\mathrm{m}},\mathrm{g})P_a & P(\overline{\mathrm{m}},\mathrm{d})P_a & P(\mathrm{m},\mathrm{g})P_b & P(\mathrm{m},\mathrm{d})P_b \\ P(\mathrm{m},\mathrm{g})P_a & P(\mathrm{m},\mathrm{d})P_a & P(\overline{\mathrm{m}},\mathrm{g})P_b & P(\overline{\mathrm{m}},\mathrm{d})P_b \\ P(\mathrm{m},\mathrm{g})P_a & P(\mathrm{m},\mathrm{d})P_a & P(\overline{\mathrm{m}},\mathrm{g})P_b & P(\overline{\mathrm{m}},\mathrm{d})P_b \end{pmatrix},$$

where P_a and P_b are the probabilities that the next job is part A or B, respectively, and $P_a + P_b = 1$. $P(\mathrm{m}, \mathrm{g})$ and $P(\mathrm{m}, \mathrm{d})$ are the probabilities that the locator has moved and is in a "good" or "bad" location (producing good or defective products), respectively. Similarly, $P(\overline{\mathrm{m}}, \mathrm{g})$ and $P(\overline{\mathrm{m}}, \mathrm{d})$ are the probabilities that the locator has not moved and is in a "good" or "bad" location, respectively.

This matrix can be simplified. For example, when the locator is in "good" location producing part A, then it does not move if the next job is still part A, and the transition probability of making a good part A (correspondingly, defective part A) will be only determined by P_a (correspondingly, 0). (Note that here we assume location error is the only source for defects.) This is because when the locator is in a good position and the next job belongs to the same type, the probability of making another good job is 1. Similarly, if it is in the "good" location producing part A, but the next job is part B, the locator will move. The probability of moving to a "good" position (making a good part B) is δ_b. Therefore the transition probability from good A location to good B location is $\delta_b P_b$. Repeat this process and finally we can obtain a simplified transition matrix:

$$P_{transition} = \begin{pmatrix} \nu_{11} & \lambda_{11} & \nu_{12} & \lambda_{12} \\ \mu_{11} & \eta_{11} & \mu_{12} & \eta_{12} \\ \nu_{21} & \lambda_{21} & \nu_{22} & \lambda_{22} \\ \mu_{21} & \eta_{21} & \mu_{22} & \eta_{22} \end{pmatrix}$$
$$= \begin{pmatrix} P_a & 0 & \delta_b P_b & (1-\delta_b)P_b \\ 0 & P_a & \delta_b P_b & (1-\delta_b)P_b \\ \delta_a P_a & (1-\delta_a)P_a & P_b & 0 \\ \delta_a P_a & (1-\delta_a)P_a & 0 & P_b \end{pmatrix}. \quad (19)$$

With above relationship, we obtain values for variables λ_{ij}, μ_{ij}, ν_{ij} and η_{ij}, $i, j = 1, 2$. Then, using Theorem 1, the good part probability is obtained:

$$P(g) = \frac{\mathcal{F}}{\mathcal{F} + \mathcal{G}}, \quad (20)$$

where

$$\begin{aligned} \mathcal{F} &= (1 - P_a + \delta_a P_a)(1-\delta_a)P_a\delta_b P_b \\ &\quad + (1 - P_b + \delta_b P_b)(1-P_a)\delta_a P_a - P_a(1-\delta_a)\delta_b P_b \delta_a P_a, \\ \mathcal{G} &= \delta_a P_a \delta_b P_b (1-\delta_a)P_a - \delta_b P_b (1-P_a)(1-\delta_a)P_a \\ &\quad + [(1-P_a)(1-P_b) - \delta_b P_b \delta_a P_a][1 - P_a + (1-\delta_a)P_a]. \end{aligned}$$

After some algebric manipulation, we obtain

$$P(g) = \frac{P_a(1-P_a)[\delta_a(1-P_b) + \delta_b P_b]}{P_a P_b} = \delta_a P_a + \delta_b P_b. \quad (21)$$

Compared with the single product case (where only product A is produced), whose quality is defined by δ_a, we have

$$P(g) - \delta_a = P_a\delta_a + (1-P_a)\delta_b - \delta_a = (1-P_a)(\delta_b - \delta_a).$$

Clearly, only when a new product (B) has smaller locator discrepancy, introducing additional product can improve the overall quality performance, which agrees with intuition.

In addition, it is reasonable to assume Δ_a and Δ_b would be the same in many cases. Therefore $\delta_a = \delta_b = \delta$, and we obtain $P(g) = \delta$, i.e., the probability of making a good part depends only on the flexible locators, which is consistent with our intuition.

C. Extensions to Three- and Multiple-Product Cases

Applying the same concept to three-product case, we assume three products A, B and C are manufactured with the flexible locator. For simplicity here we only consider the case of $\delta_a = \delta_b = \delta_c = \delta$. We compose the matrix A in (13) and simplify it as shown in (22) on next page.

After some simplification and rearrangement (see [5]), we can finally reach

$$P(g_a) = \delta P_a, \quad P(g_b) = \delta P_b, \quad P(g_c) = \delta P_c,$$

where g_a, g_b and g_c denote that the system is in good states producing parts A, B and C, respectively. Therefore, the probability of making a good part is

$$P(g) = P(g_a) + P(g_b) + P(g_c) = \delta(P_a + P_b + P_c) = \delta.$$

This result again is consistent with the one of two-product case and matches our expectation. It also verifies the analysis presented in Section III.

$$A = \begin{pmatrix} \nu_{11}-1 & \nu_{21} & \nu_{31} & \mu_{11} & \mu_{21} & \mu_{31} \\ \nu_{12} & \nu_{22}-1 & \nu_{32} & \mu_{12} & \mu_{22} & \mu_{32} \\ \nu_{13} & \nu_{23} & \nu_{33}-1 & \mu_{13} & \mu_{23} & \mu_{33} \\ \lambda_{11} & \lambda_{21} & \lambda_{31} & \eta_{11}-1 & \eta_{21} & \eta_{31} \\ \lambda_{12} & \lambda_{22} & \lambda_{32} & \eta_{12} & \eta_{22}-1 & \eta_{32} \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} P_a-1 & \delta P_a & \delta P_a & 0 & \delta P_a & \delta P_a \\ \delta P_b & P_b-1 & \delta P_b & \delta P_b & 0 & \delta P_b \\ \delta P_c & \delta P_c & P_c-1 & \delta P_c & \delta P_c & 0 \\ 0 & (1-\delta)P_a & (1-\delta)P_a & P_a-1 & (1-\delta)P_a & (1-\delta)P_a \\ (1-\delta)P_b & 0 & (1-\delta)P_b & (1-\delta)P_b & P_b-1 & (1-\delta)P_b \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}. \quad (22)$$

For the general multiple products case, assume there are n products, and all $\delta_i = \delta$, $i = 1, \ldots, n$. By induction, we can show that $P(g) = \delta$ holds again. The idea of the proof is as follows: We first show the base case ($n = 2$) is true (equation (21)). Next we assume the case $n = k - 1$ is true. Then for case $n = k$, we can group the first $k - 1$ products into an aggregated product since they result in good part probability equals to δ. Now we only have two products, the aggregated product and product k. Using the results for $n = 2$ we prove that the case $n = k$ is also true, which will lead to the good part probability equals to δ for n products as well.

It is not surprising that the probability of making a good part is not dependent on the number of products nor the penetration of each product, since we assume the quality is only determined by the locators with the same δ. This implies that once we can control the flexible fixture (locator), introducing more products will not hurt product quality. However, when δs are not identical for different products, then the system quality performance will be dependent on the number of products, their respective δs, and different ratio of product mix.

V. CONCLUSIONS

The quality performance a flexible manufacturing system is less studied. In this paper, we develop a quantitative model to evaluate the quality performance of a flexible machining system based on discrete Markov chain. We derive closed formulas to calculate good part probability and compare the results under different sequencing policies. The case study of a flexible fixture presented here illustrates the applicability of the method and verifies the results obtained in the paper.

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